Odd Cycle Transversal on P_5-free Graphs in Polynomial Time

Recent Trends in Algorithms, 2024

Indian Association of the Cultivation of Sciences, Kolkata

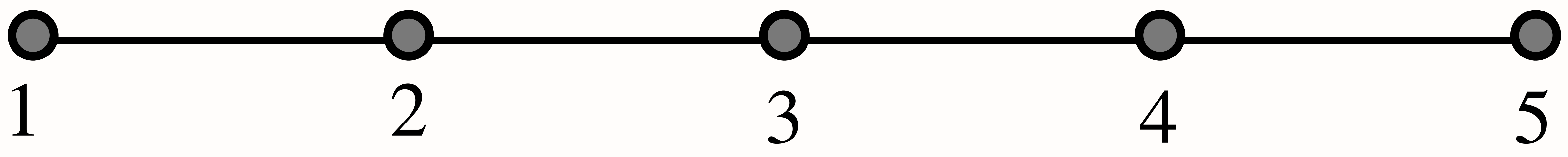
Akanksha Agrawal

(Joint work with P. Lima, D. Lokshtanov, P. Rzążewski, S. Saurabh, R. Sharma)

Graph P_i

For $i \in \mathbb{N}$, P_i is the graph with:

- $V(P_i) = \{1, 2, \cdots, i\}$
- $E(P_i) = \{\{j, j+1\} \mid j \in \{1, 2, \cdots, i-1\}\}$



P_5

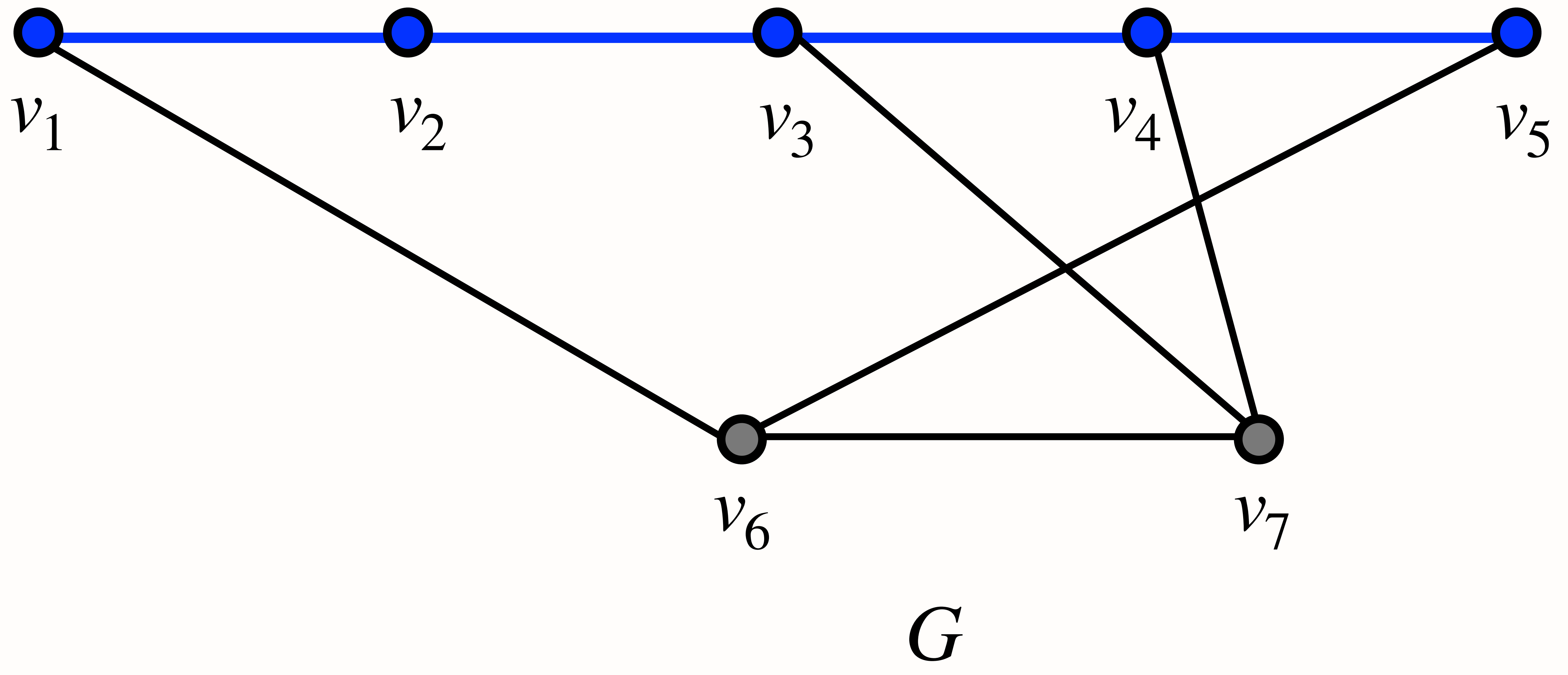
H-free Graphs

A graph $G = (V, E)$ is ***H-free*** if G does not have H as an induced subgraph, i.e., there is no $S \subseteq V(G)$ and a bijective function $\phi : V(H) \to S$ such that for $h, h' \in V(H)$, $\{h, h'\} \in E(H)$ if and only if $\{\phi(h), \phi(h')\} \in E(G)$.

H-free Graphs

A graph $G = (V, E)$ is ***H*-free** if G does not have H as an induced subgraph, i.e., there is no $S \subseteq V(G)$ and a bijective function $\phi : V(H) \to S$ such that for $h, h' \in V(H)$, $\{h, h'\} \in E(H)$ if and only if $\{\phi(h), \phi(h')\} \in E(G)$.

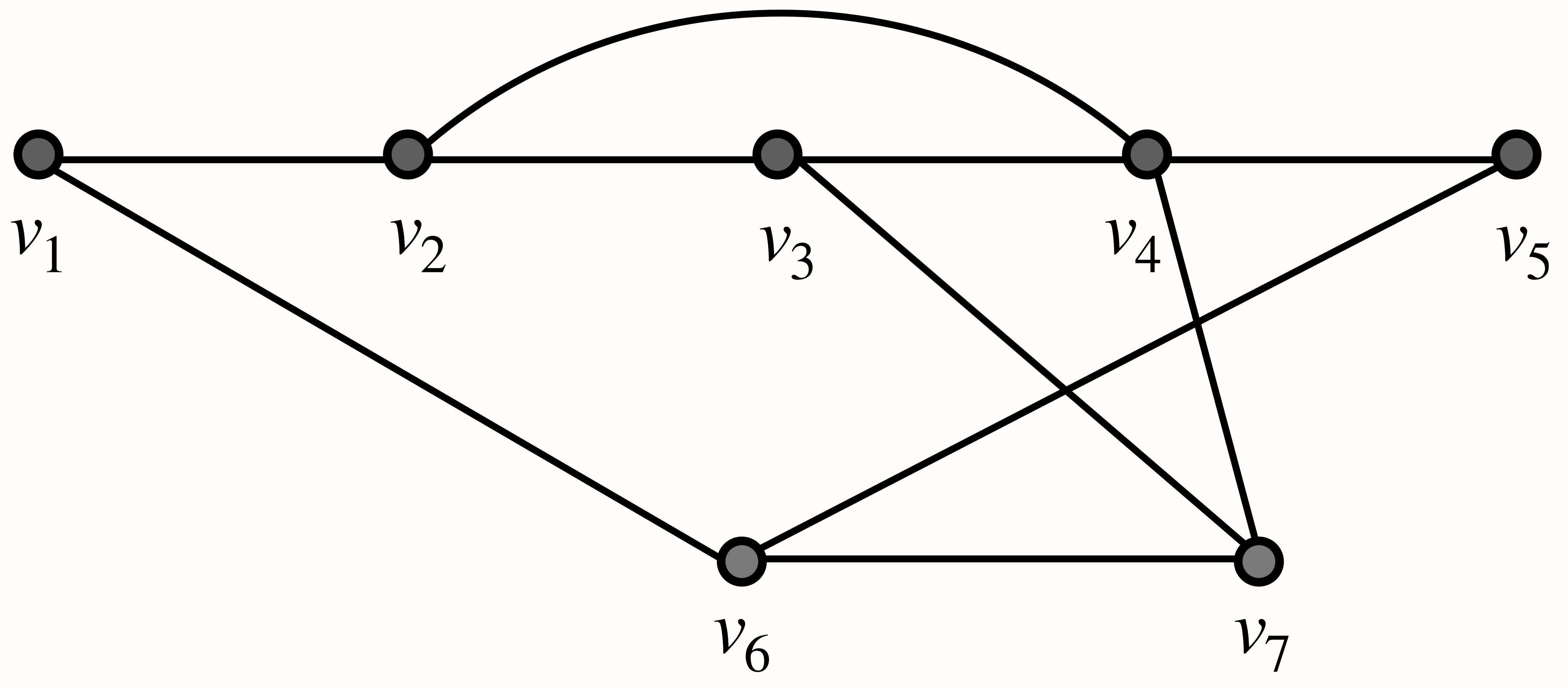
NOT P_5-free



H-free Graphs

A graph $G = (V, E)$ is ***H-free*** if G does not have H as an induced subgraph, i.e., there is no $S \subseteq V(G)$ and a bijective function $\phi : V(H) \rightarrow S$ such that for $h, h' \in V(H)$, $\{h, h'\} \in E(H)$ if and only if $\{\phi(h), \phi(h')\} \in E(G)$.

P_5-free



G

H-free Graphs

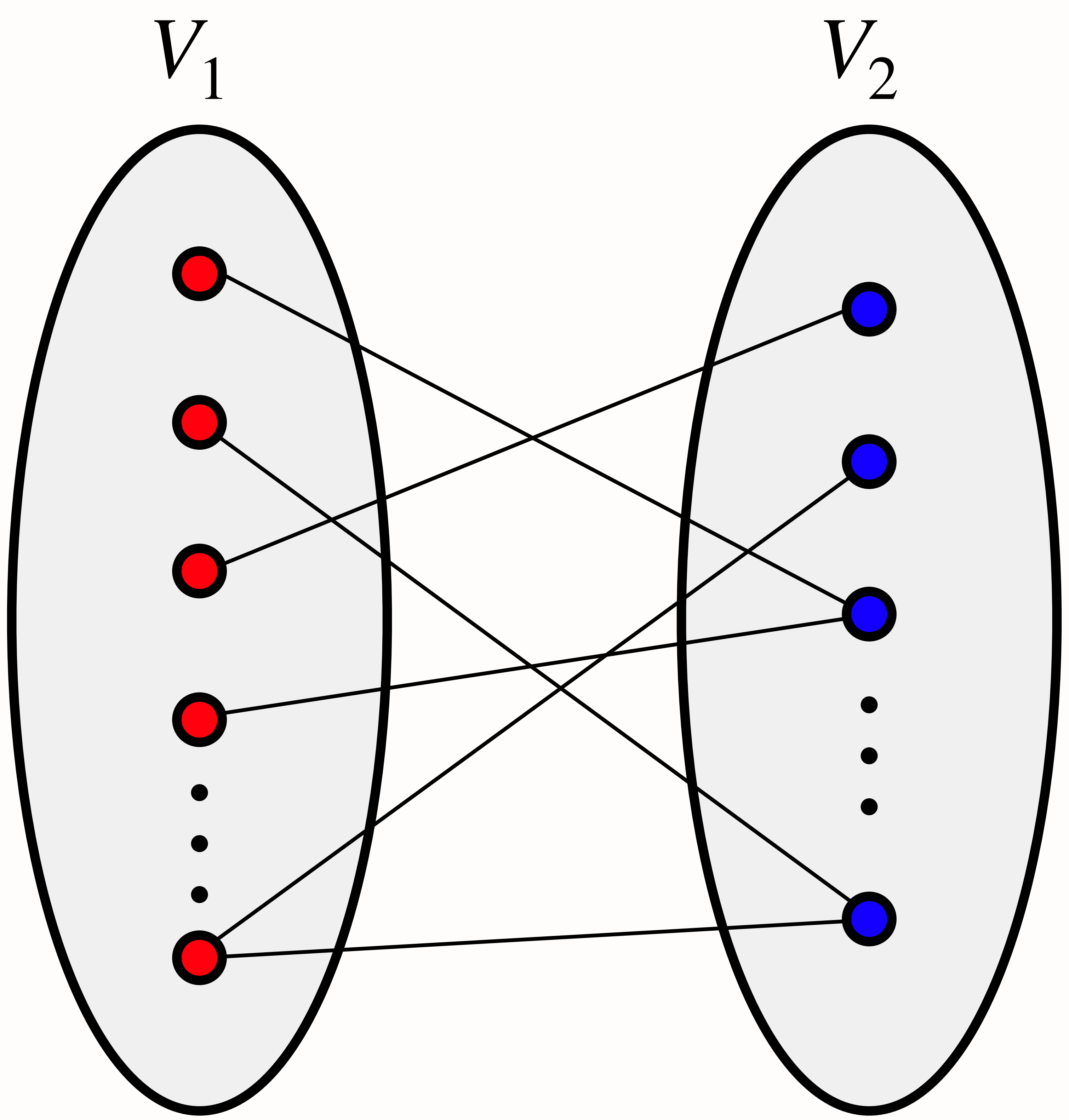
A graph $G = (V, E)$ is ***H*-free** if G does not have H as an induced subgraph, i.e., there is no $S \subseteq V(G)$ and a bijective function $\phi : V(H) \to S$ such that for $h, h' \in V(H)$, $\{h, h'\} \in E(H)$ if and only if $\{\phi(h), \phi(h')\} \in E(G)$.

P_3-free graphs are disjoint union of cliques!

