# SEARCH METHODS IN AI

## STATE SPACE SEARCH

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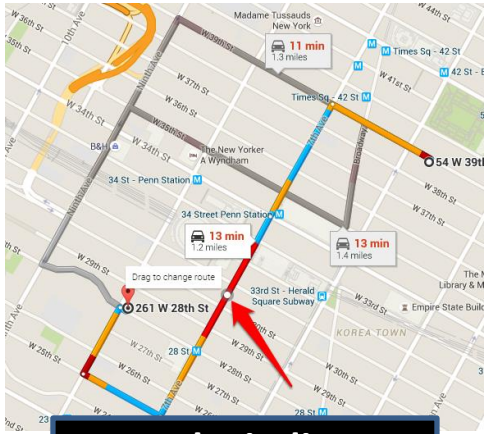
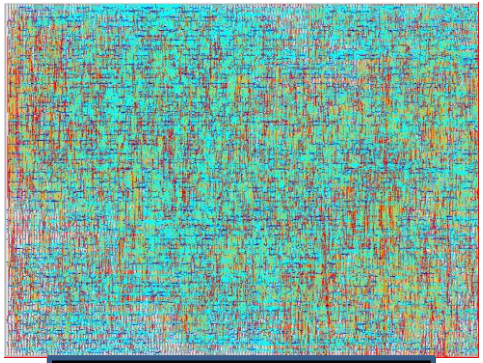
# COMPLEX PROBLEMS & SOLUTIONS



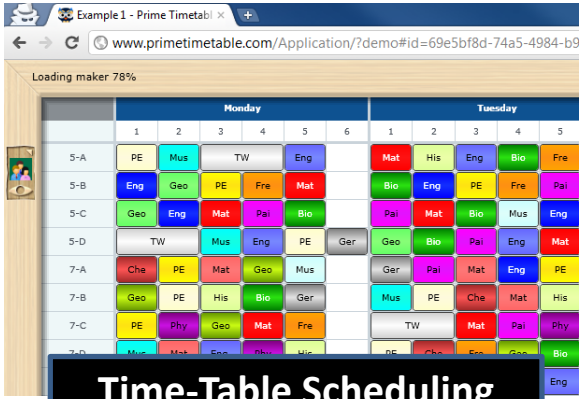

**Path Finding**


**Chess Playing**

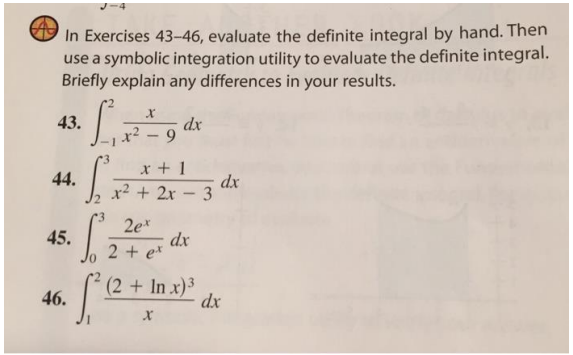
**Robot Assembly**



**VLSI Chip Design**




**Time-Table Scheduling**




**Symbolic Integration**

# AUTOMATED PROBLEM SOLVING BY SEARCH

- Generalized Techniques for Solving Large Classes of Complex Problems
- Problem Statement is the Input and solution is the Output, sometimes even the problem specific algorithm or method could be the Output
- Problem Formulation by AI Search Methods consists of the following key concepts
  - Configuration or State
  - Constraints or Definitions of Valid Configurations
  - Rules for Change of State and their Outcomes
  - Initial or Start Configurations
  - Goal Satisfying Configurations
  - An Implicit State or Configuration Space
  - Valid Solutions from Start to Goal in the State Space
  - General Algorithms which SEARCH for Solutions in this State Space
- ISSUES
  - Size of the Implicit Space, Capturing Domain Knowledge, Intelligent Algorithms that work in reasonable time and Memory, Handling Incompleteness and Uncertainty

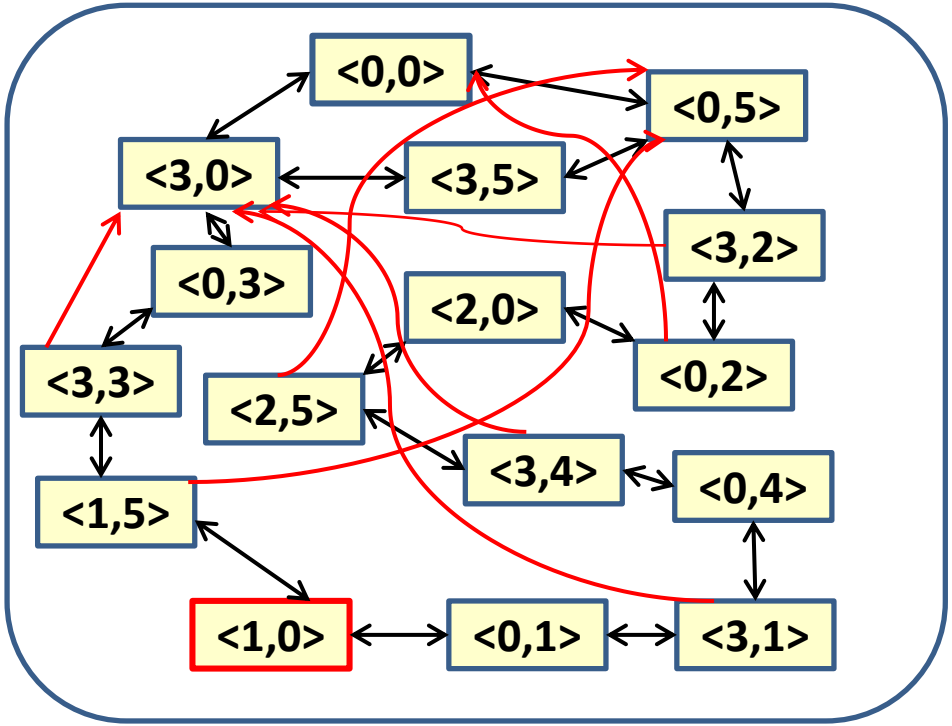
# BASICS OF STATE SPACE MODELLING

- **STATE or CONFIGURATION:**
  - A set of variables which define a state or configuration
  - Domains for every variable and constraints among variables to define a valid configuration
- **STATE TRANSFORMATION RULES or MOVES:**
  - A set of RULES which define which are the valid set of NEXT STATE of a given State
  - It also indicates who can make these Moves (OR Nodes, AND nodes, etc)
- **STATE SPACE or IMPLICIT GRAPH**
  - The Complete Graph produced out of the State Transformation Rules.
  - Typically too large to store. Could be Infinite.
- **INITIAL or START STATE(s), GOAL STATE(s)**
- **SOLUTION(s), COSTS**
  - Depending on the problem formulation, it can be a PATH from Start to Goal or a Sub-graph of And-ed Nodes
- **SEARCH ALGORITHMS**
  - Intelligently explore the Implicit Graph or State Space by examining only a small sub-set to find the solution
  - To use Domain Knowledge or HEURISTICS to try and reach Goals faster

