Solutions to Tutorial 1
(Complexity and Order)
Date: Sep 5 – September – 2020

1. $f3 < f7 < \{f2, f4\} < f6 < f5 < f1$
2. (a) False. Use $f(n) = n$ and $g(n) = n^2$.
   (b) True, Hint: if $X(n) \leqslant c * Y(n)$ then $X(n) = O(Y(n))$. Show that this holds true for the given set of functions.
   (c) True. Hint: $f(n) \geqslant cg(n)$ for all $n >= n'$ implies $g(n) \leqslant \frac{1}{c} f(n)$ for all $n >= n'$. $(c > 0)$
3. $\Theta(n^2)$
4. Firstly, $n^{1.2^{0.4166667}} \approx \sqrt{(n)}$ Consider $n = 2^{2^k}$. Then, the recurrence relation is $T(2^{2^k}) = T(\lfloor\sqrt{2^{2^k}}\rfloor) + 1$
5. $T(n) = \Theta(n^2)$
6. Hint:
$$3^n = \begin{cases} 3^{\frac{n}{2}} \times 3^{\frac{n}{2}} & \text{n is even} \\ 3^{\frac{n}{2}} \times 3^{\frac{n}{2}} \times 3 & \text{n is odd} \end{cases}$$
7. $O(nmr)$ (For naive matrix multiplication algorithm with 3 nested for loops running over 1:n, 1:m and 1:r)
8. $g(n) = 1 \forall n \in \mathbf{Z}^+$ and
$$f(n) = \begin{cases} n^2 & \text{n is even}, n \in \mathbf{Z}^+ \\ 0 & \text{n is odd}, n \in \mathbf{Z}^+ \end{cases}$$
9. $T(n) = 2T(n/2) + cn^2$ for $n \geqslant 2$, c a positive constant.
   $\implies T(n) = \Theta(n^2)$