FOCS-Autumn 2017-Class Test 2 Solution Outline

1. (a) TRUE

If $\mathcal{L} \in P$ then $\exists M$, a poly-time deterministic TM to <u>decide</u> $\mathcal{L}$.

$\Rightarrow$ Every DTM is also an NTM.

$\Rightarrow$ To verify $w \in \mathcal{L}$ we can use M
To verify $w \in \bar{\mathcal{L}}$ we can also use M | Since M is det. poly time. The Yes answer can be verified using M in P.

(b) TRUE

$\forall\, w_1 \in \mathcal{L}_1$ and $w_2 \in \mathcal{L}_2$, $\exists\, M_1$ to decide $w_1 \in \mathcal{L}_1$ and $\exists\, M_2$ to decide $w_2 \in \mathcal{L}_2$, where $\mathcal{L}_1, \mathcal{L}_2 \in$ TIME

If $f_1(w_1)$ and $f_2(w_2)$ are complexities of M_1, M_2 then computing $w \in \mathcal{L}_1 \cap \mathcal{L}_2$ can be achieved as follows:

Run w_1 on M_1 — $f_1(w_1)$
w_2 on M_2 — $f_2(w)$

If both return a Yes, return a Yes else return a No. — $O(1)$

Total time $\underbrace{f_1(w)}_{\text{Polytime}} + \underbrace{f_2(w)}_{\text{Polytime}}$

$\therefore\ \mathcal{L}_1 \cap \mathcal{L}_2 \in P$

(c) False.

SAT $\in$ NP-Complete

Graph coloring for interval graphs $\in P$.

$GCI \leq_p SAT$ For k colors, for each vertex $v_i \in G$, $j \in \{1, 2, \ldots, k\}$

Create propositions c_{ij}

Constraints: (a) $(c_{ij} \wedge \overline{c_{hj}}) \vee (\overline{c_{ij}} \wedge c_{hj}) \vee (\overline{c_{ij}} \wedge \overline{c_{hj}})$ for $(v_i, v_h) \in E$

(b) For each vertex a color is assigned, and only one color is given

(d) TRUE

$B \in$ NP-Complete
$A \in P$
$B \leq_p A$

Hence a polytime function $f(w)$ exists transforming an instance $w \in B$ into $f(w) \in A$, solved using a solver for A in polytime. Hence $B \in P$

Since $B \in P$, now all other problems in NP reduce to B and therefore to A.

$\therefore NP = P$.

(e) TRUE

An $O(f(n))$ space NTM can be simulated by an $O(f^2(n))$ space DTM, where n is the input size

Problems in NP take polytime to solve. In polytime $g(n)$, space used (cells of the TM) must be $f(n) \leq g(n)$, since the TM cannot have visited more cells than $g(n)$.

$\therefore$ NP $\subseteq$ PSPACE

2. (a) SAT $\in$ NP

   (b) VALIDITY $\in$ CO-NP

   (c) k-CUT $\in$ P

   (d) Non-Validity $\in$ NP

   (e) k-REGALLOC $\in$ P

3. (a) SET-COVER $\in$ NP

Certificate is a cover C.
Compute union of sets in C (size of C) | Polytime. Verifier.
Check in union is the universe (size of U)

Compute C Non-Deterministically.
For each $S_i \in S$, non-deterministically decide if $S_i \in C$ or not. | Size of S

Reduction VERTEX-COVER $\leq_p$ SET-COVER

Consider graph $G = (V, E)$

Vertex set Edge Set.

For each edge $e_i \in E$ construct U as follows

$$\mathbf{U} = \{e_i \mid e_i \in E\}$$

Construct S as follows.

For each vertex $v_j \in V$, V_j is added to S, where V_j contains all edges incident on vertex $v_j \in V$

PART-1 · VERTEX COVER has a solution if the instance of SET COVER has a solution

Consider a k-sized set cover C.

Each subset in C corresponds to a unique vertex in the Graph.

i.e V_j corresponds to v_j.

Choose v_j to be in the vertex cover, if $V_j \in C$

$$\bigcup_{V \in C} V = \mathbf{U}$$ the universal set of all edges.

$\therefore$ By choosing v_j in the vertex cover all edges are covered.

PART 2 : If VERTEX-COVER has a solution then SET-COVER has a sol^n.

Consider a k-sized VC and for each vertex v_j in the vertex cover pick the subset V_j and add V_j to C. Since all edge are covered by the VC, the union of subsets in C cover all objects in $\mathbf{U}$.

But VERTEX-COVER $\in$ NP-Complete. $\therefore$ SET-COVER $\in$ NP-Complete.

(b) The problem is in Difference Polynomial.

$k\text{-}SC = \{\langle U, S, k\rangle \mid U$ has a cover of size k from ~~subsets~~ ~~of~~ in $S\}$

$\overline{k\text{-}SC} = \{\langle U, S, k\rangle \mid U$ has no cover of size $k-1$ from ~~sub~~sets in $S\}$

$k\text{-}MIN\text{-}SC = k\text{-}SC \cap \overline{k\text{-}SC}$