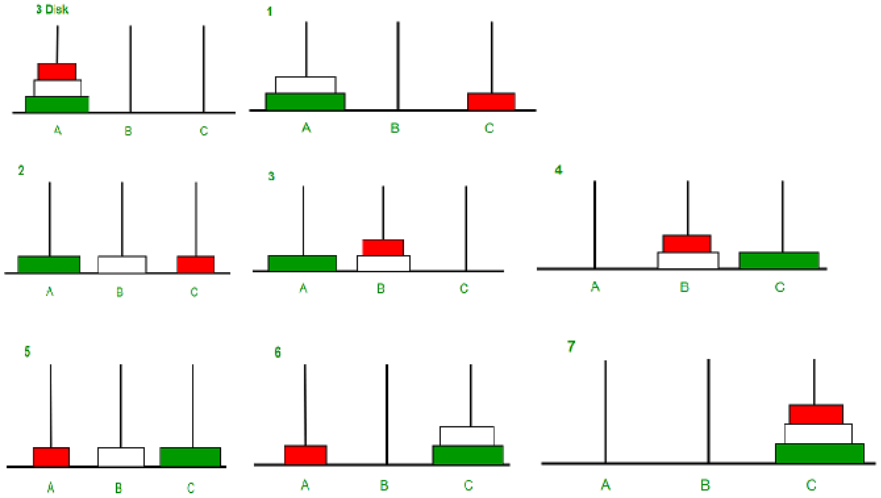
# TWO JUG PROBLEM

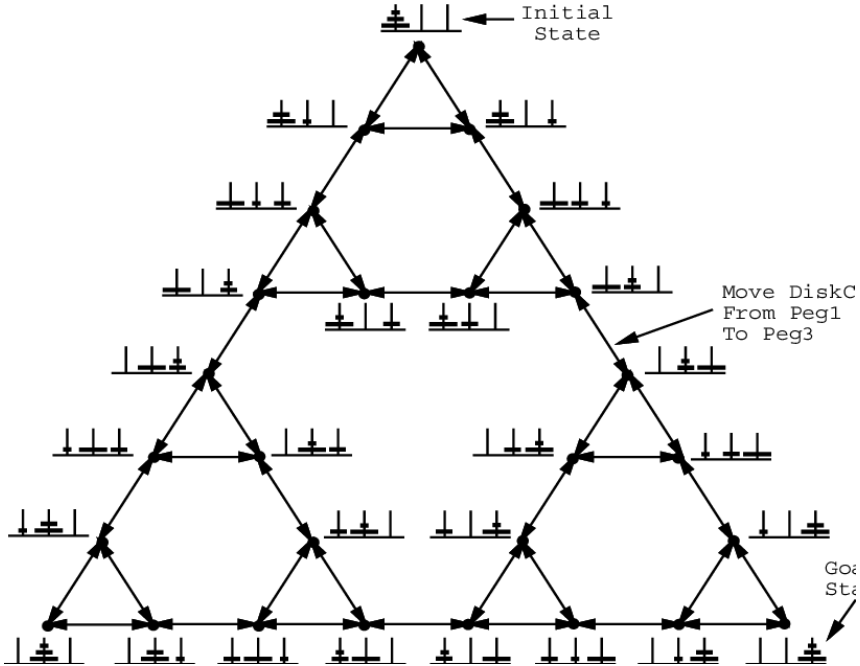
- There is a large bucket B full of water and Two (02) jugs, J1 of volume 3 litre and J2 of volume 5 litre. You are allowed to fill up any empty jug from the bucket, pour all water back to the bucket from a jug or pour from one jug to another. The goal is to have jug J1 with exactly one (01) litre of water
- State Definition: <J1, J2>
- Rules:
  - Fill (J1): <J1, J2> to <3,J2>
  - Fill (J2): <J1, J2> to <J1, 5>
  - Empty (J1), Empty (J2): Similarly defined
  - Pour (J1, J2): <J1, J2> to <X,Y>, where
    - X = 0 and Y = J1 + J2 if J1+J2 ≤ 5,
    - Y = 5 and X = (J1+ J2) – 5, if J1+J2 > 5
  - Pour (J2, J2): Similarly defined
- Start: <0,0>, Goal: <1,0>
- Part of State Space Shown on the right

(Not all Links shown here)



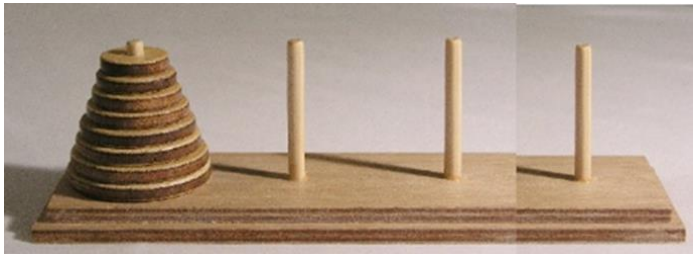
# 3 DISK, 3 PEG TOWER of HANOI STATE SPACE

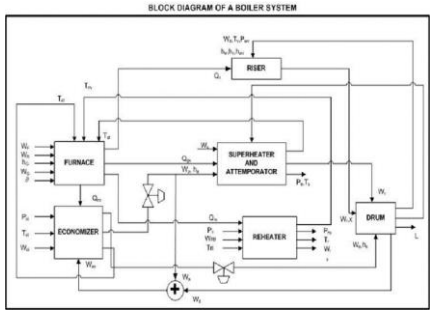


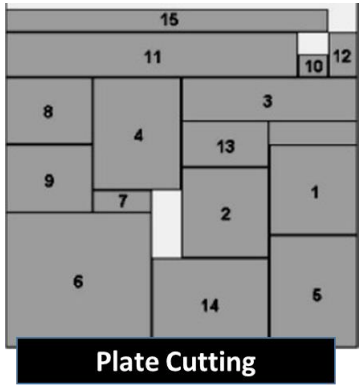



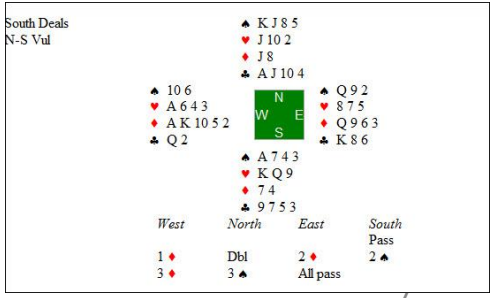
# STATES, SPACES, SOLUTIONS, SEARCH

- States
  - Full / Perfect Information and Partial Information States
- State Transformation Rules
  - Deterministic Outcomes
  - Non-Deterministic / Probabilistic Outcomes
- State Spaces As Generalized Games
  - Single Player: OR Graphs
  - Multi-Player: And / Or, Adversarial, Probabilistic Graphs
- Solutions
  - Paths
  - Sub-graphs
  - Expected Outcomes
- Costs
- Sizes
- Domain Knowledge
- Algorithms for Heuristic Search

