INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……*FN / AN* Time: *2 hours* Full marks: 50 No. of students: 66

Autumn Mid Semester Exams, 2010 Dept: *Comp. Sc & Engg.* Sub No: CS60007

M.Tech (Core) Sub Name: ***Foundations of Computing Science***

Instructions: *Answer all four questions*

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1. (a) Give the formal definition of a *countable set*. Show that the set of odd natural numbers is countable.

   (b) Show that the power set, $2^S$, of a countably infinite set, S, is uncountable.

   [Hint: *Develop a correspondence with infinite binary sequences.*]

   [ 5 + 6 = 11 marks]

2. Consider the following languages over $\Sigma$ = {a, b, c}:

   $L_1 = \{ a^n \mid n \geq 0 \}$

   $L_2 = \{ a^n b^n \mid n \geq 0 \}$

   $L_3 = \{ a^n b^n c^n \mid n \geq 0 \}$

   (a) Draw the hierarchy of languages showing the regular, context-free, Turing decidable, and Turing recognizable classes, and place $L_1$, $L_2$, $L_3$ in the appropriate classes.

   (b) Design an appropriate acceptor (DFA/NFA/PDA etc) for each language.

   [ 5 + 10 = 15 marks ]

3. (a) Give the formal definition of a *context free grammar*

   (b) Consider the following CFG:

   S → LSR | SS | ∈

   L → ( | {

   R → ) | }

   Explain why this is not a correct grammar for matching brackets. Note that we have used two types of brackets, namely the first brackets, ( ), and the curly brackets, { }.

   (c) Modify the CFG of part (b) to develop a correct grammar for matching brackets.

   (d) Draw a Push-down automaton (PDA) from the grammar for accepting the same language.

   [ 2 + 2 + 3 + 5 = 12 marks ]

4. For each of the following statements, indicate whether the statement is True or False. For the True-ones give a brief justification (you can use known results), and for the False-ones give a counter-example. Your answer must begin with True/False.
   (a) $L_2$ is a given regular language. $L_3$ is a language such that each string $w \in L_3$ also belongs to $L_2$. Then $L_3$ is also a regular language.
   (b) The complement of a Turing-recognizable language cannot be Turing-recognizable.
   (c) There exists Turing-machines for deciding every regular language, but it is not always possible to decide whether the language accepted by a given Turing machine is regular.

[ 4 + 4 + 4 = 12 marks ]