LOGICAL DEDUCTION IN AI

PREDICATE LOGIC FUNDAMENTALS

Partha P Chakrabarti
Indian Institute of Technology Kharagpur

Predicate Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

New Additions in Proposition (First Order Logic)

Function

Variables, Constants, Predicate Symbols and New Connectors: ∃ (there exists), ∀(for all)

all Some

Wherever Mary goes, so does the Lamb. Mary goes to School. So the Lamb goes to School.

Predicate: goes(x,y) to represent x goes to y

New Connectors: ∃ (there exists), ∀(for all)

F1: $\forall x(\text{goes}(\text{Mary}, x) \rightarrow \text{goes}(\text{Lamb}, x))$ ←

x is bound by ∀x

quantified

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: (F1 ∧ F2) → G) is always true

Use of Quantifiers

EXAMPLES:

Someone likes everyone

Everyone likes someone

There is someone whom everyone likes

Everyone likes everyone

If everyone likes everyone then someone likes everyone

If there is a person whom everyone likes then that person likes himself

Exercise

LAWS of NEGATION:

$\exists x \; \vee$

$\forall \; \bigwedge_{i=1}$ $\quad \forall x \; \bigwedge_{i \in x}$

$\neg \forall x (p(x)) \equiv \exists x (\neg p(x))$

$\forall x\, p(x) \quad \bigwedge_x p(x)$

likes(x,y) : x likes y

$\boxed{\exists x (\forall y (\text{likes}(x,y)))}$ F1

$\forall x (\exists y (\text{likes}(x,y)))$

y depends on x

$\boxed{\exists y \forall x (\text{likes}(x,y))}$

y is independent of x

F2 $\boxed{\forall x \forall y (\text{likes}(x,y))}$ $\quad \forall y \forall x \, \text{likes}(x,y)$

$\boxed{F2 \rightarrow F1}$

scope rules

$\exists x_1 (\forall y_1 (\text{likes}(x_1,y_1)))$ $\quad (\forall x \forall y (\text{likes}(x,y)))$ F2

De Morgan's laws

Use of Function Symbols

If x is greater than y and y is greater than z then x is greater than z.

The age of a person is greater than the age of his child.

Therefore the age of a person is greater than the age of his grandchild.

The sum of ages of two children are never more than the sum of ages of their parents.

variables
Constant → Functions
propositions → predicates
$\forall$, $\exists$

$g(x,y)$ x is greater than y

$\forall x \forall y \forall z\, ((g(x,y) \wedge g(y,z)) \rightarrow g(x,z))$

Constant Symbol A → B(x)

→ Age(x) → Function Symbol → variable

→ child (x,y) : x is a child of y

$\boxed{\forall x \forall y\, (child(x,y) \rightarrow g(Age(y), Age(x)))}$

Age(x) → returns a value

child (x,y) → returns TRUE or FALSE

Sum (x,y) → Function symbol

parent (x,y) : use the child predicate

Variables and Predicate / Function Symbols

Variables, Free variables, Bound variables

Symbols – proposition symbols, constant symbols, function symbols, predicate symbols ✓

Variables can be quantified in first order predicate logic ✓

Symbols cannot be quantified in first order predicate logic

Interpretations are mappings of symbols to relevant aspects of a domain

$\cancel{\exists p\,(\forall x\{p(x)\})}$

Not in predicate logic

$\forall x\,(p(x, y))$

free variable,

x is a bound variable

bound

$\forall x\,\{p(x, y) \wedge \exists z\, q(x, y, z, w)\}$

bound

free

free bound

$\forall x\,\{p(x, y) \wedge \exists z\, \exists y\, q(x, y, z, w)\}$

free

Terminology for Predicate Logic

Domain: D

Constant Symbols: M, N, O, P, ….

Variable Symbols: x,y,z,….

Function Symbols: F(x), G(x,y), H(x,y,z)

Predicate Symbols: p(x), q(x,y), r(x,y,z),

Connectors: ~, Λ, V, →, ∃, ∀

Terms:

Well-formed Formula: WFF

Free and Bound Variables:

Interpretation, Valid, Non-Valid, Satisfiable, Unsatisfiable

SYNTAX

SEMANTICS

$p(x,..)$… where x is free in p

$\forall x\, p(x,..)$ $\exists x\, p(x,..)$

D : Domain D will specified for every interpretation

Term: – variables are terms
– constant symbols are terms
– if $t_1, t_2, \ldots t_k$ is a Term
and $F(x_1, x_2, \ldots, x_k)$ is a Fn symbol
then $F(t_1, t_2, \ldots, t_k)$ is also a term
– if $p(x_1, x_2, \ldots, x_k)$ is a predicate symbol
$p(t_1, t_2, \ldots, t_k)$

WFF: propositions are wffs,

p is a WFF → $\neg p$ is a WFF

p & q are WFF → $p \wedge q$, $p \vee q$, $p \to q$

Validity, Satisfiability, Structure

F1: ∀x(goes(Mary, x) → goes(Lamb, x))

F2: goes(Mary, School)

G: goes(Lamb, School)

To prove: (F1 ∧ F2) → G) is always true

Is the same as:

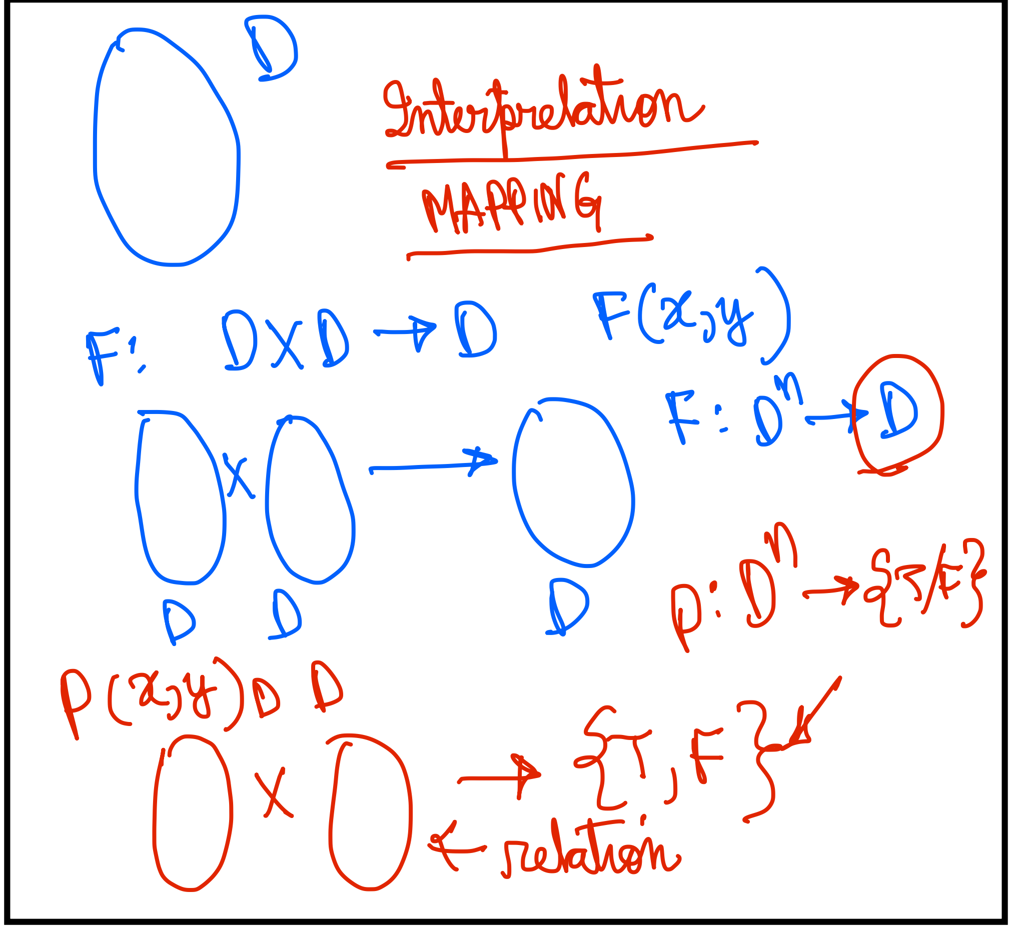
F1: ∀x(www(M, x) → www(L, x))

F2: www(M, S)

G: www(L, S)

To prove: (F1 ∧ F2) → G) is always true

predicate symbol



Interpretations

What is an Interpretation? Assign a domain set D, map constants, functions, predicates suitably.
The formula will now have a truth value

Example:

F1: $\forall x(g(M, x) \rightarrow g(L, x))$

F2: $g(M, S)$

G: $g(L, S)$

$F_1 \quad F_2 \quad G$

$F1 \wedge F2 \rightarrow G$

Interpretation 1: D = {Akash, Baby, Home, Play, Ratan, Swim}, etc.,

Interpretation 2: D = Set of Integers, etc.,

How many interpretations can there be?

To prove Validity, means $(F1 \wedge F2) \rightarrow G)$ is true under all interpretations

To prove Satisfiability means $(F1 \wedge F2) \rightarrow G)$ is true under at least one interpretation.

if it is true under SOME interpretation

D: Assign the Domain

F: Assign Constants or F's from the Domain

P: Assign a specific relation from the Domain

M: Home, L: Akash, S: Akash

$g(x, y)$: for all pairs in D we have say whether $g(x, y)$ is T/F

D: set of Integers L, M, S

$g(x, y)$: x divides y

$F1 \wedge F2 \rightarrow G$

A formula is said to be valid if it is true FOR ALL interpretations

In Its Power Lies Its Limitations

<u>Russell's Paradox</u> (The barber shaves all those who do not shave themselves. Does the barber shave himself?)

- There is a single barber in town.
- Those and only those who do not shave themselves are shaved by the barber.
- Who shaves the barber?

Checking Validity of First order logic is undecidable but partially decidable (semi-decidable) {Robinson's Method of Resolution Refutation}

Higher order predicate logic - can quantify symbols in addition to quantifying variables.

$$\forall p((p(0) \wedge (\forall x(p(x) \rightarrow p(S(x))) \rightarrow \forall y(p(y))$$

NOT IN 1st order logic

Power? Computation

Predicate Logic can model any computable function TURING M/C

Undecidability | HALTING

Barber shaves himself X
Someone else shaves the Barber X

Unsolvable Problem ✓

if the answer is YES then there is a Method SEMI-DECIDABLE ✓

HIGHER ORDER LOGIC

what this query tries to say

Which Theorem

Thank you