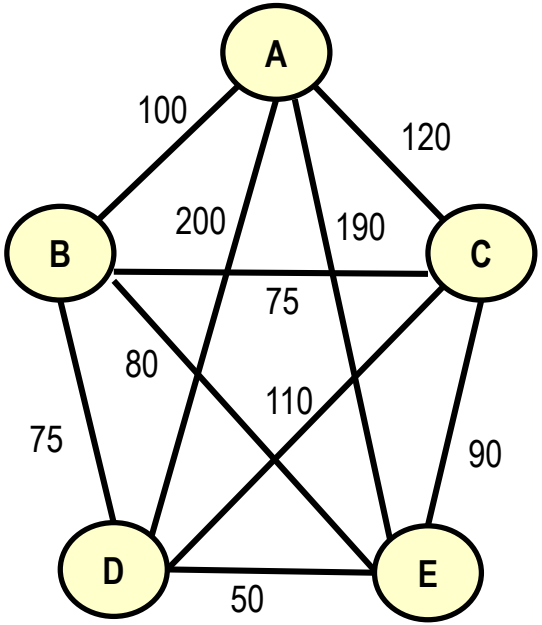


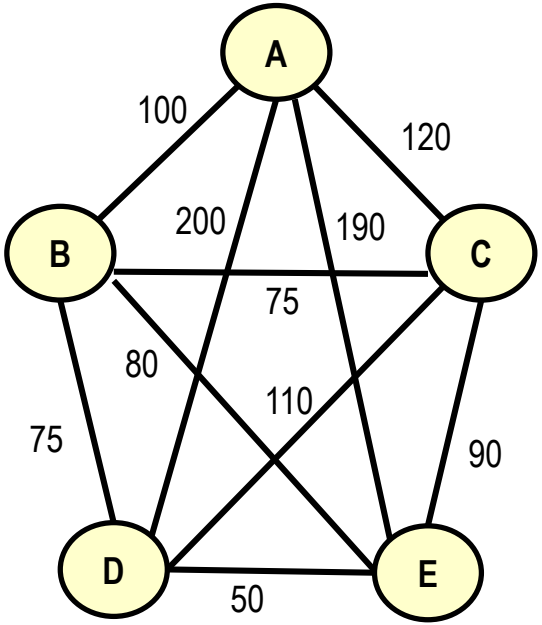



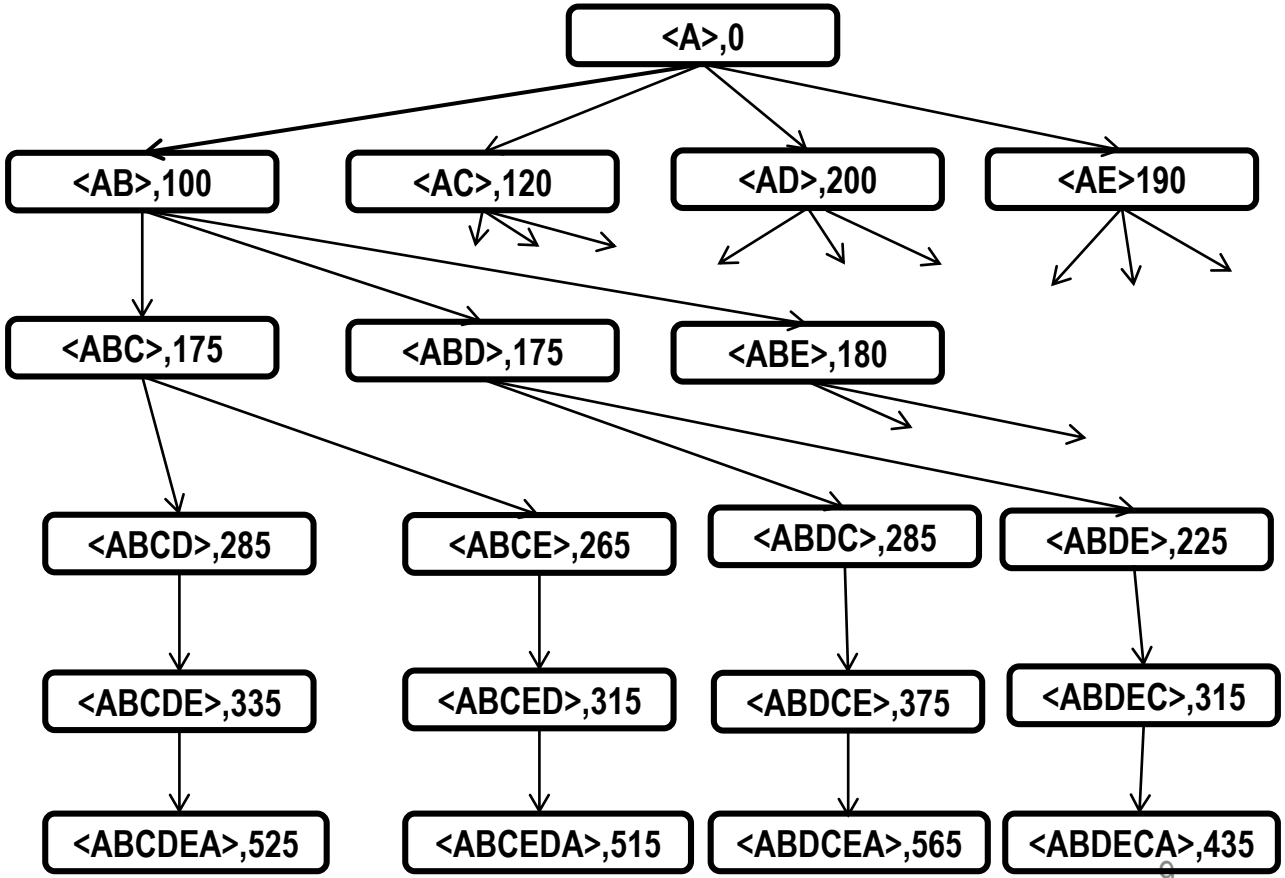



# OR-Graph: TRAVELLING SALESPERSON PROBLEM



# OR-Graph: TRAVELLING SALESPERSON PROBLEM



# MODELLING AND/OR GRAPHS:

OR Nodes are ones for which one has a choice.