Bipartite Graphs

A graph $G = (V, E)$ is ***bipartite*** if V can be partitioned into two sets, V_1 and V_2, s.t., for each $\{u, v\} \in E(G)$,

- $u \in V_1$ and $v \in V_2$, or
- $v \in V_1$ and $u \in V_2$

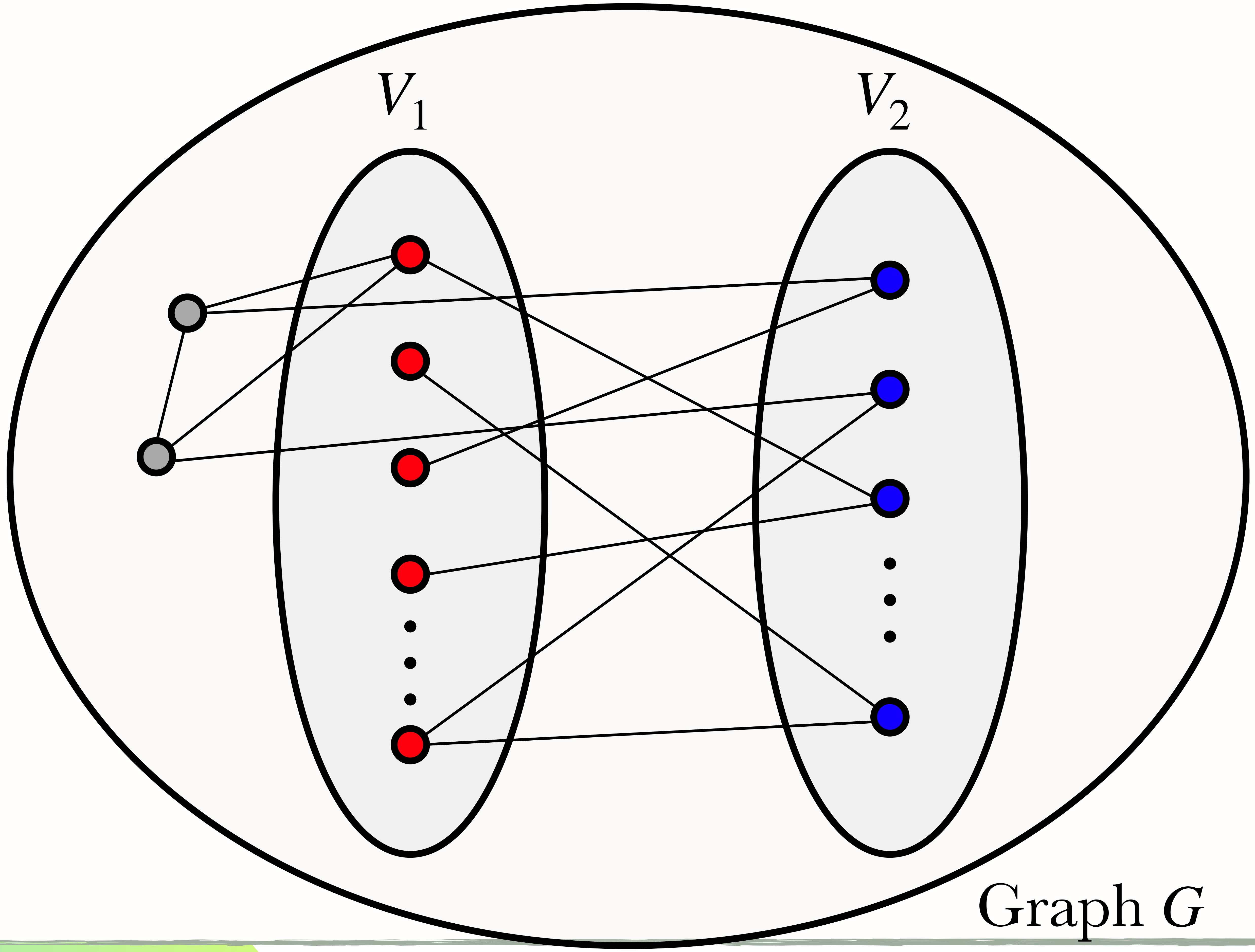


Graph $G = (V, E)$

Odd Cycle Transversal

Input: A graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$

Task: Find a maximum weight induced bipartite subgraph of G, i.e., find $S \subseteq V(G)$ of maximum total weight such that $G[S]$ is bipartite.



Some Relevant Results

- [Chiarelli et al.] Odd Cycle Transversal is NP-hard on graph classes line graphs and graphs of small girth.

  —*Thus, also* NP-hard *on graphs with no claws or a fixed length cycle.*

Some Relevant Results

- [Chiarelli et al.] Odd Cycle Transversal is NP-hard on graph classes line graphs and graphs of small girth.

  —*Thus, also* NP-hard *on graphs with no claws or a fixed length cycle.*

- [Chiarelli et al.] The problem is polynomial time solvable on H-free graphs where H is the graph with a matching, or $(sP_1 + P_3)$.

Some Relevant Results

- [Chiarelli et al.] Odd Cycle Transversal is NP-hard on graph classes line graphs and graphs of small girth.

  —*Thus, also* NP-hard *on graphs with no claws or a fixed length cycle.*

- [Chiarelli et al.] The problem is polynomial time solvable on H-free graphs where H is the graph with a matching, or $(sP_1 + P_3)$.

- [Okrasa et al.] On P_4-free graphs the problem is polynomial time solvable.

Some Relevant Results

- [Chiarelli et al.] Odd Cycle Transversal is NP-hard on graph classes line graphs and graphs of small girth.
  —*Thus, also* NP-hard *on graphs with no claws or a fixed length cycle.*

- [Chiarelli et al.] The problem is polynomial time solvable on H-free graphs where H is the graph with a matching, or $(sP_1 + P_3)$.

- [Okrasa et al.] On P_4-free graphs the problem is polynomial time solvable.

- [Dabrowski et al.] On P_6-free graphs the problem is NP-hard.

Some Relevant Results

- [Chiarelli et al.] Odd Cycle Transversal is NP-hard on graph classes line graphs and graphs of small girth.
  —*Thus, also* NP-hard *on graphs with no claws or a fixed length cycle.*
- [Chiarelli et al.] The problem is polynomial time solvable on H-free graphs where H is the graph with a matching, or $(sP_1 + P_3)$.
- [Okrasa et al.] On P_4-free graphs the problem is polynomial time solvable.
- [Dabrowski et al.] On P_6-free graphs the problem is NP-hard.
- The above results imply (prior to our work), the only connected graph H for which the status of Odd Cycle Transversal on H-free graphs was unknown was $H = P_5$.

Our Result

THEOREM

Odd Cycle Transversal admits a polynomial time algorithm on P_5-free graphs.

Two Ingredients

THEOREM

Odd Cycle Transversal admits a polynomial time algorithm on P_5-free graphs.

Ingredient 1: A polynomial-sized ***solution covering family***.

Ingredient 2: Translating solution to finding ***independent sets on a P_5-free auxiliary graph*** over a solution covering family.

Two Ingredients

THEOREM

Odd Cycle Transversal admits a polynomial time algorithm on P_5-free graphs.

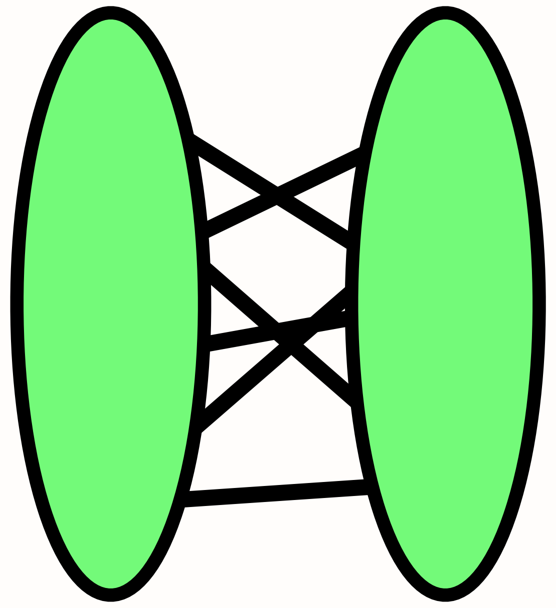
???

Ingredient 1: A polynomial-sized ***solution covering family***.

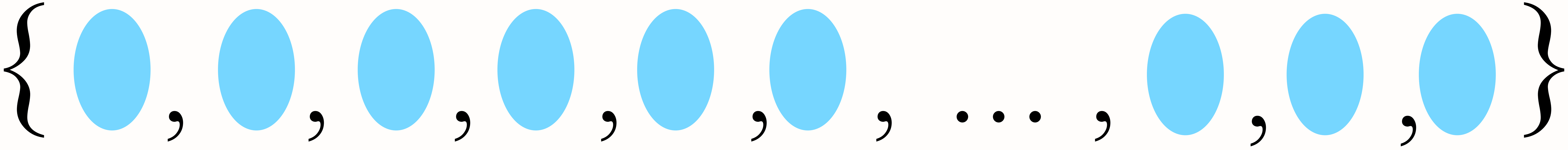
Ingredient 2: Translating solution to finding ***independent sets on a P_5-free auxiliary graph*** over a solution covering family.

Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

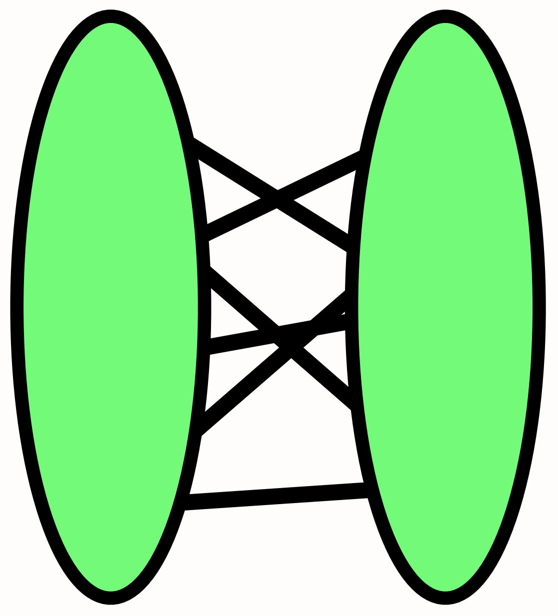


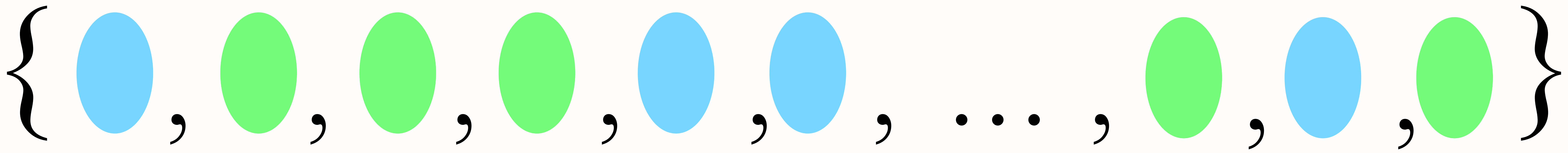
$G[S]$ is bipartite



Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:





Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$

Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$
- Sets in $\mathscr{C}'$ are pairwise disjoint

Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$
- Sets in $\mathscr{C}'$ are pairwise disjoint
- No edge between different sets in $\mathscr{C}'$, i.e., for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$
- Sets in $\mathscr{C}'$ are pairwise disjoint
- No edge between different sets in $\mathscr{C}'$, i.e., for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \varnothing$

LEMMA

Given a P_5-free graph G on n vertices and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, there is a polynomial-time algorithm that outputs a solution covering family of size $O(n^6)$.

Two Ingredients

THEOREM

Odd Cycle Transversal admits a polynomial time algorithm on P_5-free graphs.

Ingredient 1: A polynomial-sized ***solution covering family***.

Ingredient 2: Translating solution to finding ***independent sets on a P_5-free auxiliary graph*** over a solution covering family.

Extracting Solution From Solution Covering Family

RECALL

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$,
- Sets in $\mathscr{C}'$ are pairwise disjoint
- for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

Extracting Solution From Solution Covering Family

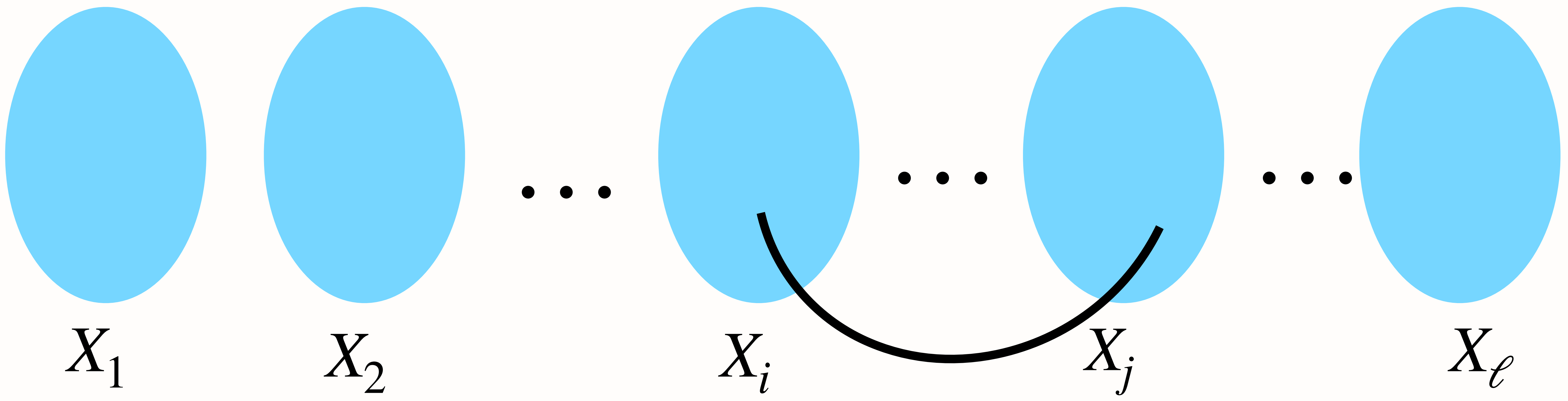
RECALL

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$,
- Sets in $\mathscr{C}'$ are pairwise disjoint
- for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

Consider a P_5-free graph G and $\mathsf{w} : V(G) \to \mathbb{Q}$ and its ***solution covering family*** $\mathscr{C} = \{X_1, X_2, \cdots X_\ell\}$.

