

AI COURSE TUTORIALS: October 21, 2022

1. a. $P(\neg E, \neg S, \neg M, \neg B) = P(\neg E) P(\neg M) P(\neg S \mid \neg E, \neg M) P(\neg B \mid \neg M) = (1 - 0.4) * (1 - 0.1) * (1 - 0.1) * (1 - 0.1) = 0.4374$

b. $P(B) = P(B \mid M) P(M) + P(B \mid \neg M) P(\neg M) = 1.0 * 0.1 + 0.1 * (1 - 0.1) = 0.19$

c. $P(M \mid B) = P(B \mid M) P(M) / P(B) = (1.0)(0.1) / (0.19) \approx 0.5263$

d. $P(M \mid S, B, E) = P(M, S, B, E) / P(S, B, E) = P(M, S, B, E) / [P(M, S, B, E) + P(\neg M, S, B, E)]$

$$P(M, S, B, E) = P(E) P(M) P(S \mid E, M) P(B \mid M) = 0.4 * 0.1 * 1.0 * 1.0 = 0.04$$

$$P(\neg M, S, B, E) = P(E) P(\neg M) P(S \mid E, \neg M) P(B \mid \neg M) = 0.4 * 0.9 * 0.8 * 0.1 = 0.0288$$

$$\text{Therefore, } P(M \mid S, B, E) = 0.04 / [0.04 + 0.0288] = 0.5814$$

e. $P(E \mid M) = P(E) = 0.4$

In the absence of evidence on S, the node S d-separates E and M. Therefore, E and M are conditionally independent.

2. a.

F	C	
T	T	$0.1 * 0.8 = 0.08$
T	F	$0.1 * 0.2 = 0.02$
F	T	$0.9 * 0.3 = 0.27$
F	F	$0.9 * 0.7 = 0.63$

b. No; because $P(C) = 0.35$ but $P(C \mid F) = 0.8$

c. $P(C) = 0.08 + 0.27 = 0.35$

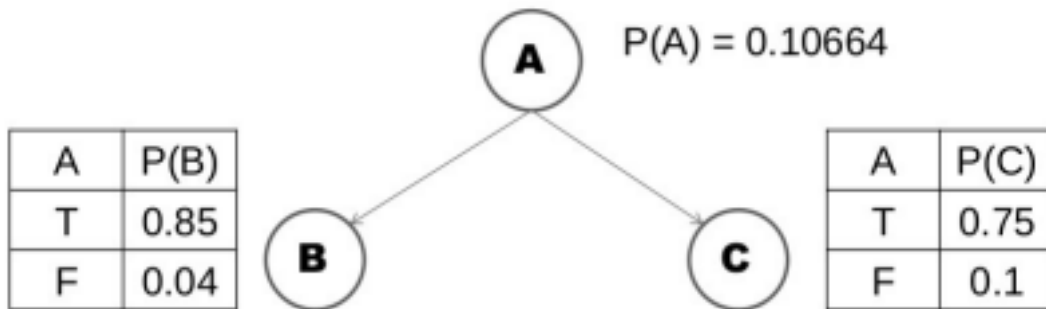
$$P(F \mid C) = P(F, C) / P(C) = 0.08 / 0.35 \approx 0.23$$

$$P(F \mid \neg C) = P(F, \neg C) / P(\neg C) = 0.02 / 0.65 \approx 0.03$$

$$3. P(A | \neg B, C) = P(\neg B, C | A) P(A) / P(\neg B, C)$$

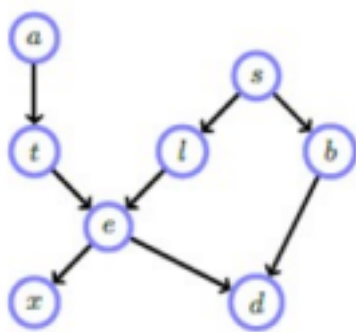
$$= P(\neg B, C | A) P(A) / (P(\neg B, C | A) P(A) + P(\neg B, C | \neg A) P(\neg A))$$