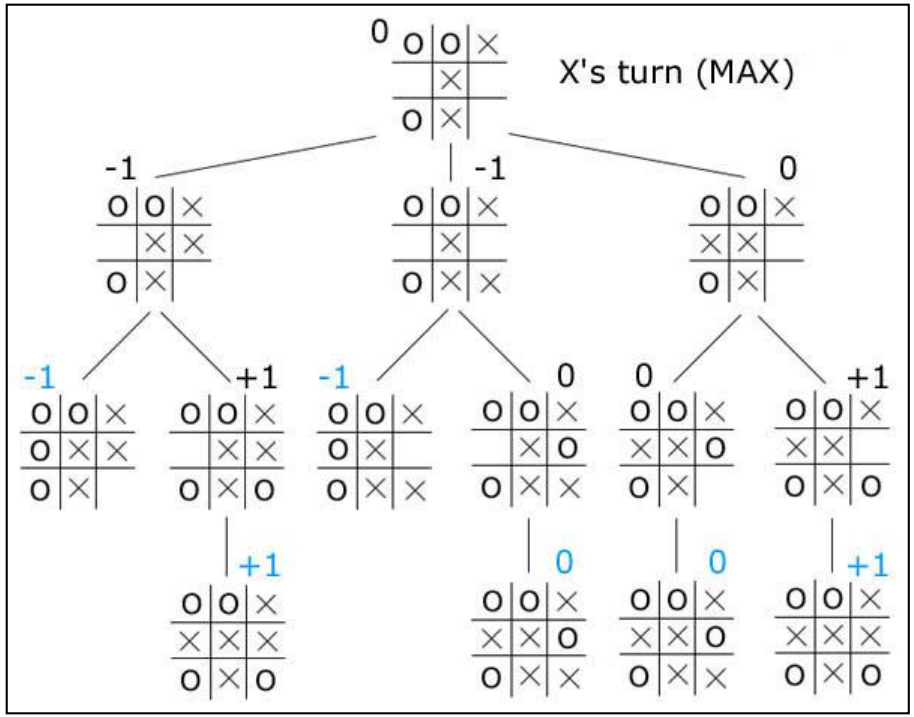
The AND nodes could be compositional (sum, product, min, max, etc., depending on the way the sub-problems are composed),

Adversarial (game where the other parties have a choice)

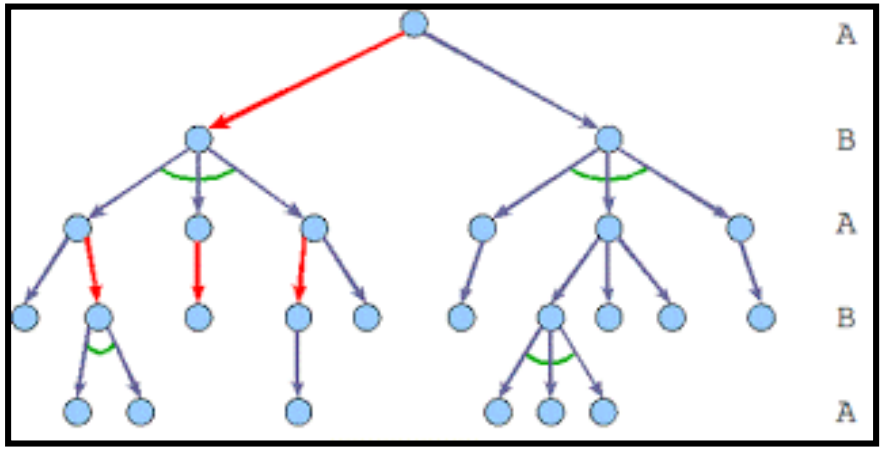
or

Probabilistic (Environmental Actions)

# AND/OR GRAPHS: ADVERSARIAL



# AND/OR GRAPHS: COMPOSITIONAL / ADVERSARIAL / PROBABILISTIC



# COMPOSITIONAL AND/OR GRAPHS – MATRIX CHAIN MULTIPLICATION

**(M1 X (M2 X (M3 X M4))) = ((M1 x M2) X (M3 X M4)) = (((M1 x M2) X M3) X M4) = (M1 X (M2 X M3) ) X M4)**

**BUT THE NUMBER OF MULTIPLICATIONS TO GET THE ANSWER DIFFER !!**

Let A be a [p by q] Matrix and B be a [q by r] Matrix. The number of multiplications needed to compute A X B = p*q*r

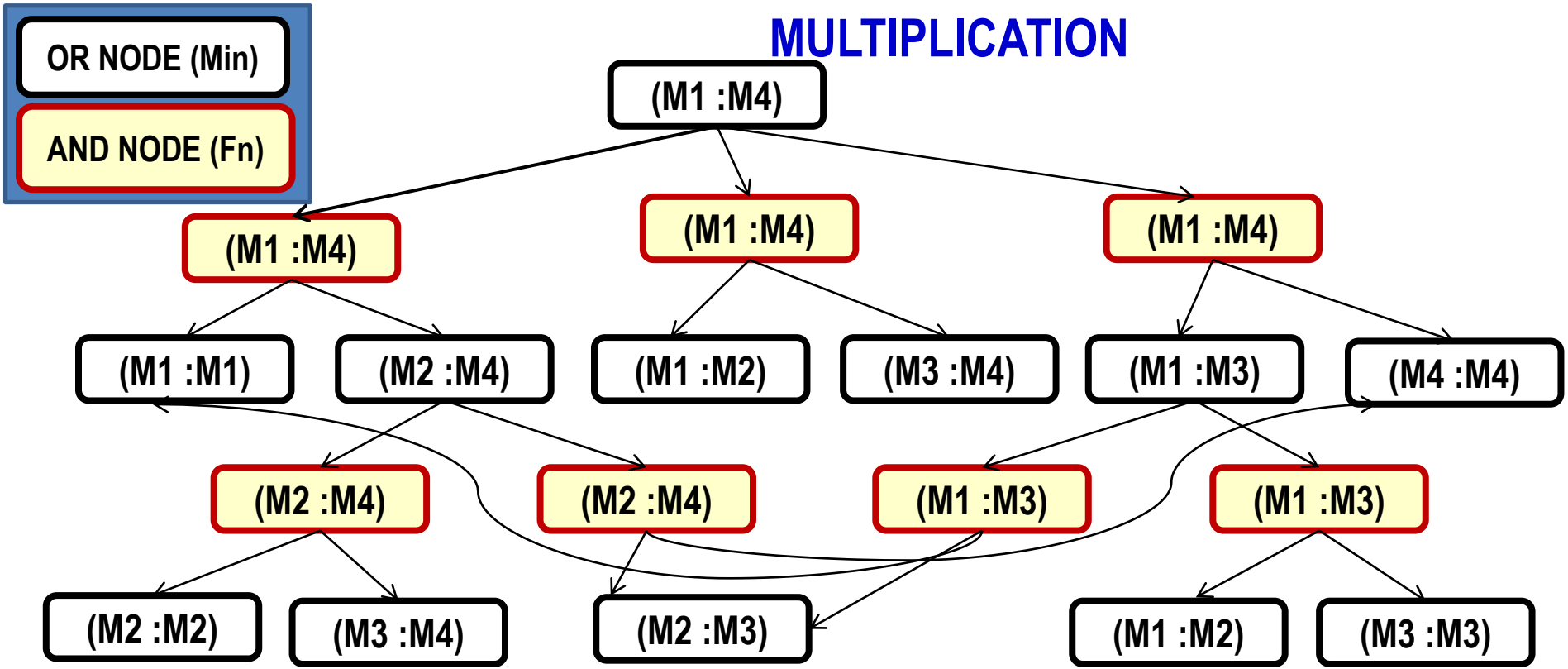
Thus if M1 be a [10 by 30] Matrix, M2 be a [30 by 5] Matrix and M3 be a [5 by 60] Matrix