Extracting Solution From Solution Covering Family



Add the edge if and only if
$X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$

RECALL

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is $\mathscr{C}' \subseteq \mathscr{C}$ such that:

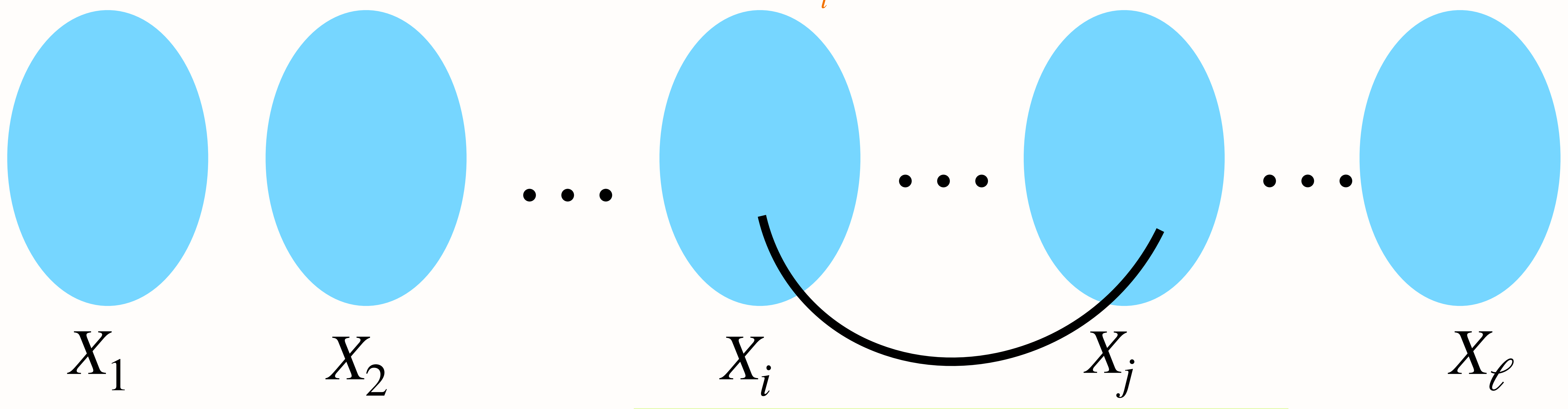
- $S = \cup_{X \in \mathscr{C}'} X$,
- Sets in $\mathscr{C}'$ are pairwise disjoint
- for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

Consider a P_5-free graph G and $\mathsf{w} : V(G) \to \mathbb{Q}$ and its ***solution covering family*** $\mathscr{C} = \{X_1, X_2, \cdots X_\ell\}$.

Create a graph $H_{\mathscr{C}}$ with $V(H_{\mathscr{C}}) = \mathscr{C}$, where, for distinct $X_i, X_j \in \mathscr{C}$, add $\{X_i, X_j\}$ to $E(H_{\mathscr{C}})$ if and only if:
$X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$.

Extracting Solution From Solution Covering Family

$\mathsf{q}(X_i) = \sum_{v \in X_i} \mathsf{w}(v)$



Add the edge if and only if
$X_i \cap X_j \neq \emptyset$ OR $E(X_i, X_j) \neq \emptyset$

RECALL

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is $\mathscr{C}' \subseteq \mathscr{C}$ such that:

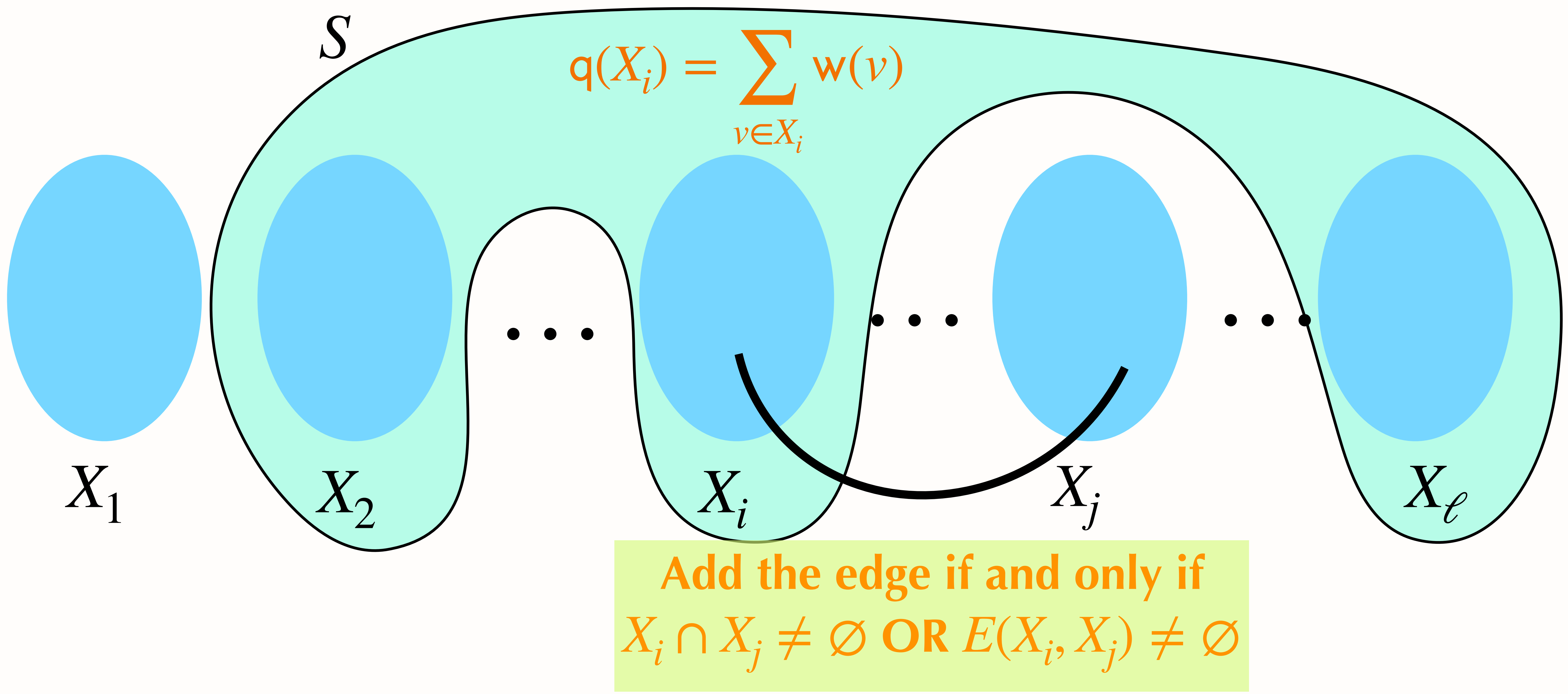
- $S = \cup_{X \in \mathscr{C}'} X$,
- Sets in $\mathscr{C}'$ are pairwise disjoint
- for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

Consider a P_5-free graph G and $\mathsf{w} : V(G) \to \mathbb{Q}$ and its ***solution covering family*** $\mathscr{C} = \{X_1, X_2, \cdots X_\ell\}$.

Create a graph $H_{\mathscr{C}}$ with $V(H_{\mathscr{C}}) = \mathscr{C}$, where, for distinct $X_i, X_j \in \mathscr{C}$, add $\{X_i, X_j\}$ to $E(H_{\mathscr{C}})$ if and only if:
$X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$.

Create a weight function $\mathsf{q} : V(H_{\mathscr{C}}) \to \mathbb{Q}$, where for $X \in \mathscr{C}$, $\mathsf{q}(X) = \sum_{v \in X} \mathsf{w}(v)$.

Extracting Solution From Solution Covering Family



RECALL

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is $\mathscr{C}' \subseteq \mathscr{C}$ such that:

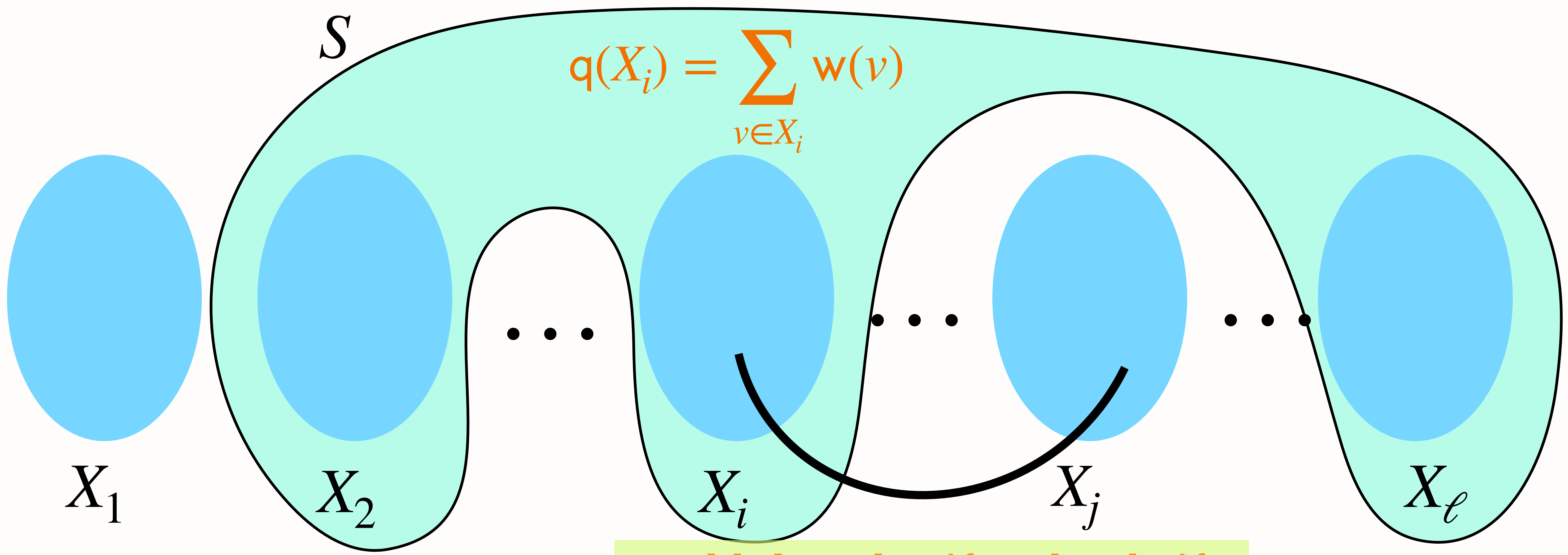
- $S = \cup_{X \in \mathscr{C}'} X$,
- Sets in $\mathscr{C}'$ are pairwise disjoint
- for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

Consider a P_5-free graph G and $\mathsf{w} : V(G) \to \mathbb{Q}$ and its ***solution covering family*** $\mathscr{C} = \{X_1, X_2, \cdots X_\ell\}$.

Create a graph $H_{\mathscr{C}}$ with $V(H_{\mathscr{C}}) = \mathscr{C}$, where, for distinct $X_i, X_j \in \mathscr{C}$, add $\{X_i, X_j\}$ to $E(H_{\mathscr{C}})$ if and only if: $X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$.

Create a weight function $\mathsf{q} : V(H_{\mathscr{C}}) \to \mathbb{Q}$, where for $X \in \mathscr{C}$, $\mathsf{q}(X) = \sum_{v \in X} \mathsf{w}(v)$.

Extracting Solution From Solution Covering Family



Add the edge if and only if
$X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$

RECALL

For a graph G and a weight function $w : V(G) \rightarrow \mathbb{Q}$, a family $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$,
- Sets in $\mathscr{C}'$ are pairwise disjoint
- for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

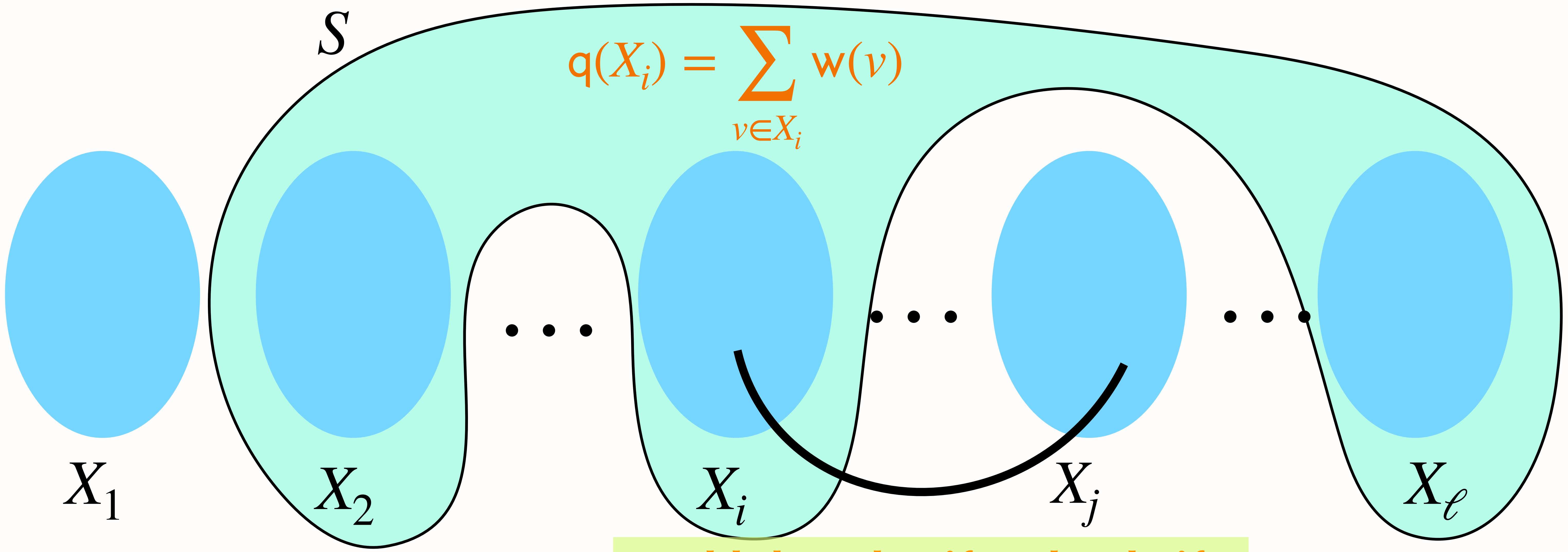
For $\mathscr{C}' \subseteq \mathscr{C}$, $S = \cup_{X \in \mathscr{C}'} X$ induces bipartite graph in G of weight t if and only if $\mathscr{C}'$ is a independent set in $H_{\mathscr{C}}$ of weight t.

Consider a P_5-free graph G and $w : V(G) \rightarrow \mathbb{Q}$ and its ***solution covering family*** $\mathscr{C} = \{X_1, X_2, \cdots X_\ell\}$.

Create a graph $H_{\mathscr{C}}$ with $V(H_{\mathscr{C}}) = \mathscr{C}$, where, for distinct $X_i, X_j \in \mathscr{C}$, add $\{X_i, X_j\}$ to $E(H_{\mathscr{C}})$ if and only if:
$X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$.

Create a weight function $q : V(H_{\mathscr{C}}) \rightarrow \mathbb{Q}$, where for $X \in \mathscr{C}$, $q(X) = \sum_{v \in X} w(v)$.

Extracting Solution From Solution Covering Family



Add the edge if and only if
$X_i \cap X_j \neq \emptyset$ OR $E(X_i, X_j) \neq \emptyset$

How do we find max. wt. independent set in $H_{\mathscr{C}}$?

