FOCS Tutorial-1

Foundations of Computing Science

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Questions

1. Construct a DFA that will accept the following languages over the alphabet {0, 1}
   - (a) All strings that start with 0 and have odd length OR start with 1 and have even length.
   - (b) The set of all strings with number of 0's divisible by 3 and number of 1's divisible by 2.

2. Write regular expressions for the following language
   - (a) The set of all strings of 0's and 1's with at least one 0 and one 1.
   - (b) The set of all strings of 0's and 1's whose fourth symbol from the right end is 1.
   - (c) The set of all strings with an equal number of 0's and 1's, such that no prefix has two more 0's tan 1's, nor two more 1's than 0's.

Questions

3. One of the following languages is regular, and the other one is not regular. Identify which is which with respective proofs:

$$L_a = \{a^i b^j \mid i,j>=0 \text{ and } i+j>=10\}$$
$$L_b = \{a^i b^j \mid i,j>=0 \text{ and } i-j>=10\}$$

4. TRUE / FALSE (with reasons)

a) If L is a regular language and F is a finite language (i.e. a language with finite number of words), then L U F is regular.

b) Regular expression that do not contain the star operator can represent only finite languages.

c) Define EVEN(w), for a finite string w, to be the string consisting of the symbols of w in even-numbered positions.
   For example, EVEN(1011010) = 011.
   If L is a regular language, then {EVEN(w) : w ∈ L} must be regular.

d) For every pair of regular expressions R and S, the languages denoted by R(SR)* and (RS)*R are the same.

5. Prove the following:

Prove that the following language L over the alphabet {a, b, c} is not regular.
L = { wcx : w, x ∈ {a, b}* and the number of a's in w is equal to the number of b's in x.}

For example, the word abababcbbb is in L.

6. Use pumping lemma to prove the language is not regular.

$$L = \{a^{n!} : n \geq 0\}$$