Then the number of computations for

(M1 X M2) X M3 = 10*30*5 for P = (M1 X M2) and 10*5*60 for P X M3. Total = 4500

M1 X (M2 X M3) = 30*5*60 for Q = (M2 X M3) and 10*30*60 for M1 X Q. Total = 27000

# COMPOSITIONAL AND/OR GRAPHS: MATRIX CHAIN MULTIPLICATION




(M1 X (M2 X (M3 X M4))) = ((M1 x M2) X (M3 X M4)) = (((M1 x M2) X M3) X M4) = (M1 X (M2 X M3) ) X M4)

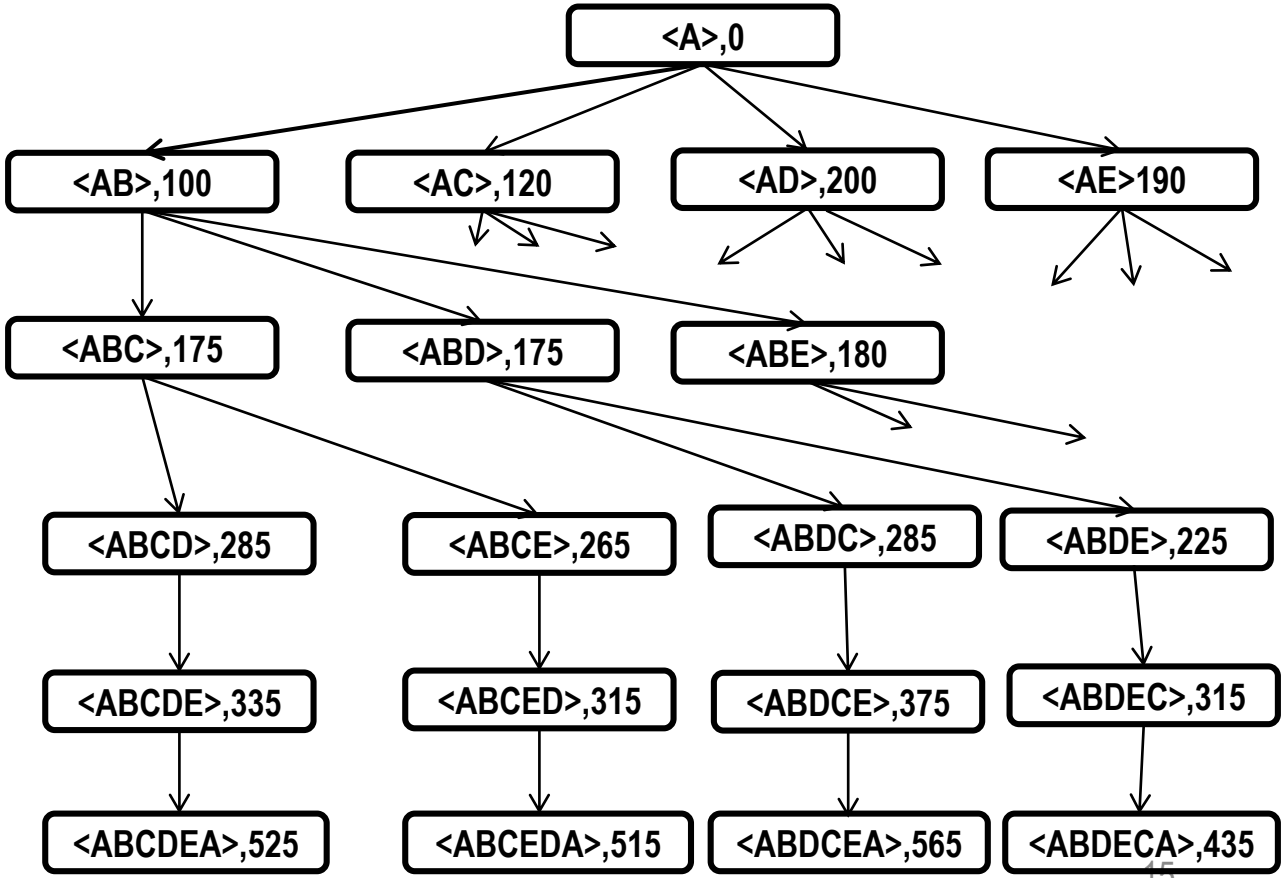
# SEARCHING IMPLICIT GRAPHS

Given the start state the SEARCH Algorithm will create successors based on the State Transformation Rules and make part of the Graph EXPLICIT.

It will EXPAND the Explicit graph INTELLGENTLY to rapidly search for a solution without exploring the entire Implicit Graph or State Space

For OR Graphs, the solution is a PATH from start to Goal.