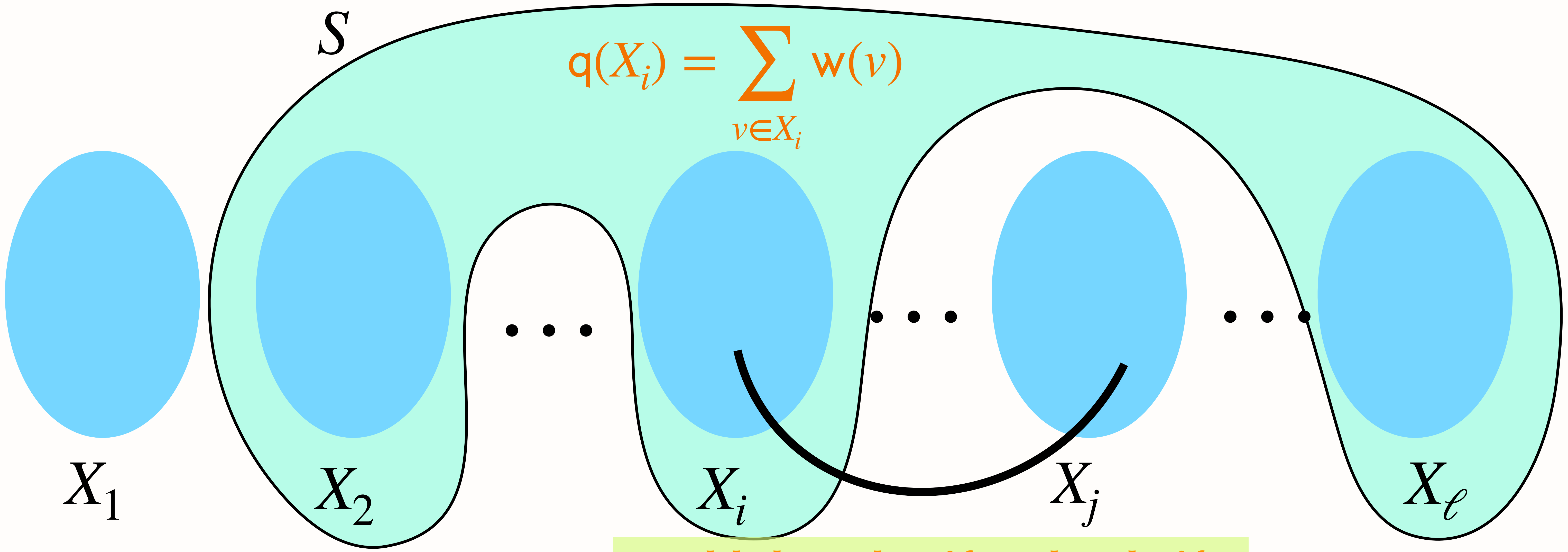
For $\mathscr{C}' \subseteq \mathscr{C}$, $S = \cup_{X \in \mathscr{C}'} X$ induces bipartite graph in G of weight t if and only if $\mathscr{C}'$ is a independent set in $H_{\mathscr{C}}$ of weight t.

Consider a P_5-free graph G and $\mathsf{w} : V(G) \to \mathbb{Q}$ and its ***solution covering family*** $\mathscr{C} = \{X_1, X_2, \cdots X_\ell\}$.

Create a graph $H_{\mathscr{C}}$ with $V(H_{\mathscr{C}}) = \mathscr{C}$, where, for distinct $X_i, X_j \in \mathscr{C}$, add $\{X_i, X_j\}$ to $E(H_{\mathscr{C}})$ if and only if:
$X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$.

Create a weight function $\mathsf{q} : V(H_{\mathscr{C}}) \to \mathbb{Q}$, where for $X \in \mathscr{C}$, $\mathsf{q}(X) = \sum_{v \in X} \mathsf{w}(v)$.

Extracting Solution From Solution Covering Family



Add the edge if and only if
$X_i \cap X_j \neq \emptyset$ OR $E(X_i, X_j) \neq \emptyset$

How do we find max. wt. independent set in $H_{\mathscr{C}}$?

- G is P_5-free then so is $H_{\mathscr{C}}$
- Max. Wt. Independent Set on P_5-free graphs has a polynomial time algorithm (Lokshtanov et al.)

For $\mathscr{C}' \subseteq \mathscr{C}$, $S = \cup_{X \in \mathscr{C}'} X$ induces bipartite graph in G of weight t if and only if $\mathscr{C}'$ is a independent set in $H_{\mathscr{C}}$ of weight t.

Consider a P_5-free graph G and $w : V(G) \rightarrow \mathbb{Q}$ and its ***solution covering family*** $\mathscr{C} = \{X_1, X_2, \cdots X_\ell\}$.

Create a graph $H_{\mathscr{C}}$ with $V(H_{\mathscr{C}}) = \mathscr{C}$, where, for distinct $X_i, X_j \in \mathscr{C}$, add $\{X_i, X_j\}$ to $E(H_{\mathscr{C}})$ if and only if:
$X_i \cap X_j \neq \emptyset$ **OR** $E(X_i, X_j) \neq \emptyset$.

Create a weight function $q : V(H_{\mathscr{C}}) \rightarrow \mathbb{Q}$, where for $X \in \mathscr{C}$, $q(X) = \sum_{v \in X} w(v)$.

Two Ingredients

THEOREM

Odd Cycle Transversal admits a polynomial time algorithm on P_5-free graphs.

Ingredient 1: A polynomial-sized ***solution covering family***.

✓ *Ingredient 2:* Translating solution to finding ***independent sets on a P_5-free auxiliary graph*** over a solution covering family.

Two Ingredients

THEOREM

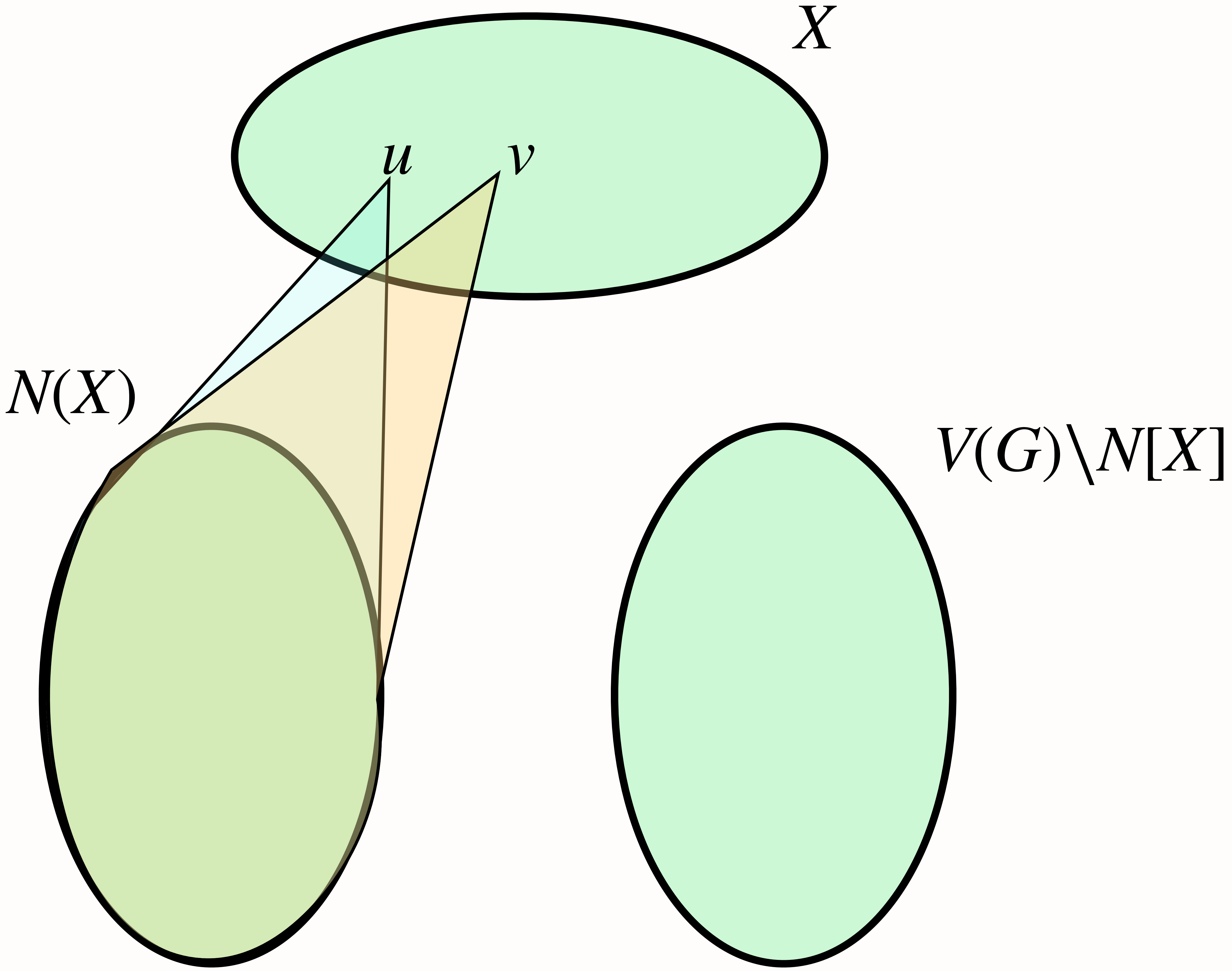
Odd Cycle Transversal admits a polynomial time algorithm on P_5-free graphs.

Ingredient 1: A polynomial-sized ***solution covering family***.

Ingredient 2: Translating solution to finding ***independent sets on a P_5-free auxiliary graph*** over a solution covering family.

Useful Definition

For a graph G, a set $X \subseteq V(G)$ is a ***module*** in G if for all $u, v \in X$, $N_G(u) \setminus X = N_G(v) \setminus X$.



Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$
- Sets in $\mathscr{C}'$ are pairwise disjoint
- No edge between different sets in $\mathscr{C}'$, i.e., for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

LEMMA

Given a P_5-free graph G on n vertices and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, there is a polynomial-time algorithm that outputs a solution covering family of size $O(n^6)$.

Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for some $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

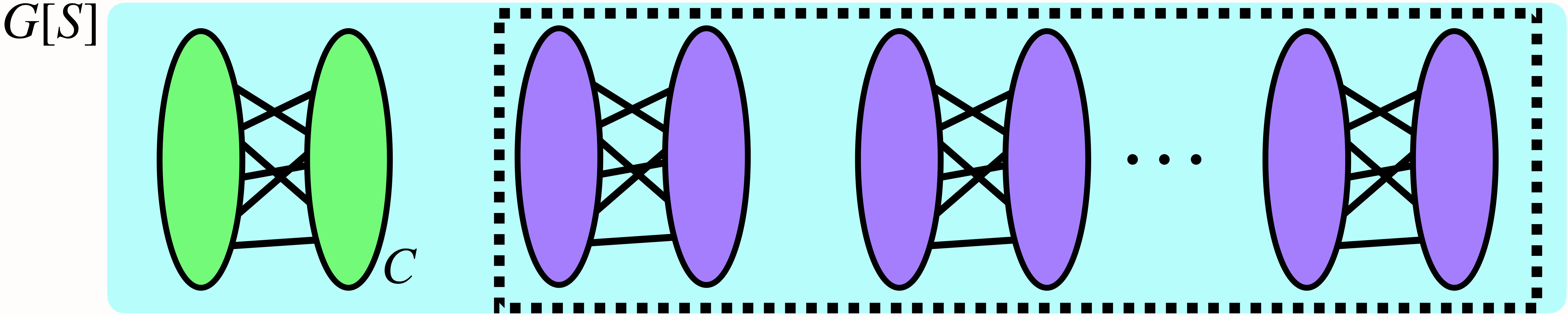
Consider some $S \subseteq V(G)$ where $G[S]$ is bipartite.

- $S = \cup_{X \in \mathscr{C}'} X$
- Sets in $\mathscr{C}'$ are pairwise disjoint
- No edge between different sets in $\mathscr{C}'$, i.e., for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

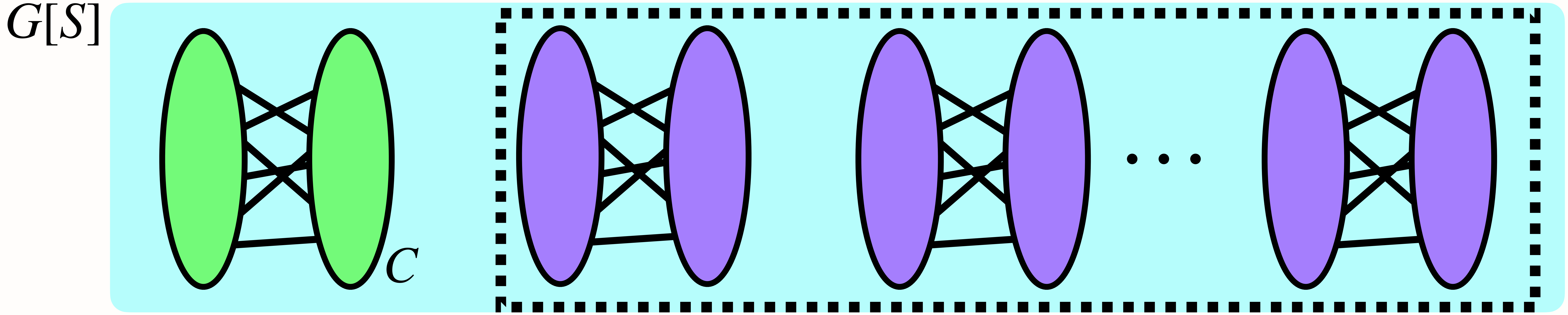
LEMMA

Given a P_5-free graph G on n vertices and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, there is a polynomial-time algorithm that outputs a solution covering family of size $O(n^6)$.

