INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……*FN / AN* Time: *3 hrs* Full marks: 100 No. of students: 41

Autumn End Semester Exams, 2012 Dept: *Comp. Sc & Engg.* Sub No: CS60005

M.Tech (Core) Sub Name: ***Foundations of Computing Science***

Instructions: *Part-A must be answered on the question paper itself.*

Part-B must be answered on the answer book.

Answer all parts of a question in the same place

PART- A (36 marks) Answer all questions in this part

Roll No: Name:

1. Indicate whether the following statements are True/False/Unknown in the box provided with an accurate reason in the line provided. [10 x 3 = 30 marks]

(a) NP is closed under union

(b) NP is closed under complementation

(c) If language $L \in NP$, and $L_1 \subseteq L$, then $L_1 \in NP$

(d) All problems in NP can be solved using a *deterministic* Turing machine in *polynomial space*

(e) The set of positive rational numbers is uncountable

(f) If NP ≠ coNP then P ≠ NP

(g) Deterministic PDAs recognize fewer languages than non-deterministic PDAs

(h) Deterministic Turing machines recognize fewer languages than non-deterministic Turing machines

(i) A propositional logic formula is either valid or unsatisfiable

(j) It is not possible to develop any algorithm for deciding in general whether a post correspondence problem has a solution

2. Draw the polynomial hierarchy of languages inside the box.
[6 marks]

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PART- B (64 marks) Answer any four from this part

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3. [Mapping Reduction]

(a) Define the 3-SAT problem.

(b) The k-SAT problem is a generalization of the 3-SAT problem where each clause has exactly k literals. Describe a polynomial time mapping reduction from the k-SAT problem to the 3-SAT problem.

[Hint: ($x_1 \vee \ldots \vee x_i \vee x_{i+1} \vee \ldots \vee x_k$) can be rewritten as

($x_1 \vee \ldots \vee x_i \vee z$) $\wedge$ ($x_{i+1} \vee \ldots \vee x_k \vee \neg z$) where z is a new variable]

(c) Estimate the length of the 3-SAT formula corresponding to a k-SAT formula having *m* clauses. How many new variables does it have?

[3+6+7 = 16 marks]

4. [Complexity] In mobile telephony, the frequency allocation problem is stated as follows. There are a number of transmitters deployed and each of them can transmit on any of a given set of frequencies. Different transmitters have different frequency sets. Some transmitters are so close that they cannot transmit at the same frequency, because then they would interfere with each other.

You are given the frequency range of each transmitter and the pairs of transmitters that can interfere if they use the same frequency. The problem is to determine if there is any possible choice of frequencies so that no transmitter interferes with any other.

(a) Show that the frequency allocation problem is in NP

(b) Given that the Graph Coloring problem is known to be NP-complete, show that the frequency allocation problem is NP-complete. Indicate which problem is being reduced to which problem and clearly show the steps in reducing one problem to the other

(c) Consider the following variant of the frequency allocation problem: Is the minimum number of frequencies required to get a proper allocation exactly equal to k? What can you say about the complexity of this problem?

[3+10+3 = 16 marks]

5. [Logic]

(a) Define the *resolution rule* for first order logic

(b) What is the meaning of the statement? – *Resolution is a sound and complete inference rule for first order logic.* Answer this question in the light of the known fact that the validity of a statement in first order logic is not decidable in general.

(c) Find the most general unifier for the following set of predicates:

$$\{ p(a, X, h(g(Z))), p(Z, h(Y), h(Y)) \}$$

(d) *Anyone is unfortunate who bears the same name as a person who commits a crime. A burglary is a crime*.

i. Encode the above statements in first-order logic

ii. Use resolution refutation to prove the goal: *Anyone who commits a burglary is unfortunate*.

[2+2+2+10 = 16 marks]

6. [Miscellaneous]

(a) Define a Turing machine as a 7-tuple $\langle Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject} \rangle$. Explain each element of this 7-tuple

(b) What is the input to a Turing machine? When does a Turing machine accept its input? Define a single step in the computation of a Turing machine

(c) A state of the Turing Machine is said to be useless if it is never entered on any input string. Prove that the following language L is undecidable:

$$L = \{ \langle M, q \rangle \mid M \text{ is a TM that has a useless state } q \}$$

(d) State Savitch's theorem

[3+3+8+2 = 16 marks]

7. [Miscellaneous]

(a) If $M_1 = \langle Q_1, \Sigma, \delta_1, q_1, F_1 \rangle$ and $M_2 = \langle Q_2, \Sigma, \delta_2, q_2, F_2 \rangle$ are two DFAs accepting languages L_1 and L_2 respectively, then define the DFA, $M = \langle Q, \Sigma, \delta, q, F \rangle$ for accepting the language $L_1 \cap L_2$

(b) Define Chomsky Normal Form

(c) If G is a CFG in Chomsky Normal Form, and $w \in L(G)$ is a string of length $n \geq 1$, then how many steps are required for any derivation of w? Justify your answer in brief

(d) Suppose $A \leq_m B$, that is language A is mapping reducible to language B. Answer the following questions with a brief justification

i. If A is decidable, then what can we say about the decidability of B?

ii. If A is undecidable, then what can we say about the decidability of B?

[4+2+4+6 = 16 marks]