Cost is usually sum of the edge costs on the path, though it could be something based on the problem



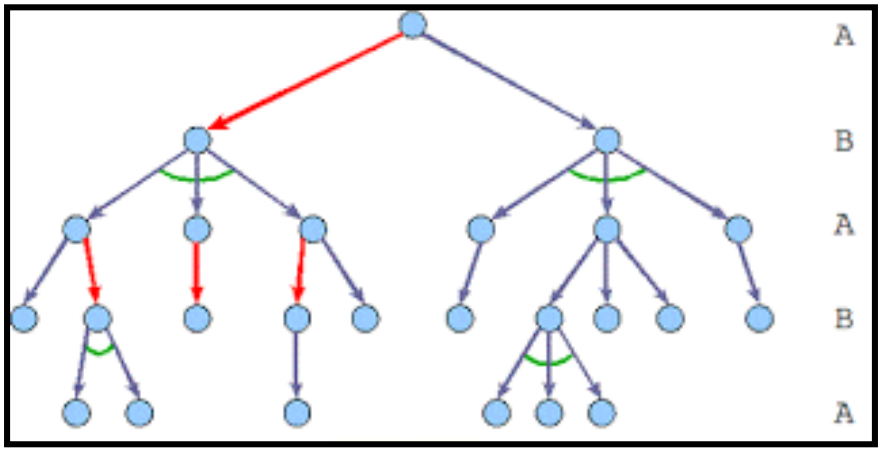
# SEARCHING IMPLICIT GRAPHS

For And/OR Graphs, the Solution is an AND Subgraph rooted at the Start and each leaf is a Goal Node.

The Cost of OR Node is usually a Min or Max.

The Cost at the AND Node depends on the type of Node (Compositional, Adversarial, Probabilistic).

For Adversarial two player games, Max / Min is used at AND Node (reverse of Or Node)



# SEARCHING IMPLICIT GRAPHS

**The various Search Algorithms include**

- **BASIC Algorithms: Depth-First (DFS), Breadth-first (BFS), Iterative Deepening (IDS)**
- **COST-based Algorithms: Depth-First Branch-and-Bound, Best First Search, Best-First Iterative Deepening**
- **Widely Used Algorithms: A* and IDA* (Or Graphs), AO* (And/Or Graphs), Alpha-beta Pruning (Game-Trees)**

# BASIC ALGORITHMS in OR GRAPHS: DFS

1. [Initialize] Initially the OPEN List contains the Start Node s. CLOSED List is Empty.
2. [Select] Select the first Node n on the OPEN List. If OPEN is empty, Terminate
3. [Goal Test] If n is Goal, then decide on Termination or Continuation / Cost Updation
4. [Expand]
   a) Generate the successors n_1, n_2, …. n_k, of node n, based on the State Transformation Rules
   b) Put n in LIST CLOSED
   c) For each n_i, not already in OPEN or CLOSED List, put n_i in the FRONT of OPEN List
   d) For each n_i already in OPEN or CLOSED decide based on cost of the paths
5. [Continue] Go to Step 2

# BASIC ALGORITHMS in OR GRAPHS: IDS

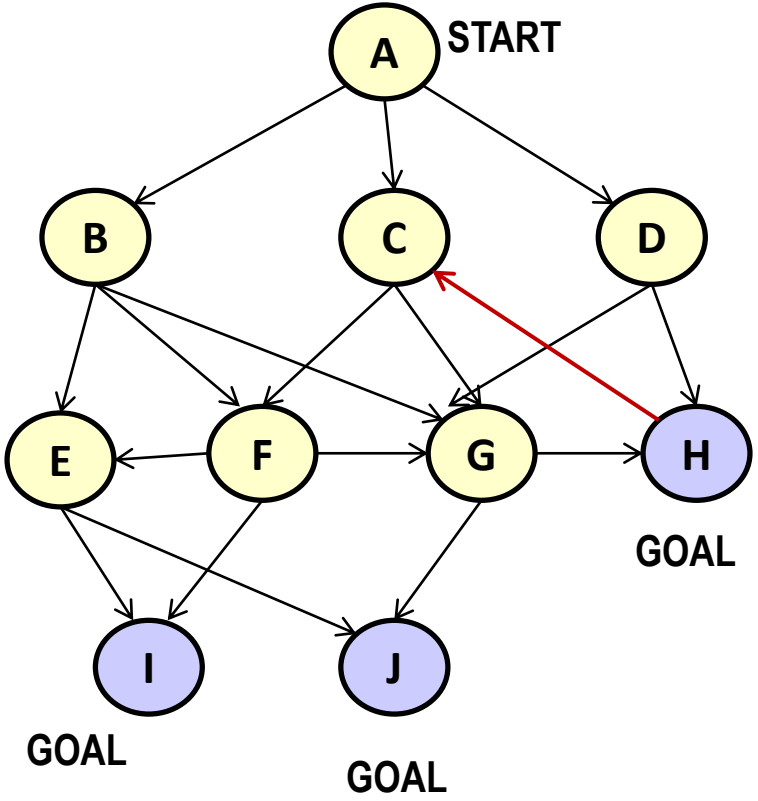
1. [Initialize] Initially the OPEN List contains the Start Node s. CLOSED List is Empty.
2. [Select] Select the first Node n on the OPEN List. If OPEN is empty, Terminate
3. [Goal Test] If n is Goal, then decide on Termination or Continuation / Cost Updation
4. [Expand]
   a) Generate the successors n_1, n_2, …. n_k, of node n, based on the State Transformation Rules
   b) Put n in LIST CLOSED
   c) For each n_i, not already in OPEN or CLOSED List, put n_i in the FRONT of OPEN List
   d) For each n_i already in OPEN or CLOSED decide based on cost of the paths
5. [Continue] Go to Step 2

**Algorithm IDS Performs DFS Level by Level Iteratively (DFS (1), DFS (2), ……. and so on)**

# BASIC ALGORITHMS in OR GRAPHS: BFS

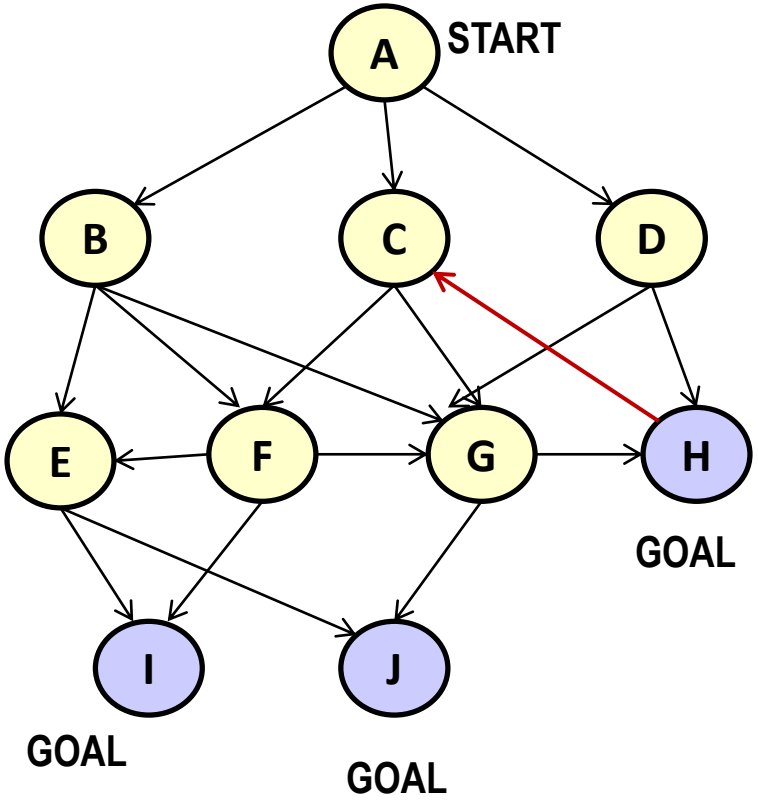
1. [Initialize] Initially the OPEN List contains the Start Node s. CLOSED List is Empty.
2. [Select] Select the first Node n on the OPEN List. If OPEN is empty, Terminate
3. [Goal Test] If n is Goal, then decide on Termination or Continuation / Cost Updation
4. [Expand]
   a) Generate the successors n_1, n_2, …. n_k, of node n, based on the State Transformation Rules
   b) Put n in LIST CLOSED
   c) For each n_i, not already in OPEN or CLOSED List, put n_i in the END of OPEN List
   d) For each n_i already in OPEN or CLOSED decide based on cost of the paths
5. [Continue] Go to Step 2