Solution Covering Family

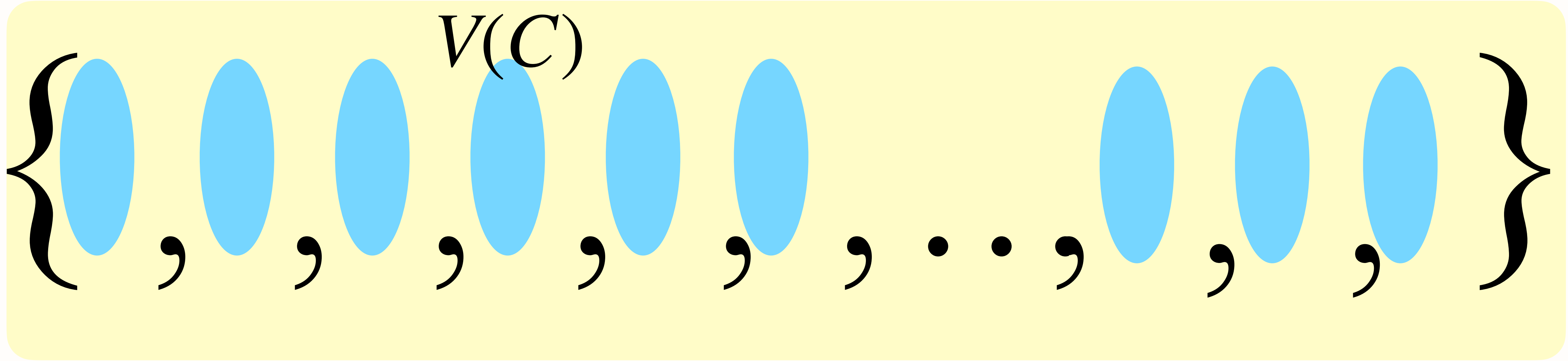


Solution Covering Family



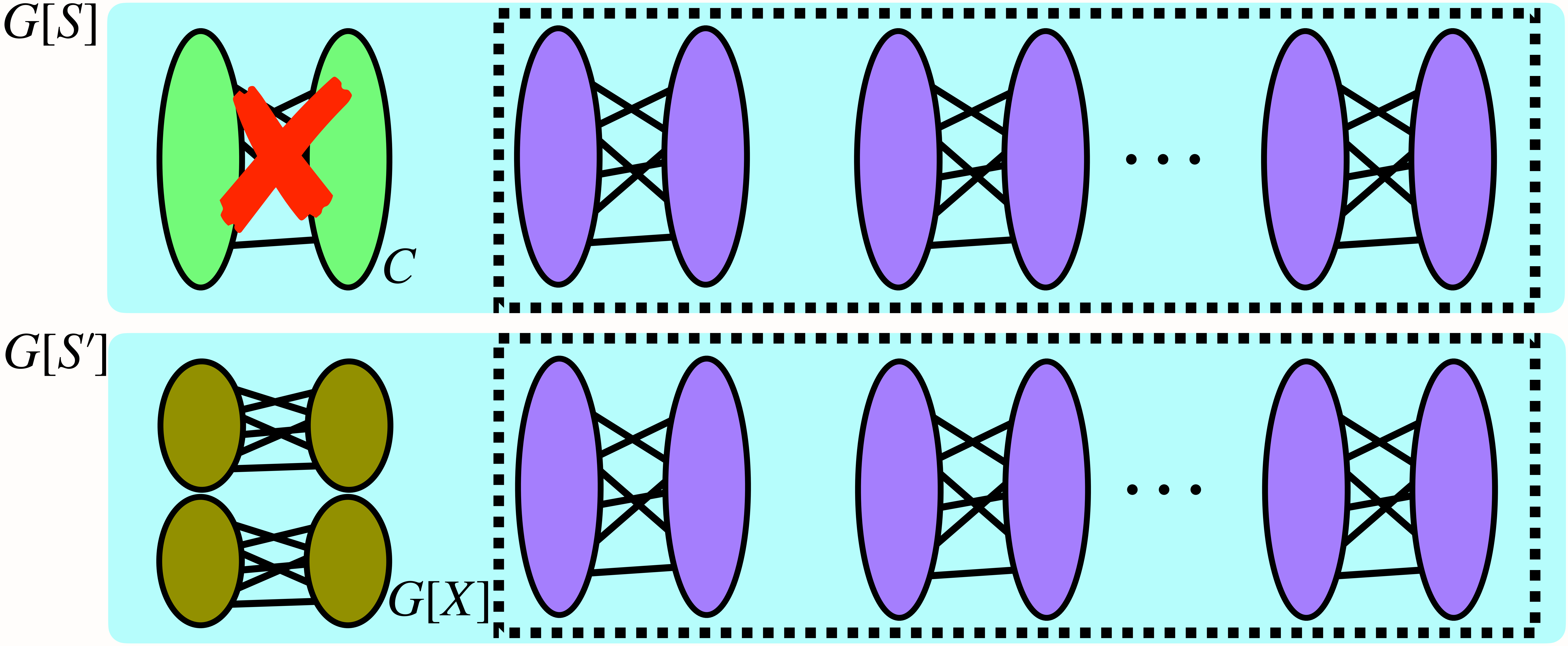
$V(C) \in \mathscr{C}$

OR



solution covering family $\mathscr{C}$

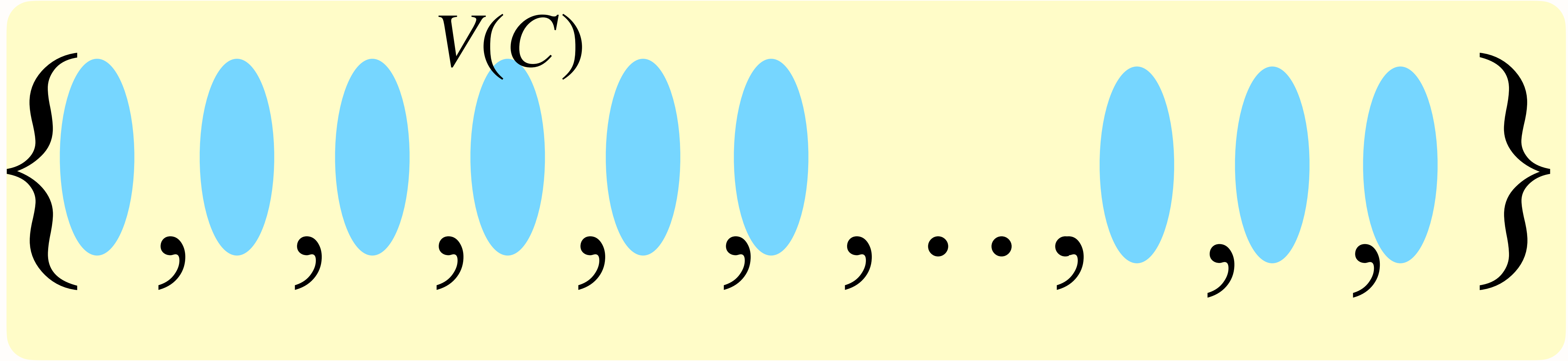
Solution Covering Family



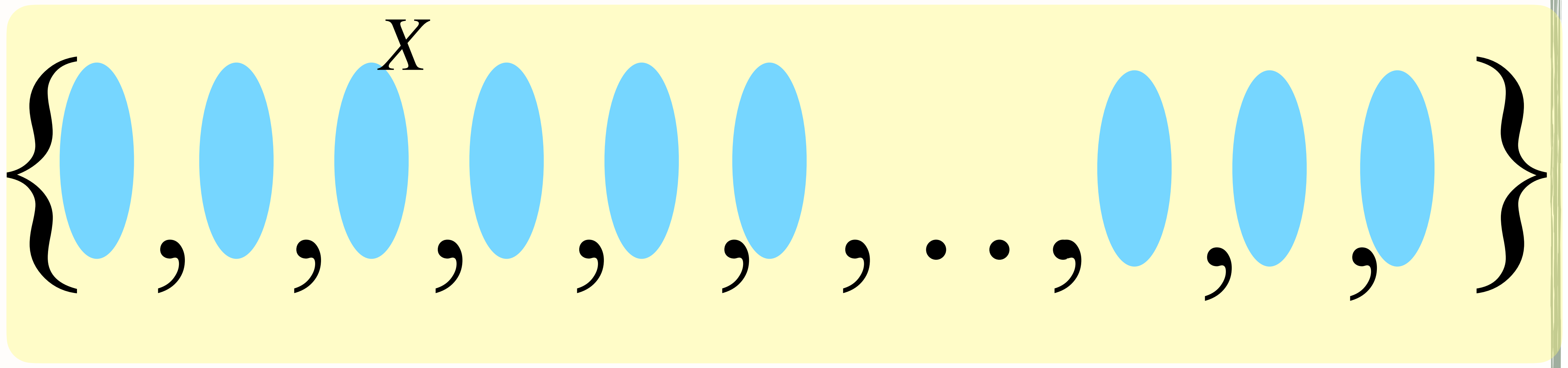
$V(C) \in \mathscr{C}$

OR

$\exists X \in \mathscr{C}$, for $S' = (S \setminus V(C)) \cup X$, $G[S']$ is bipartite and $\mathrm{w}(S') \geq \mathrm{w}(S)$

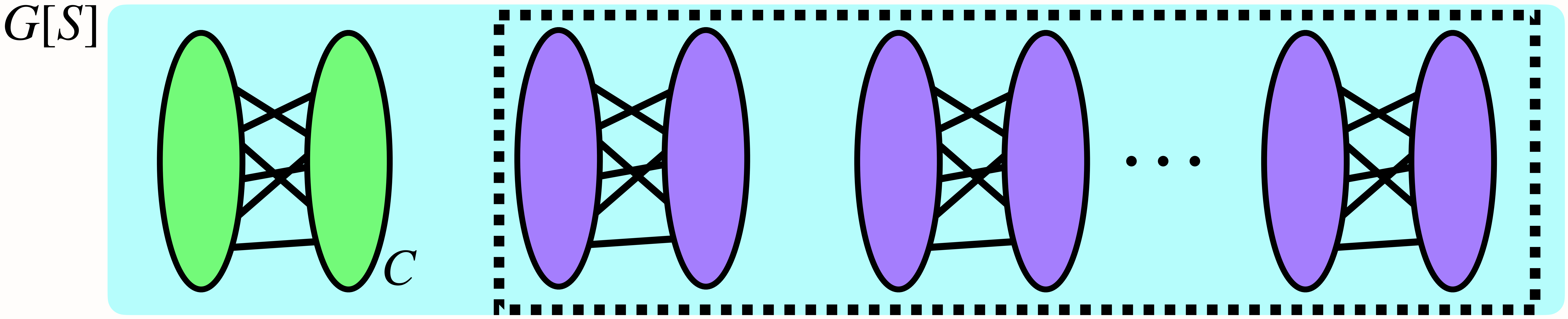


solution covering family $\mathscr{C}$



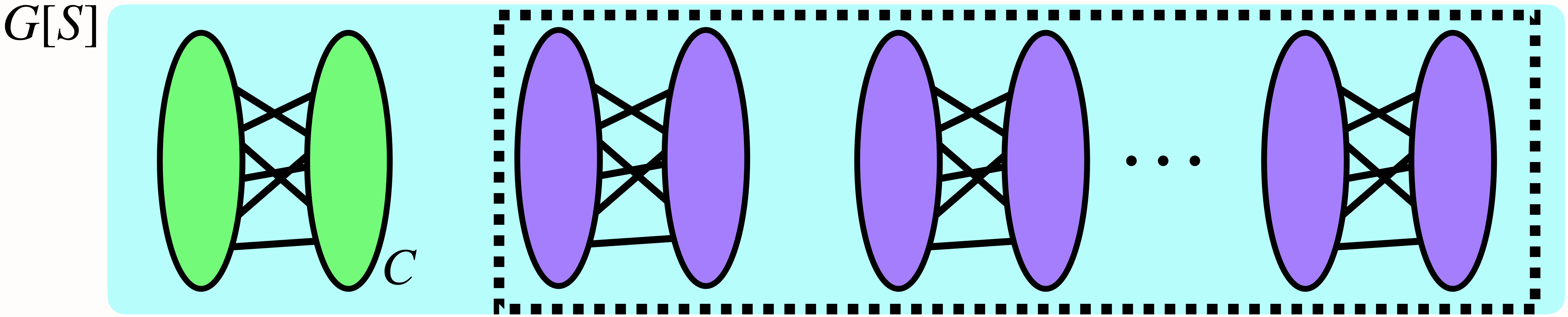
solution covering family $\mathscr{C}$

Solution Covering Family



- **[Trivial Case: $V(C) = 1$]** Add $\{v\}$ to $\mathscr{C}$, for each $v \in V(G)$.

Solution Covering Family

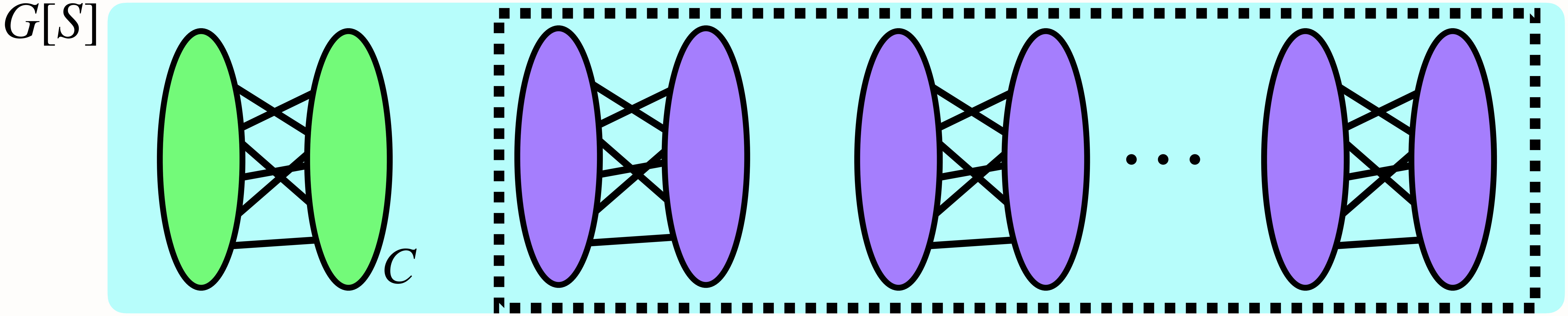


✱ **[Handling $|V(C)| \geq 2$]**

Known Property of conn. P_5-free graphs

C has a dominating P_2 or P_3, i.e., $D \subseteq V(C)$, such that $V(C) \subseteq N_G[D]$.

