

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: 24-08-2017

Time: 60 min

Full marks: 50

No. of students: 70

Autumn Semester Class Test 1, 2017

Dept: *Comp. Sc & Engg.*

Sub No: CS60005

M.Tech (Core)

Sub Name: **Foundations of Computing Science**

Instructions: *Answer all questions.*

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1. For each of the following, **only** answer True/False: **[5 x 2 marks]**
- (a) The intersection of two context free languages is context free.
  - (b) For an NFA that recognizes a language  $L$ , an NFA for the complement language can be obtained by swapping the initial and final states of the NFA.
  - (c) Every subset of a regular language is regular.
  - (d) The intersection of a regular and context free language is context free.
  - (e) For a regular language with a finite number of strings, no string in the language will be able to be chosen as a string for the pumping lemma.
2. Draw a DFA for the language of strings representing binary sums. A string of the language is represented as a series of triplets  $a_i b_i s_i$ , where  $s_i$  is the sum bit, and  $a_i$  and  $b_i$  are the bits being added, delivered to the DFA from LSB (bit 0) to MSB (bit  $N$ ). An example of a valid string representing  $0100 + 0101 = 1001$  would be  $011\ 000\ 110\ 001$  (spaces inserted only for readability), to be interpreted as  $0+1=1$ ,  $0+0=0$ ,  $1+1=0$  (1),  $0+0+(1)=1$ . An invalid string would be one representing an incomplete sum (like  $011000110$ ,  $01100$ ) or an incorrect sum (like  $011111001001$ ). Have as few states as you can, or the DFA will look messy. *[Hint: A bit sum produces a carry or does not – yielding two clusters of states in the DFA]*  
**[15 marks]**
3. A cassette tape reader/recorder head has two moves, namely going forward one tape cell (R) and going back by one tape cell (L). A string RLRRR represents a sequence of moves of the tape head. The tape head is initially at the beginning of the tape. A string of moves which requires the tape to move left of the beginning of the tape is an invalid string. For example, RLRL is invalid. **[10 + 10 + 5 marks]**
- (a) Consider the set of strings in which the tape has a finite number of cells,  $N$ . Is the language defined by legal moves over such a tape regular? If Yes, draw a DFA for  $N=5$ , else give a proof using the pumping lemma / closure properties for Regular Languages.
  - (b) Consider the set of all strings in which the tape is not finite and the head returns to the beginning of the tape. Show that this language is not regular.
  - (c) Give a context free grammar for the language in (b).