# EXAMPLE: SEARCHING A STATE SPACE GRAPH




- DEPTH-FIRST SEARCH (DFS)
- BREADTH-FIRST SEARCH (BFS)
- ITERATIVE DEEPENDING SEARCH (IDS)
- PROPERTIES
  - SOLUTION GUARANTEES
  - MEMORY REQUIREMENTS

# EXAMPLE: SEARCHING A STATE SPACE GRAPH



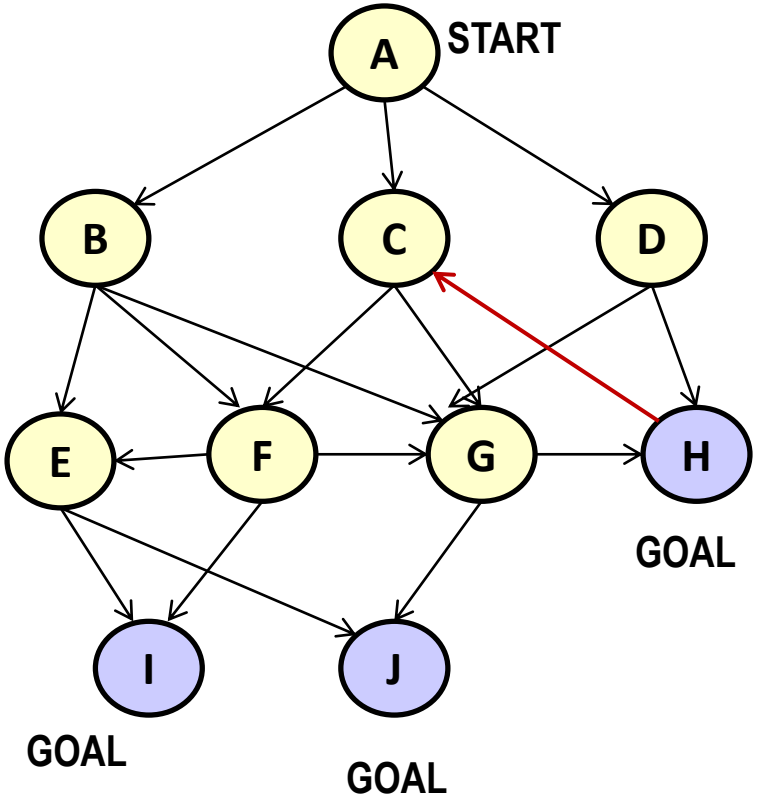

DEPTH-FIRST SEARCH:

1. OPEN ={A}, CLOSED = {}
2. OPEN = {B,C,D}, CLOSED = {A}
3. OPEN = {E,F,G,C,D}, CLOSED = (A,B}
4. OPEN = {I,J,F,G,C,D}, CLOSED = {A,B,E}
5. Goal Node I Found. Can Terminate with Path from A to I or may Continue for more Goal nodes if minimum length or cost is a criteria

DFS MAY NOT TERMINATE IF THERE IS AN INFINITE DEPTH PATH EVEN IF THERE IS A GOAL NODE AT FINITE DEPTH

DFS HAS LOW MEMORY REQUIREMENT

# EXAMPLE: SEARCHING A STATE SPACE GRAPH




ITERATIVE DEEPENING SEARCH:

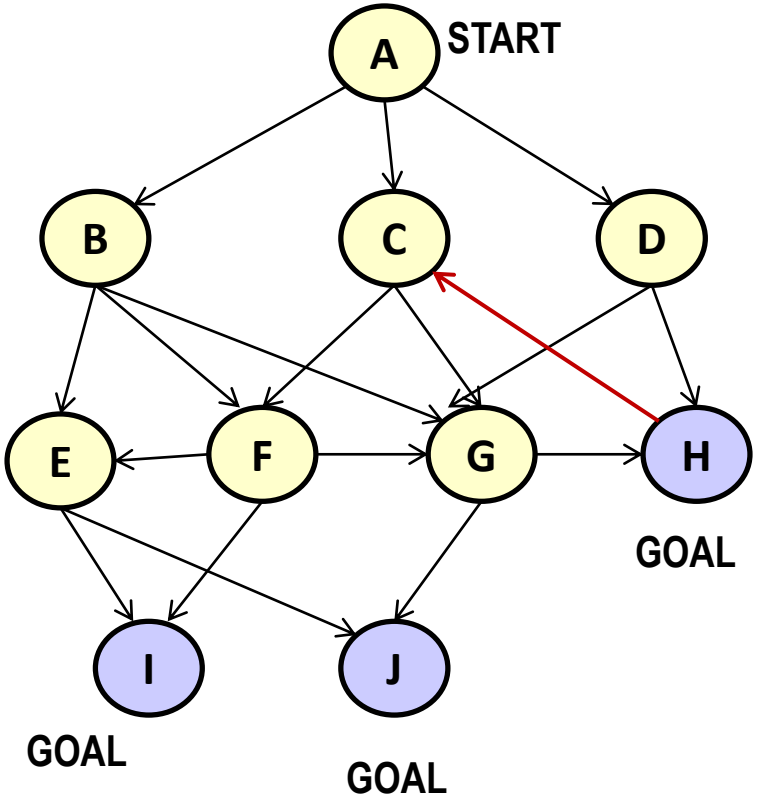
1. PERFORM DFS TILL LENGTH 1. NO SOLUTION FOUND
2. PERFORM DFS TILL LEVEL 2. GOAL NODE H REACHED.
3. Can Terminate with Path from A to H. This is guaranteed to be the minimum length path.