Solution Covering Family



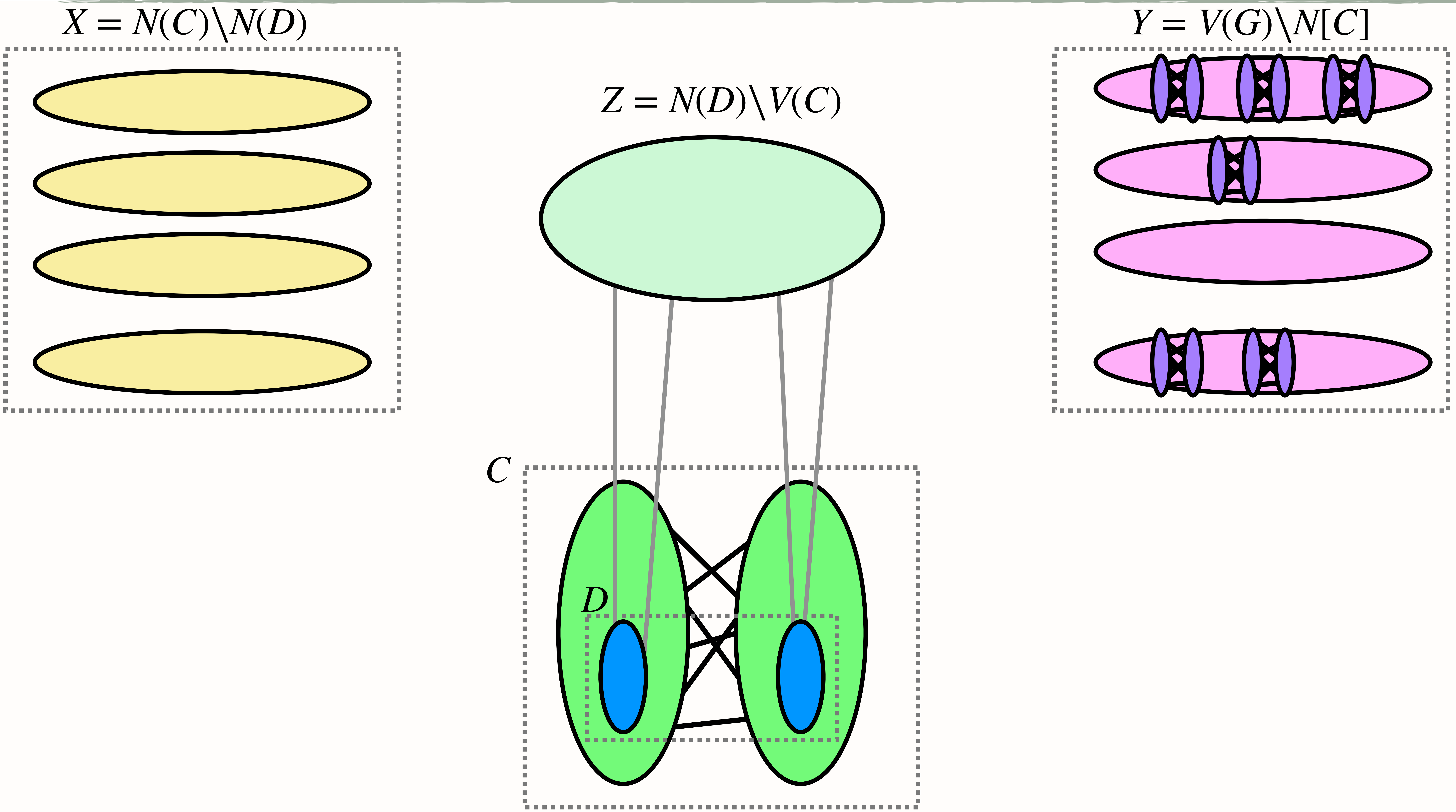
✱ **[Handling $V(C)$ ≥ 2]**

Known Property of conn. P_5-free graphs

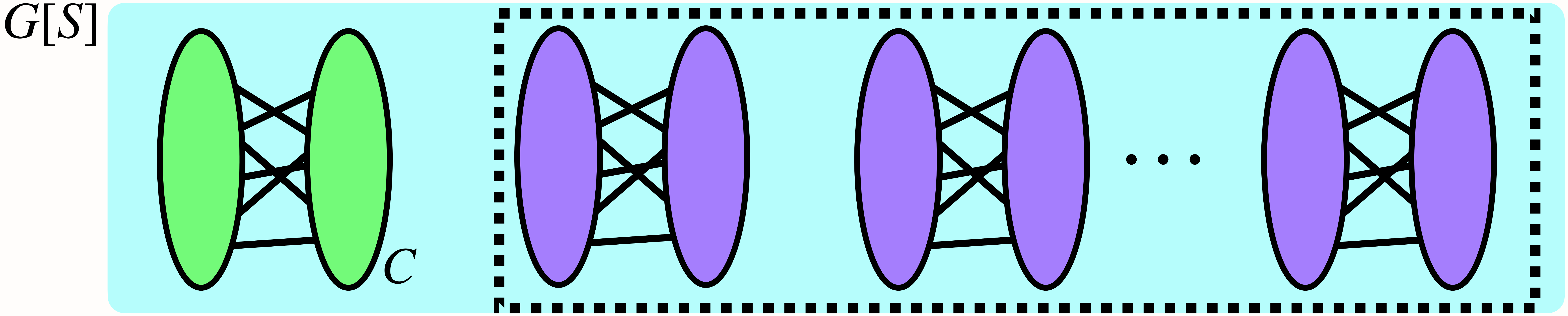
C has a dominating P_2 or P_3, i.e., $D \subseteq V(C)$, such that $V(C) \subseteq N_G[D]$.

Guess this set!

Analysing Structure



Solution Covering Family



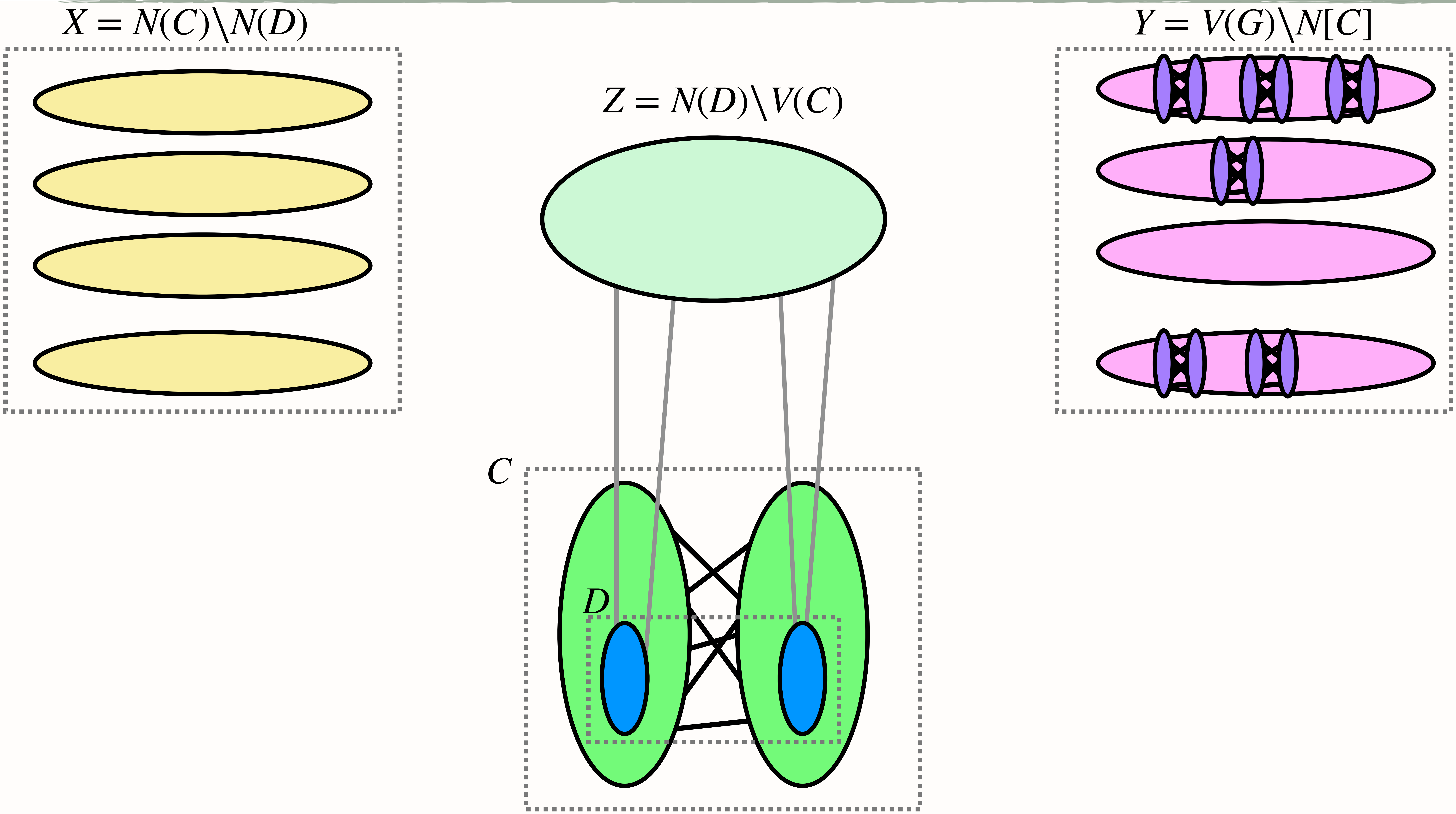
✱ **[Handling $V(C)$ ≥ 2]**

Known Property of conn. P_5-free graphs

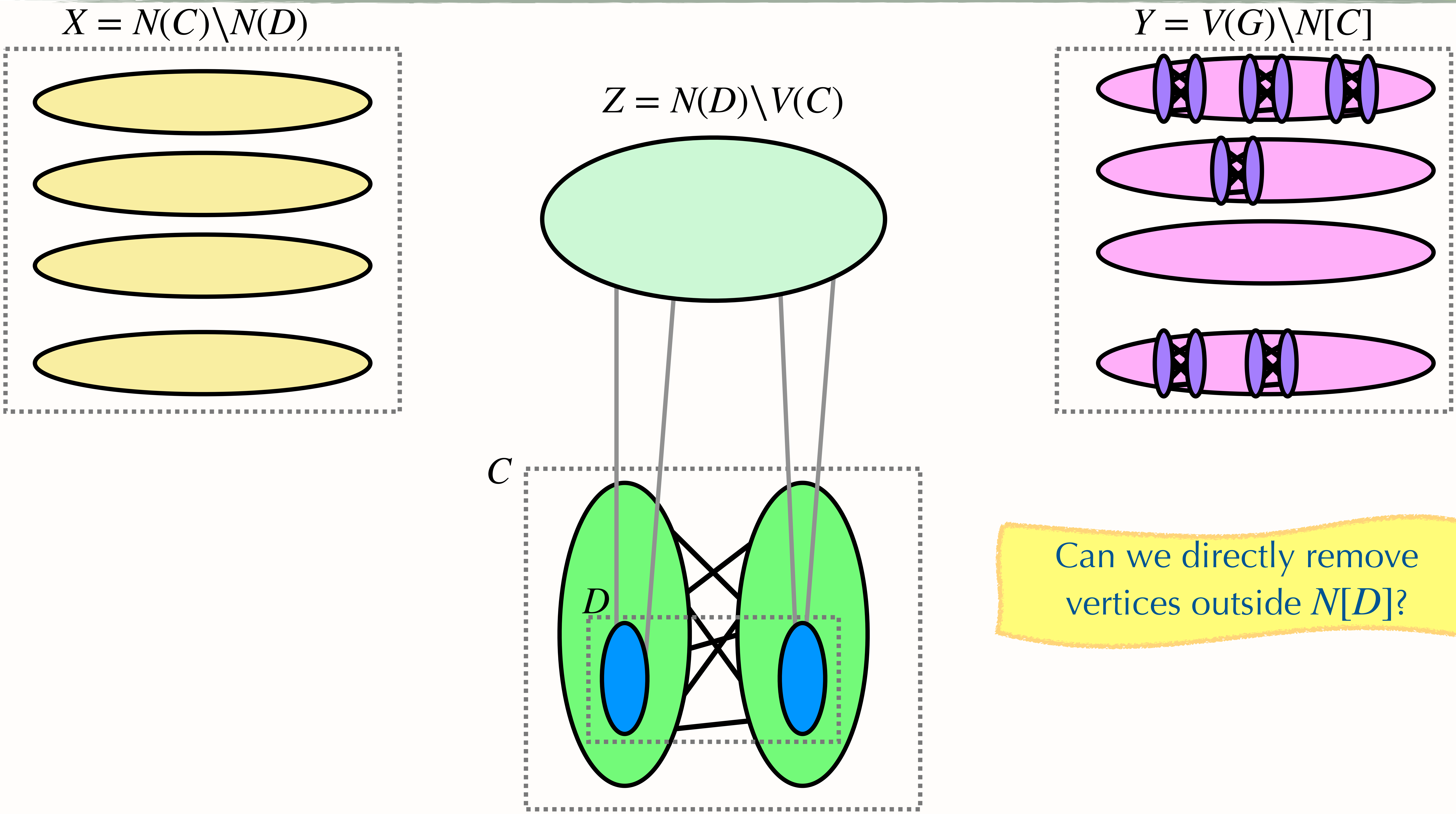
C has a dominating P_2 or P_3, i.e., $D \subseteq V(C)$, such that $V(C) \subseteq N_G[D]$.

Guess this set!

Analysing Structure

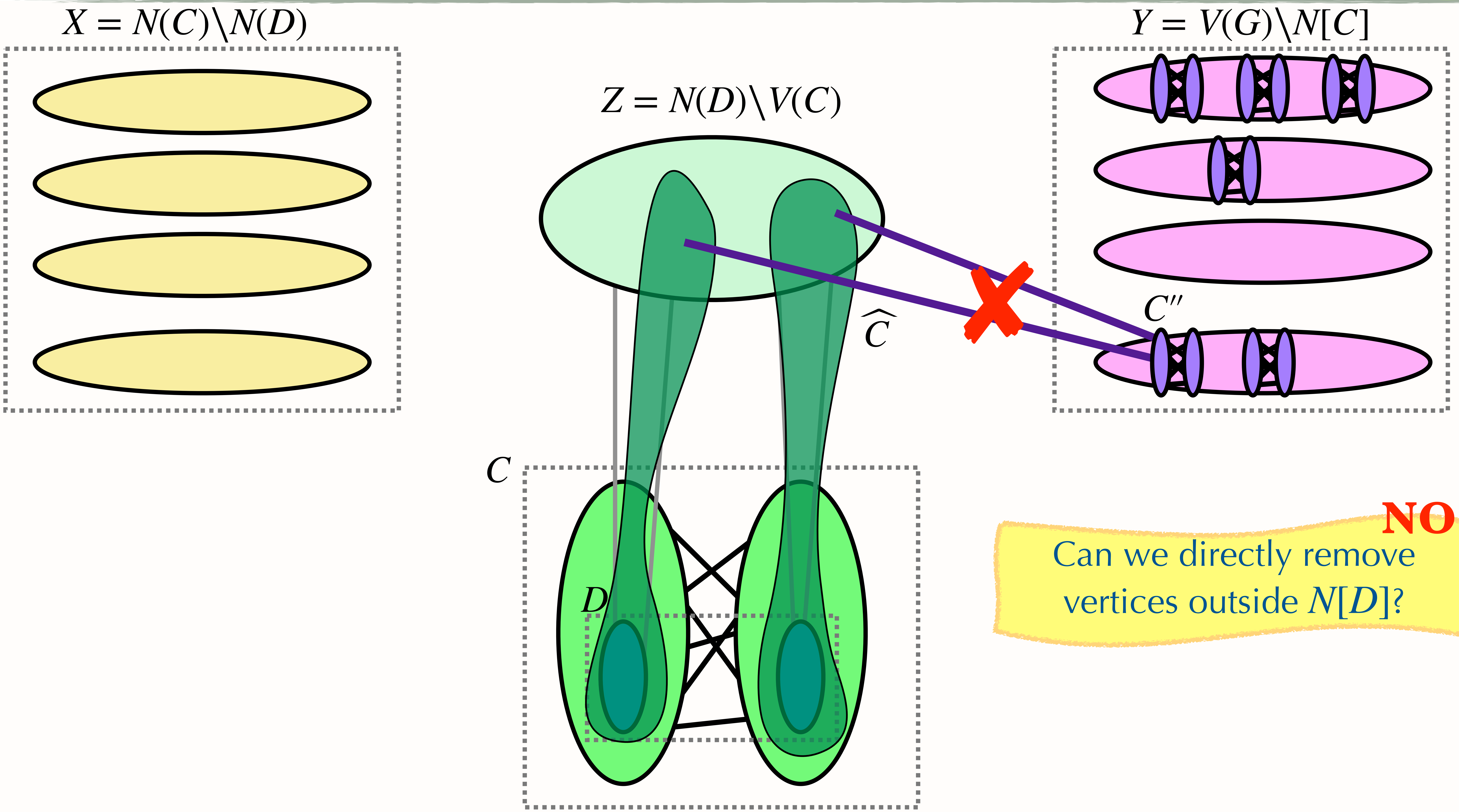


Analysing Structure



Can we directly remove vertices outside $N[D]$?