Now, $P(\neg B, C | A) = P(\neg B | A) P(C | A) = (0.15)(0.75) = 0.1125$ $P(\neg B, C | \neg A) = P(\neg B | \neg A) P(C | \neg A) = (0.96)(0.1) = 0.096$



Therefore, $P(A | \neg B, C) = (0.1125 * 0.10664) / ((0.1125 * 0.10664) + (0.096 * 0.89336)) = 0.1227$

4.



x = Positive X-ray
 d = Dyspnea (Shortness of breath)
 e = Either Tuberculosis or Lung Cancer
 t = Tuberculosis
 l = Lung Cancer
 b = Bronchitis
 a = Visited Asia
 s = Smoker

The table values are:

$$P(a) = 0.01 \quad P(s) = 0.5$$

$$P(t|a) = 0.05 \quad P(t|\neg a) = 0.01$$

$$P(l|s) = 0.1 \quad P(l|\neg s) = 0.01$$

$$P(b|s) = 0.6 \quad P(b|\neg s) = 0.3$$

$$P(x|e) = 0.98 \quad P(x|\neg e) = 0.05$$

$$P(d|e, b) = 0.1 \quad P(d|e, \neg b) = 0.7$$

$$P(d|\neg e, b) = 0.8 \quad P(d|\neg e, \neg b) = 0.1$$

$P(e|t,l) = 0$ only if both t and l are false, and 1 otherwise.

$$P(\neg s) = 0.5 \quad P(\neg a) = 0.99$$

$$P(d) = P(d,e,b) + P(d,\neg e,b) + P(d,e,\neg b) + P(d,\neg e,\neg b)$$

$$P(d,e,b) = P(d|e,b) P(e,b) = P(d|e,b) P(e) P(b)$$

$$P(e) = P(e,t,l) + P(e,\neg t,l) + P(e,t,\neg l) + P(e,\neg t,\neg l)$$

$$P(e,t,l) = P(e|t,l) P(t,l) = P(e|t,l) P(t) P(l)$$

$$P(t) = P(t|a) P(a) + P(t|\neg a) P(\neg a) = 0.05 * 0.01 + 0.01 * 0.99 =$$

$$0.0104 \quad P(\neg t) = 1 - P(t) = 1 - 0.0104 = 0.9896$$

$$P(l) = P(l|s) P(s) + P(l|\neg s) P(\neg s) = 0.1 * 0.5 + 0.01 * 0.5 =$$

$$0.055 \quad P(\neg l) = 1 - P(l) = 1 - 0.055 = 0.945$$

$$P(e) = P(e|t,l) P(t) P(l) + P(e|\neg t,l) P(\neg t) P(l) +$$

$$P(e|t,\neg l) P(t) P(\neg l) + P(e|\neg t,\neg l) P(\neg t) P(\neg l)$$

$$= 1 * P(t) P(l) + 1 * P(\neg t) P(l) + 1 * P(t) P(\neg l) + 0 * P(\neg t) P(\neg l)$$

$$= P(t) P(l) + P(\neg t) P(l) + P(t) P(\neg l) = 1 - P(\neg t) P(\neg l)$$

$$\{\text{Since, } P(t) P(l) + P(\neg t) P(l) + P(t) P(\neg l) + P(\neg t) P(\neg l) = 1\}$$

$$= 1 - 0.9896 * 0.945 = 0.064828$$

$$P(\neg e) = 1 - 0.064828 = 0.935172$$

$$P(b) = P(b|s) P(s) + P(b|\neg s) P(\neg s) = 0.6 * 0.5 + 0.3 * 0.5 =$$

$$0.45 \quad P(\neg b) = 1 - P(b) = 1 - 0.45 = 0.55$$

$$P(d) = P(d|e,b) P(e) P(b) + P(d|\neg e,b) P(\neg e) P(b) +$$

$$P(d|e,\neg b) P(e) P(\neg b) + P(d|\neg e,\neg b) P(\neg e) P(\neg b)$$

$$= 0.1 * 0.064828 * 0.45 + 0.8 * 0.935172 * 0.45 +$$

$$0.7 * 0.064828 * 0.55 + 0.1 * 0.935172 * 0.55$$

$$= 0.41597242$$