

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: .....FN / AN      Time: 2 hours      Full marks: 40      No. of students: 45  
Autumn Mid Semester Exams, 2011      Dept: Comp. Sc & Engg.      Sub No: CS60005  
M.Tech (Core)      Sub Name: **Foundations of Computing Science**  
Instructions:      Answer all questions

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1. [Context-free Languages]

- (a) State the *Pumping Lemma* for context-free languages.
- (b) Prove that the language,  $D = \{ ww \mid w \in \{0,1\}^* \}$ , is not context-free.
- (c) Suppose you are given a push down automaton with two stacks. The transition function for such an automaton is of the following form:

$$\delta : Q \times \Sigma_{\epsilon} \times \Gamma_{\epsilon}^1 \times \Gamma_{\epsilon}^2 \rightarrow P( Q \times \Gamma_{\epsilon}^1 \times \Gamma_{\epsilon}^2 )$$

In other words in a single state transition, the PDA can pop both stacks and also push into the top of both stacks. Give an informal description of how you may use a PDA with two stacks to decide the language  $D$  of Part (b). You do not have to draw the PDA, only write the broad steps.

- (d) Draw a PDA for the following language:

$$C = \{ u0v \mid u, v \in \{0,1\}^* \text{ and } u \text{ and } v \text{ have equal number of 1s} \}.$$

[ 3 + 5 + 4 + 6 = 18 marks ]

2. Let:  $\text{CONTEXT-FREE}_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is a context-free language} \}$

Show that  $\text{CONTEXT-FREE}_{\text{TM}}$  is undecidable by establishing a reduction from  $A_{\text{TM}}$ .

Recall that  $A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts } w \}$

[Hint: The reduction is very similar to the one for proving undecidability of  $\text{REGULAR}_{\text{TM}}$ ]

[ 10 marks ]

3. [Regular Languages]

- (a) Draw an NFA equivalent to the regular expression  $((00)^*(11))^*$
- (b) Prove that regular languages are closed under intersection, that is, if  $L_1$  and  $L_2$  are regular, then so is  $L_1 \cap L_2$ .

[ 5 + 7 = 12 marks ]