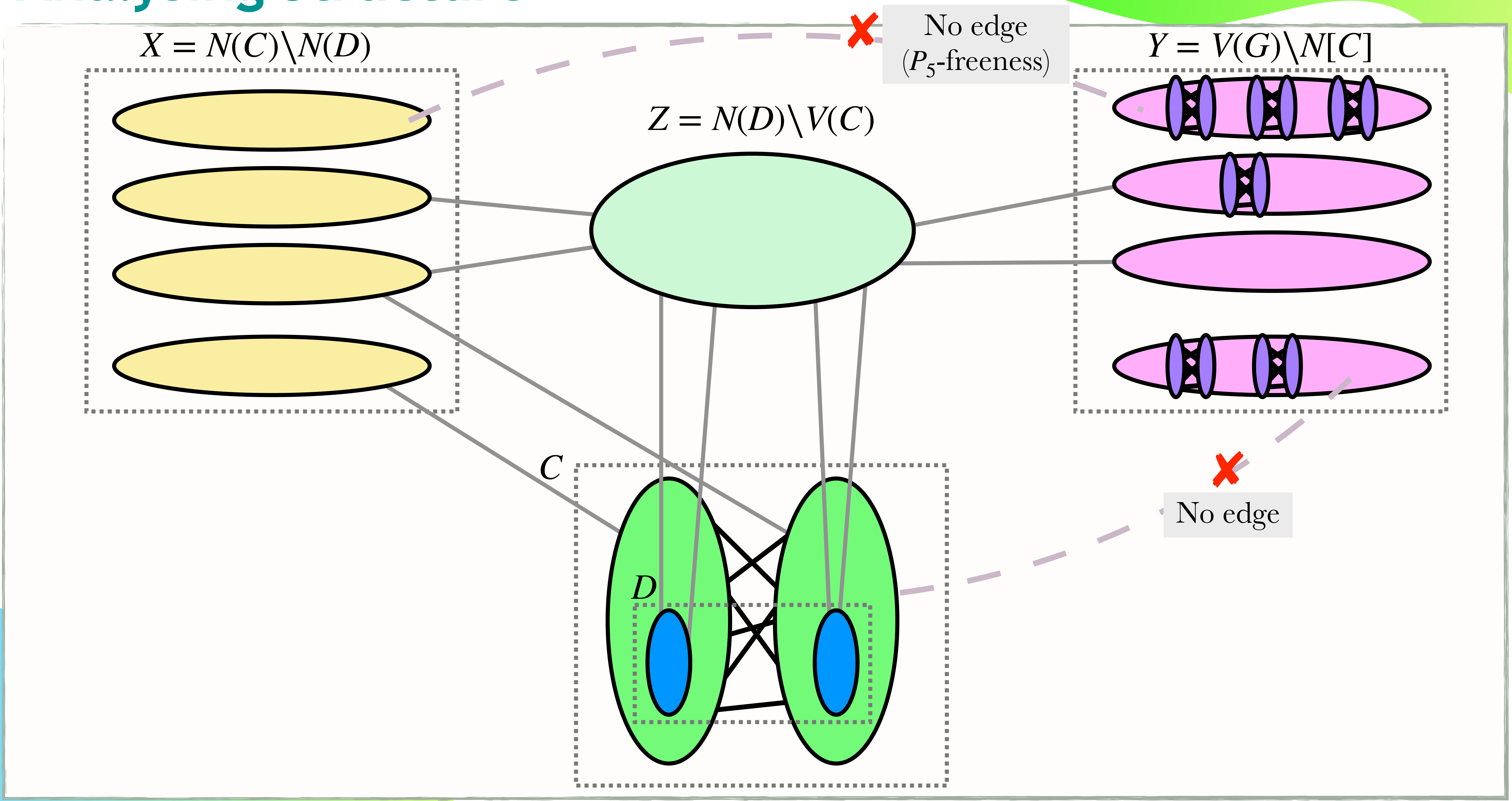
Analysing Structure



NO!

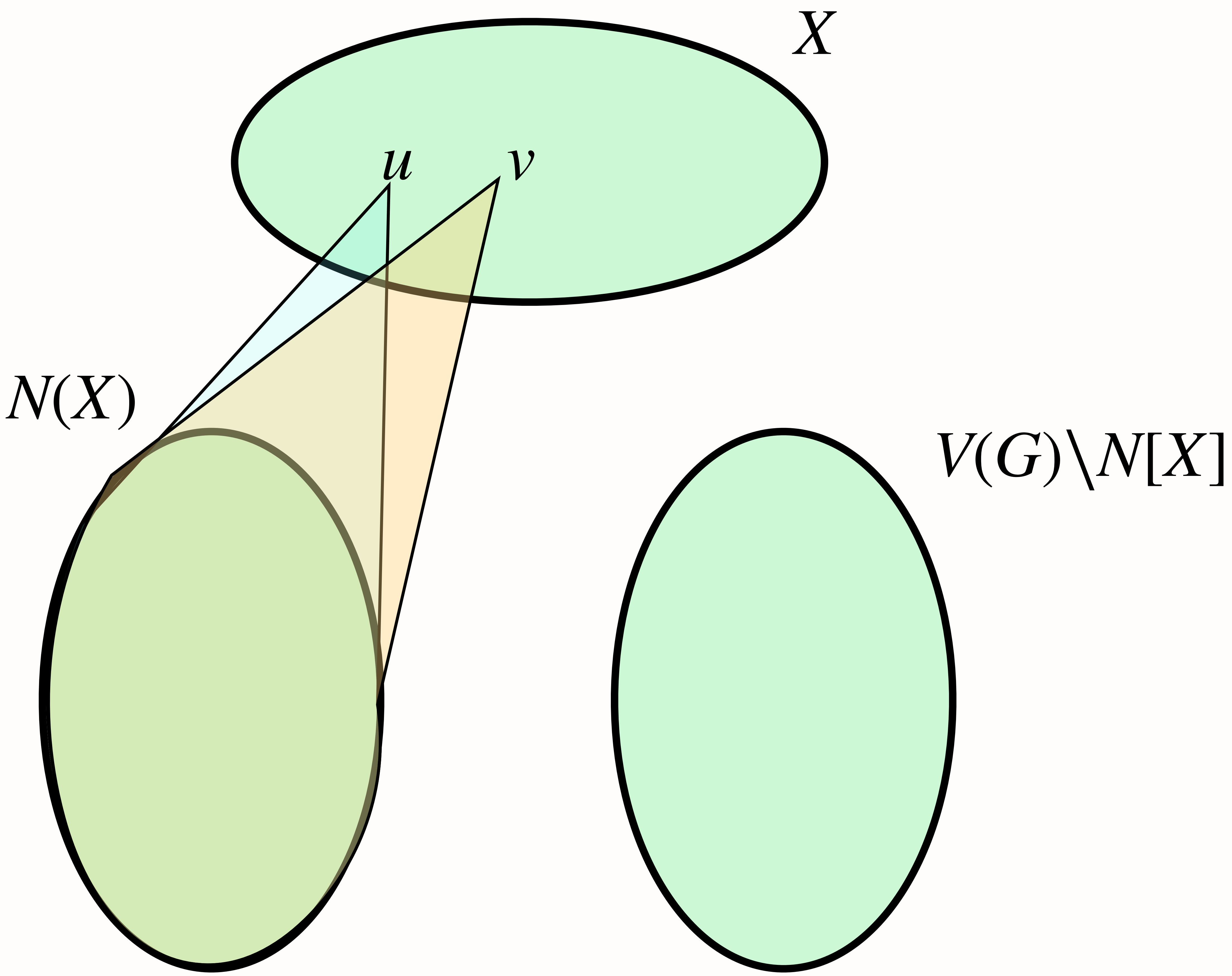
Can we directly remove vertices outside $N[D]$?

Analysing Structure



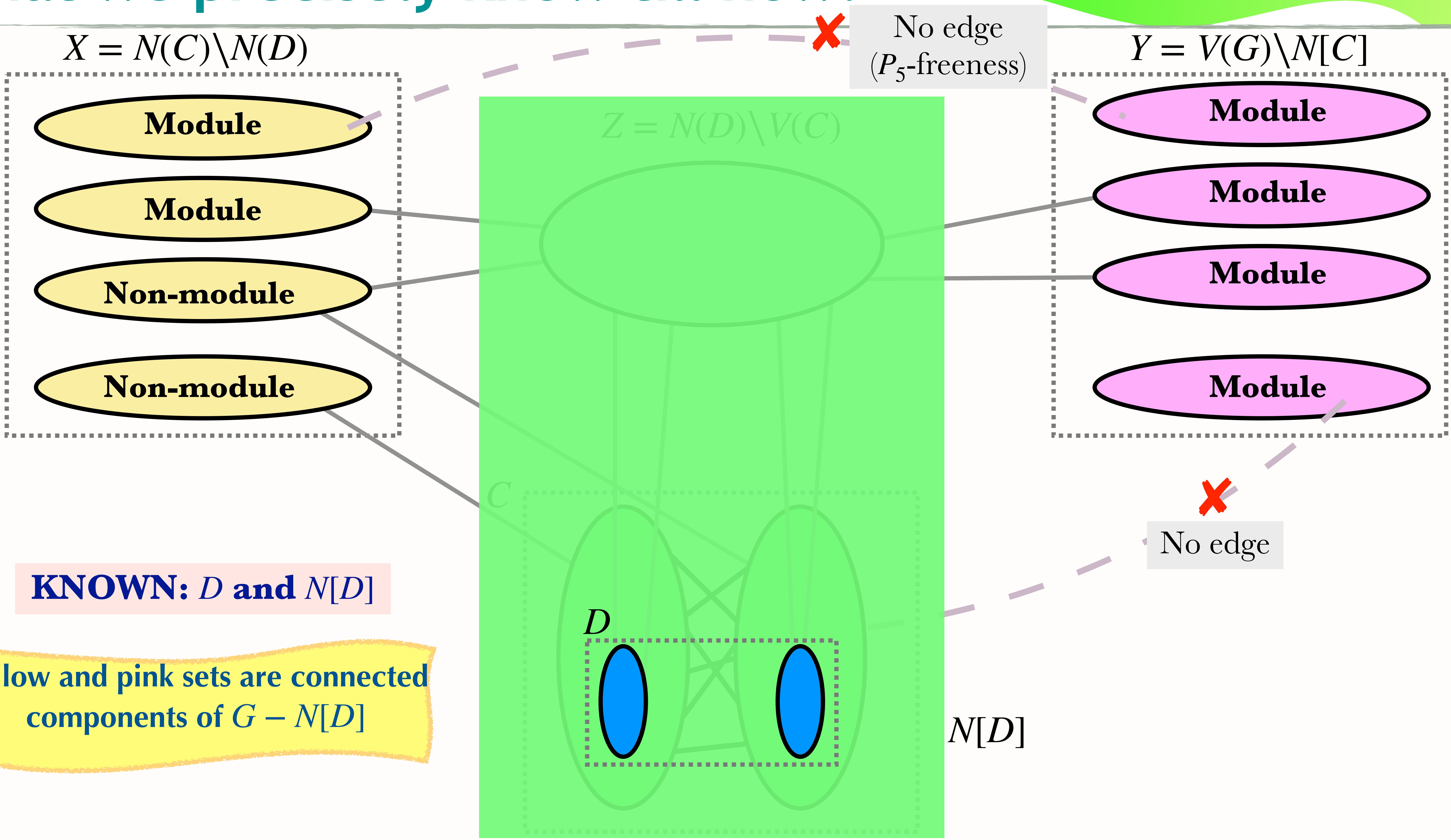
Useful Definition

For a graph G, a set $X \subseteq V(G)$ is a ***module*** in G if for all $u, v \in X$, $N_G(u) \setminus X = N_G(v) \setminus X$.



