# Trees

# Trees and Spanning Trees

- A graph having no cycles is *acyclic.*
- A *forest* is an acyclic graph.
- A *leaf* is a vertex of degree 1.
- A *spanning sub-graph* of G is a sub-graph with vertex set V(G).
- A *spanning tree* is a spanning sub-graph that is a tree.

# Distances

- If G has a *u,v*-path, then the distance from *u* to *v*, written $d_G(u,v)$ or simply $d(u,v)$, is the least length of a *u,v*-path.
  - If G has no such path, then $d(u,v) = \propto$

# Tree: Characterization

- An n-vertex graph G (with $n \geq 1$) is a tree iff:
  - G is connected and has no cycles
  - G is connected and has $n-1$ edges
  - G has $n-1$ edges and no cycles
  - For $u,v \in V(G)$, G has exactly one $u,v$-path

# Some results …

- Every tree with at least two vertices has at least two leaves.
  - Deleting a leaf from a tree with *n* vertices produces a tree with *n-1* vertices.

- If T is a tree with *k* edges and G is a simple graph with $\delta(G) \geq k$, then T is a sub-graph of G.

# Some results …

- If T and T′ are two spanning trees of a connected graph G and $e \in E(T) - E(T')$, then there is an edge $e' \in E(T') - E(T)$ such that $T - e + e'$ is a spanning tree of G.

# Diameter and Radius

- The *eccentricity* of a vertex *u*, written $\varepsilon(u)$, is the maximum of its distances to other vertices.
- In a graph G, the *diameter*, diamG, and the *radius*, radG, are the maximum and minimum of the vertex eccentricities respectively.
- The *center* of G is the subgraph induced by the vertices of minimum eccentricity.

# Counting Trees

- There are $n^{n-2}$ trees with vertex set [n].

# Prüfer Code / Sequence

Algorithm: *Production of* $f(T) = \{a_1, \ldots, a_{n-2}\}$

*Input:* A tree T with vertex set $S \subseteq \aleph$.

*Iteration:* At the $i^{th}$ step, delete the least remaining leaf, and let $a_i$ be the *neighbor* of this leaf.