INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 70 No. of students: 54

Autumn End Semester Exams, 2010 Dept: *Comp. Sc & Engg.* Sub No: CS60007

M.Tech (Core) Sub Name: ***Foundations of Computing Science***

Instructions: *Part-A must be answered on the question paper itself.*

Part-B must be answered on the answer book.

Answer all parts of a question in the same place

PART- A (25 marks)

Answer all questions

Roll No: Name:

1. Indicate whether the following statements are True/False in the box provided with an accurate reason in the line provided. [5 x 2 = 10 marks]

(a) If languages $L_1 \in NP$ and $L_2 \in coNP$, then $L_1 \cap L_2 \in P$

(b) Even if $P \neq NP$, every NP-complete problem can be solved using polynomial space.

(c) The complement of a satisfiable Boolean formula is unsatisfiable.

(d) If *p* is the pumping length for a context free language, A, then every *k,* such that $k > p$, is also a pumping length for A

(e) A Turing decidable problem A can be reduced to problem B. Then B is Turing decidable.

2. State the following : [5 X 3 = 15 marks]

(a) Savitch’s Theorem

(b) Godel’s Completeness Theorem

(c) Cook-Levin Theorem

(d) Hilbert’s Tenth Problem

(e) Pumping Lemma for Context-Free Languages

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PART- B (45 marks)

Answer any three.

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3. [Computational Complexity] [4 + 6 + 5 = 15 marks]

(a) Define the NP and coNP classes of languages.

(b) Consider the following problems:

- k-CUT: Does a given graph, G, have an edge cut of size less than k?
- ISO: Are two given graphs G_1 and G_2 isomorphic to each other?
- TQBF: Is a given totally quantified Boolean formula true?
- EULER: Does a given graph, G, have an Euler circuit?
- EQ-REX: Given two regular expressions, R_1 and R_2, are they equivalent?

Now answer the following questions carefully with brief justification (keep in mind the hierarchy of complexity classes):

i. Which of the above problems are in P?
ii. Which of the above problems are in NP?
iii. Which of the above problems are in coNP?

(c) A *coloring* of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Finding whether a graph has a *coloring* with *k* colors is NP-complete for $k \geq 3$. Now consider the following problem.

> You are given a list of final examinations $F_1, \ldots, F_k$ to be scheduled, and a list of students $S_1, \ldots, S_n$. Each student is taking some specified subset of these exams. You must schedule these exams into slots so that no student is required to take two exams in the same slot. The problem is to determine if such a schedule exists that uses only *h* slots.

Prove that the above problem is NP-complete by using the fact that graph coloring is NP-complete. Give an example which clearly demonstrates the reduction.

4. [Logic] [3 + 5 + 7 = 15 marks]

(a) Indicate which of the following first-order logic formulas are valid.

i. $\forall x\ \exists y\ P(x,y) \Rightarrow \exists y\ P(z,y)$

ii. $\forall x\ \exists y\ P(x,y) \Rightarrow \exists y\ P(y,y)$

iii. $\forall x\ \forall y\ \forall z\ [\ P(x,x) \wedge [\ P(x,z) \Rightarrow (\ P(x,y) \vee P(y,z)\ )\ ]\ ] \Rightarrow \exists y\ \forall z\ P(y,z)$

(b) The law says that it is a crime for Gauls to sell potion formulas to hostile nations. The country Rome, an enemy of Gaul, has acquired some potion formulas, and all of its formulas were sold to it by the druid, Traitorix. Traitorix is a Gaul. Write these formulas in first-order logic and prove that Traitorix is a criminal using resolution refutation.

(c) Prove that the validity problem of first-order logic is undecidable.

5. [Computability] [4 + 4 + 7 = 15 marks]

(a) In the *silly Post Correspondence Problem, SPCP,* in each pair the top string has the same length as the bottom string. Show that the SPCP problem is decidable.

(b) Suppose A and B are Turing recognizable languages. Which of these are guaranteed to be Turing recognizable? Give a brief justification for each.

i. Union of A and B

ii. Complement of A

iii. Concatenation of A and B

iv. Intersection of A and B

(c) Prove that the language $A_{TM} = \{ \langle M, w\rangle \mid M$ is a Turing Machine and M accepts $w \}$ is undecidable.

6. [Miscellaneous] [5 + 5 + 5 = 15 marks]

(a) Draw a DFA for the language, $A = \{ w \mid w$ is any string not in $(ab^+)^* \}$ [Hint: *The DFA for the complement of A is easy.*]

(b) $D_A = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $D_B = (Q_2, \Sigma, \delta_2, q_2, F_2)$ are the DFAs for accepting the regular languages, A and B. Let the perfect shuffle of A and B be the language

$$Z = \{ w \mid w = a_1b_1 \ldots a_kb_k, \text{ where } a_1 \ldots a_k \in A \text{ and } b_1 \ldots b_k \in B, \text{ each } a_i, b_i \in \Sigma \}.$$

It is known that the class of regular languages is closed under perfect shuffle. Give the definition of the DFA, $D_Z = (Q, \Sigma, \delta, q, F)$, which accepts Z. You should clearly indicate how Q, δ, q, and F are constructed from Q_1, δ_1, q_1, F_1 and Q_2, δ_2, q_2, F_2. [Hint: *It may be useful to include an additional state variable to indicate whether it is A's turn or B's turn.*]

(c) Give the formal definition of a *pushdown automaton*, $P = (Q, \Sigma, \Gamma, \delta, q_0, \{q_{accept}\})$. Given the definition of P, how can we construct the rules of the context free grammar G which accepts the same language as P?