IDS GUARANTEES SHORTEST LENGTH PATH TO GOAL

IDS MAY RE-EXPAND NODES MANY TIMES

IDS HAS LOWER MEMORY REQUIREMENT THAN BFS

# EXAMPLE: SEARCHING A STATE SPACE GRAPH

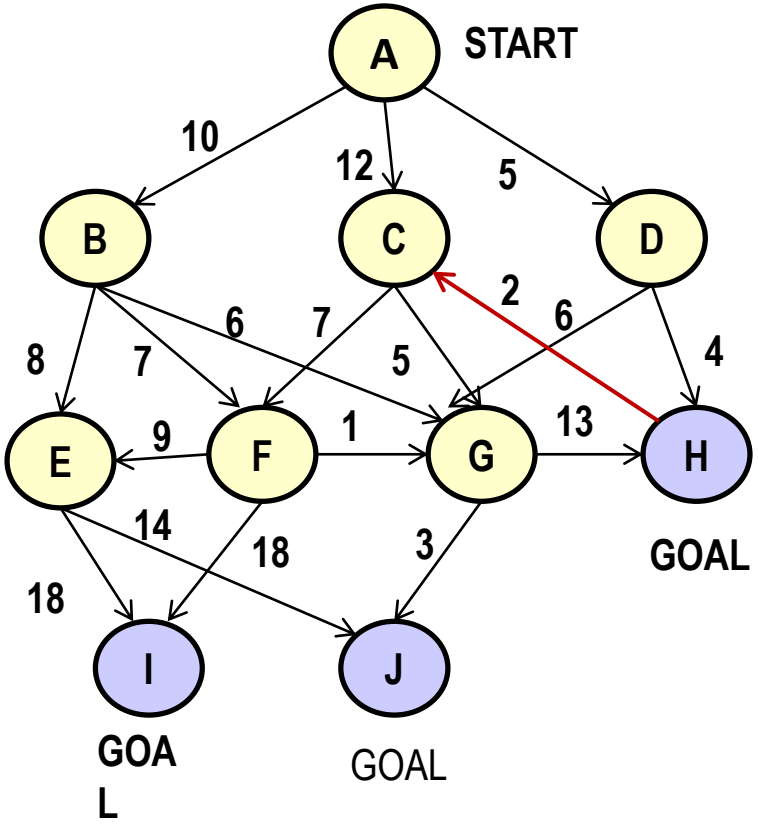



**BREADTH-FIRST SEARCH:**

1. OPEN ={A}, CLOSED = {}
2. OPEN = {B,C,D}, CLOSED = {A}
3. OPEN = {C,D,E,F,G}, CLOSED = (A,B}
4. OPEN = {D,E,F,G}, CLOSED = {A,B,C}
5. OPEN = {E,F,G,H}. CLOSED = {A,B,C,D}
6. OPEN = {F,G,H,I,J}, CLOSED = {A,B,C,D,E}
7. OPEN = {G,H,I,J}, CLOSED = {A,B,C,D,E,F}
8. OPEN = {H,I,J}, CLOSED = {A,B,C,D,E,F,G}
9. Goal Node H Found. Can Terminate with Path from A to H. This is guaranteed to be the minimum length path.

**BFS GUARANTEES SHORTEST LENGTH PATH TO GOAL BUT HAS HIGHER MEMORY REQUIREMENT**

# SEARCHING STATE SPACE GRAPHS WITH EDGE COSTS

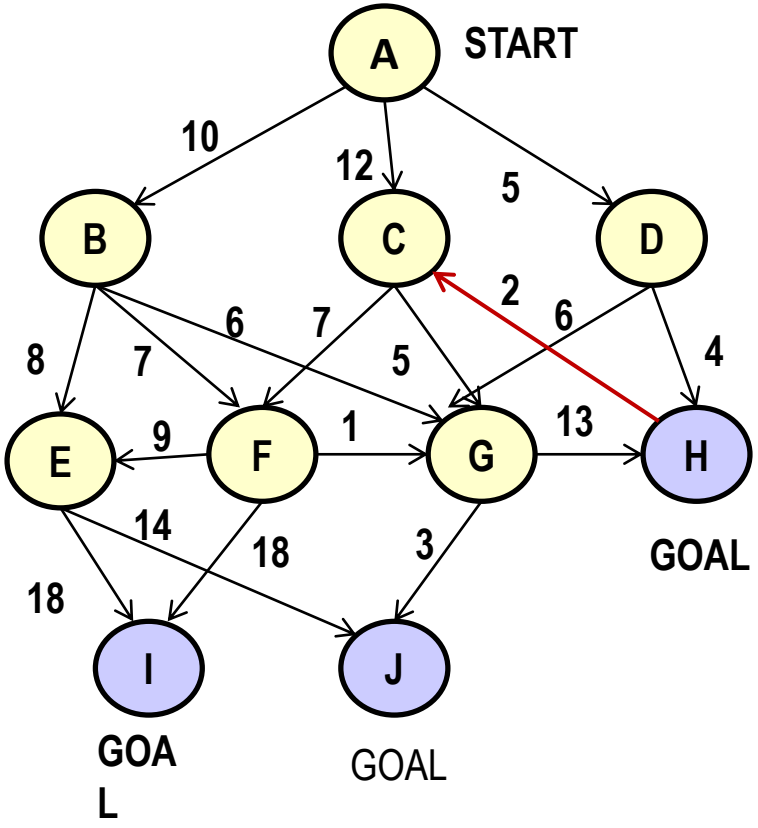


# MODIFYING BASIC ALGORITHMS TO INCORPORATE COSTS

1. [Initialize] Initially the OPEN List contains the Start Node s. CLOSED List is Empty.
2. [Select] Select the first Node n on the OPEN List. If OPEN is empty, Terminate
3. [Goal Test] If n is Goal, then decide on Termination or Continuation / Cost Updation
4. [Expand]
   a) Generate the successors n_1, n_2, …. n_k, of node n, based on the State Transformation Rules
   b) Put n in LIST CLOSED
   c) For each n_i, not already in OPEN or CLOSED List, put n_i in the FRONT (for DFS) / END (for BFS) of OPEN List
   d) For each n_i already in OPEN or CLOSED decide based on cost of the paths
5. [Continue] Go to Step 2

Algorithm IDS Performs DFS Level by Level Iteratively (DFS (1), DFS (2), ……. and so on)

# NEXT: SEARCHING STATE SPACE GRAPHS WITH EDGE COSTS




- COST ORDERED SEARCH:
  - DFBB
  - Best First Search,
  - Best First IDS
  - Use of HEURISTIC Estimates: Algorithm A* (Or Graphs), AO* (And/Or Graphs)
- PROPERTIES
  - SOLUTION GUARANTEES
  - MEMORY REQUIREMENTS

# Thank you