Analysing Structure

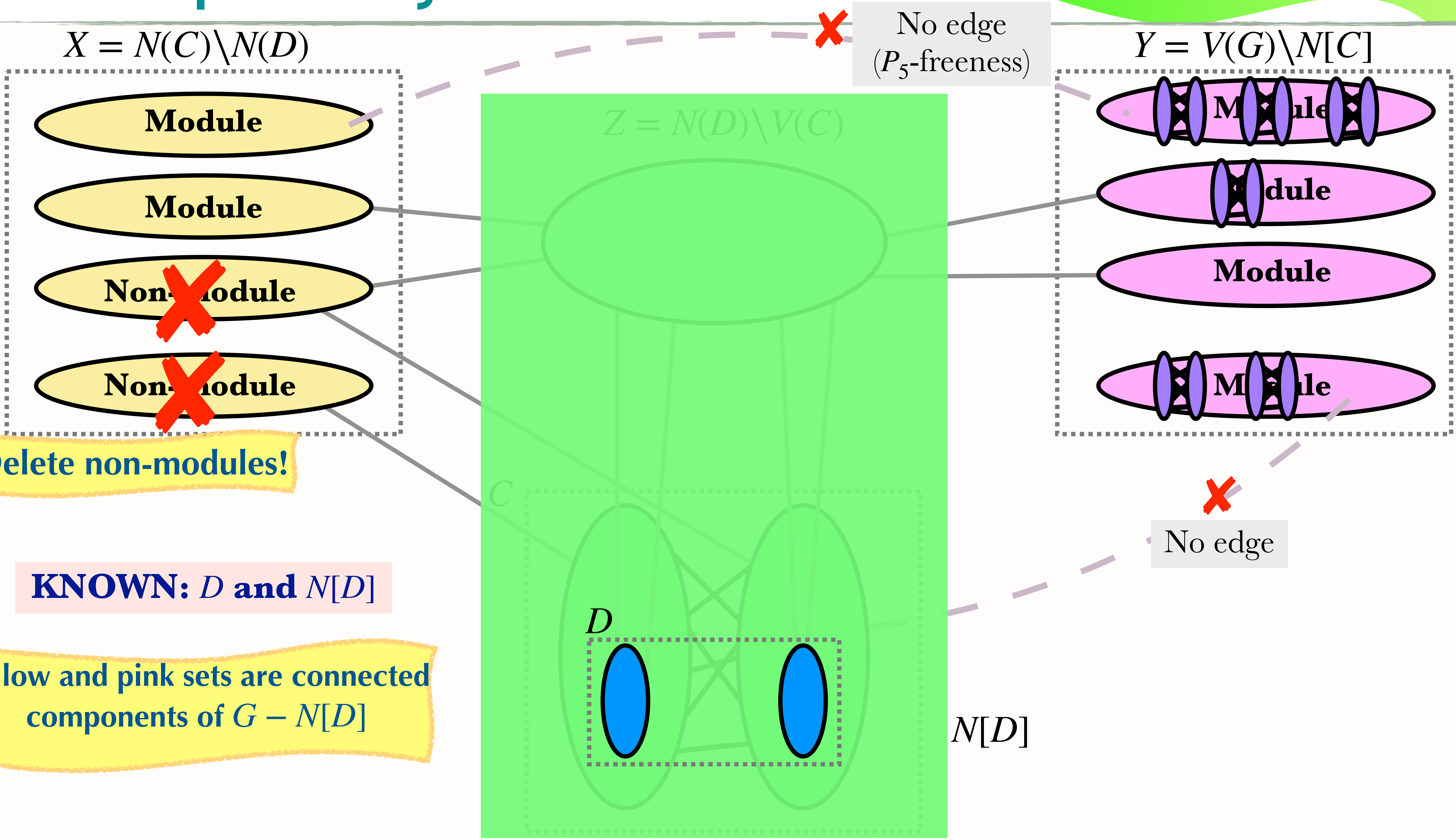
What we precisely know till now?



KNOWN: D **and** $N[D]$

Yellow and pink sets are connected components of $G - N[D]$

What we precisely know till now?

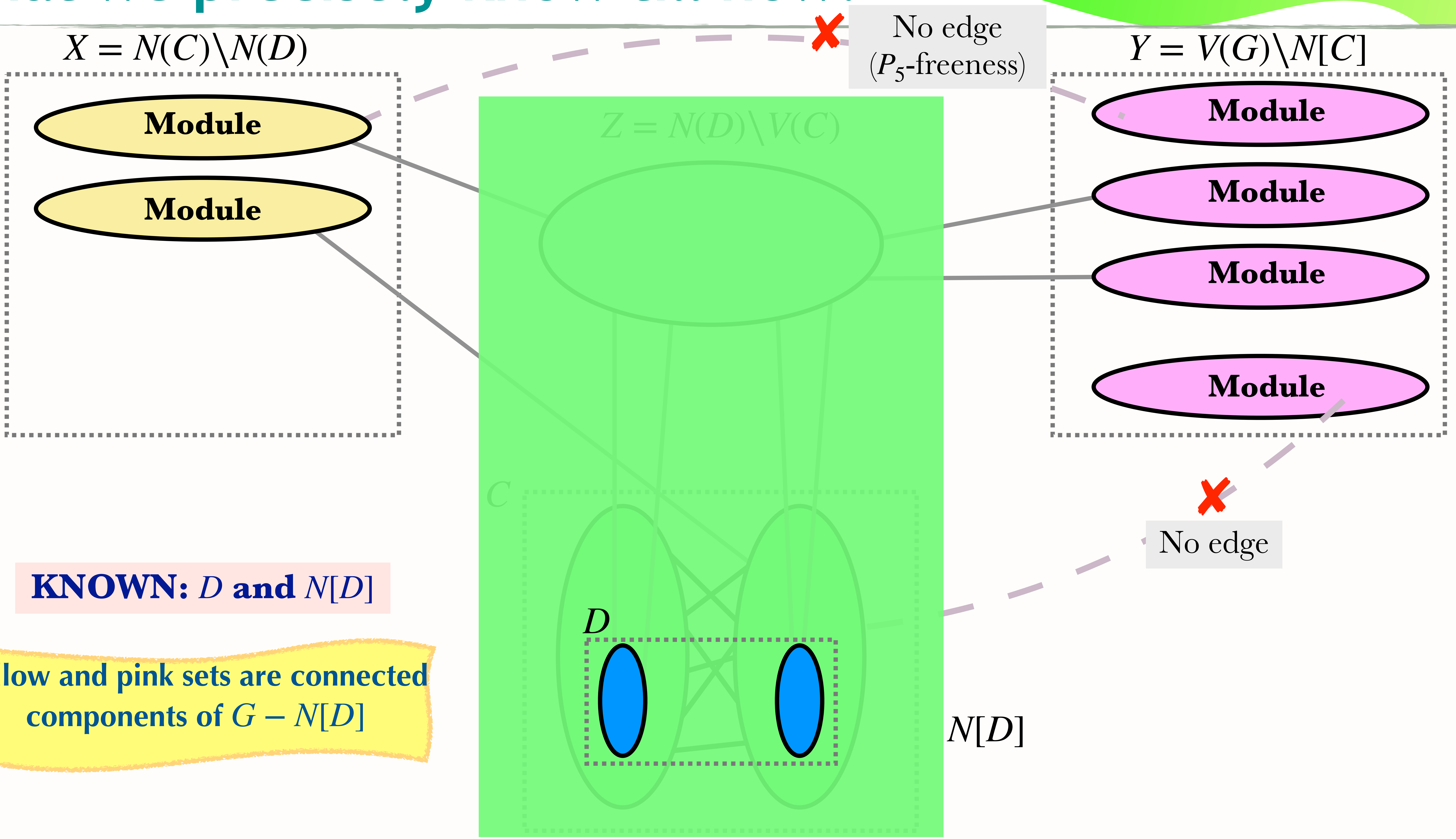


Delete non-modules!

KNOWN: D and $N[D]$

Yellow and pink sets are connected components of $G - N[D]$

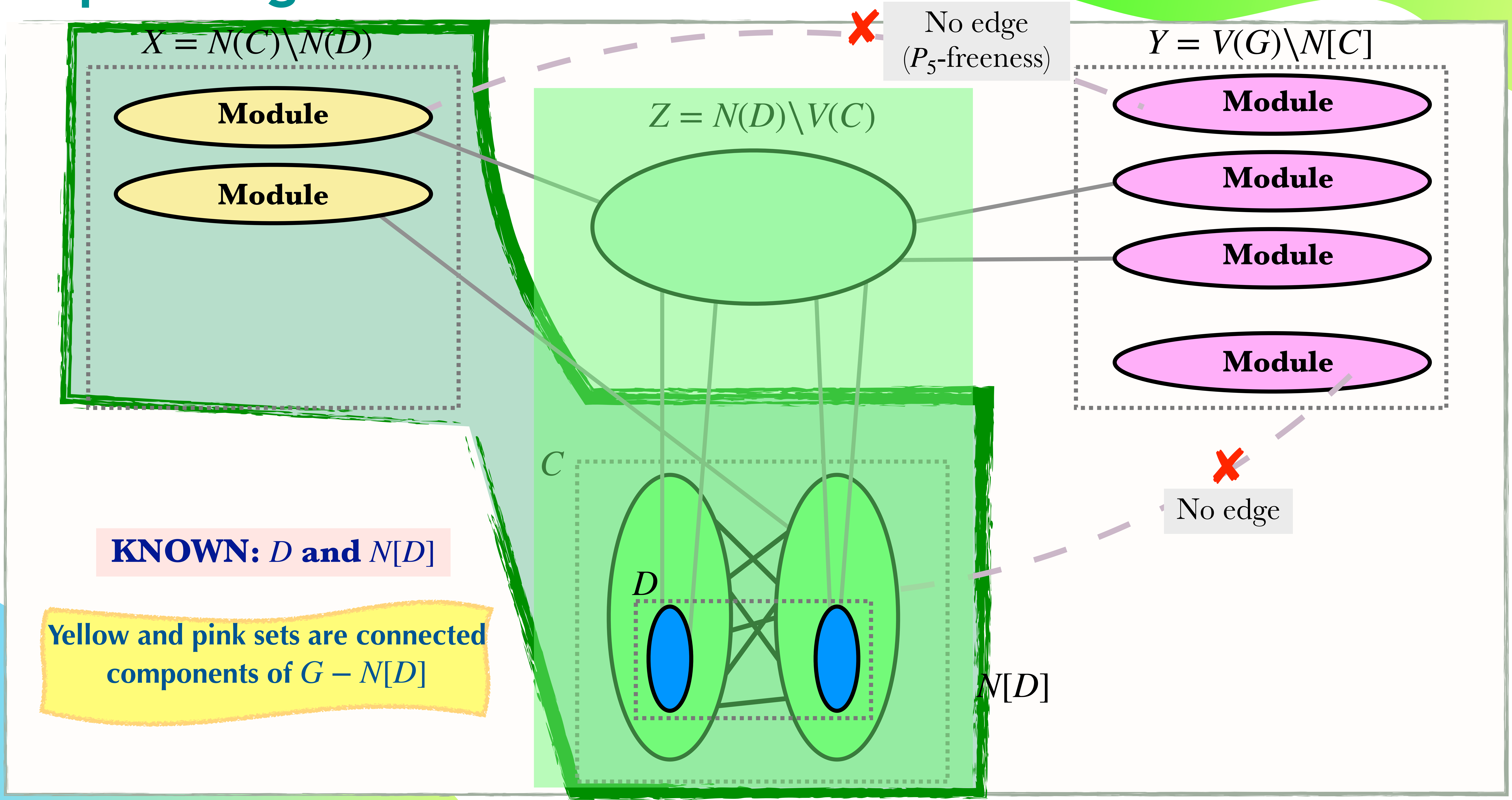
What we precisely know till now?



KNOWN: D and $N[D]$

Yellow and pink sets are connected components of $G - N[D]$

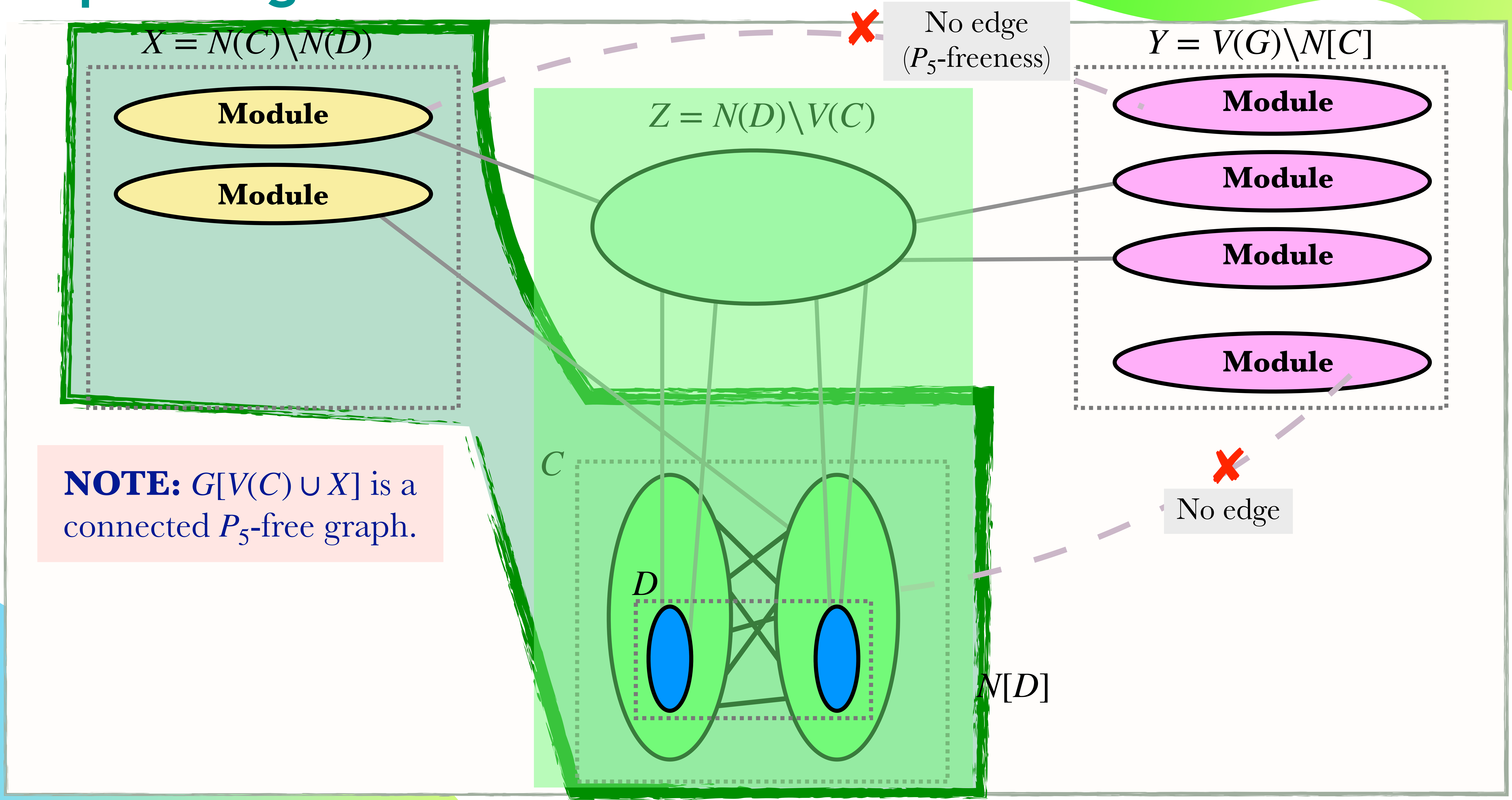
Separating Y



KNOWN: D **and** $N[D]$

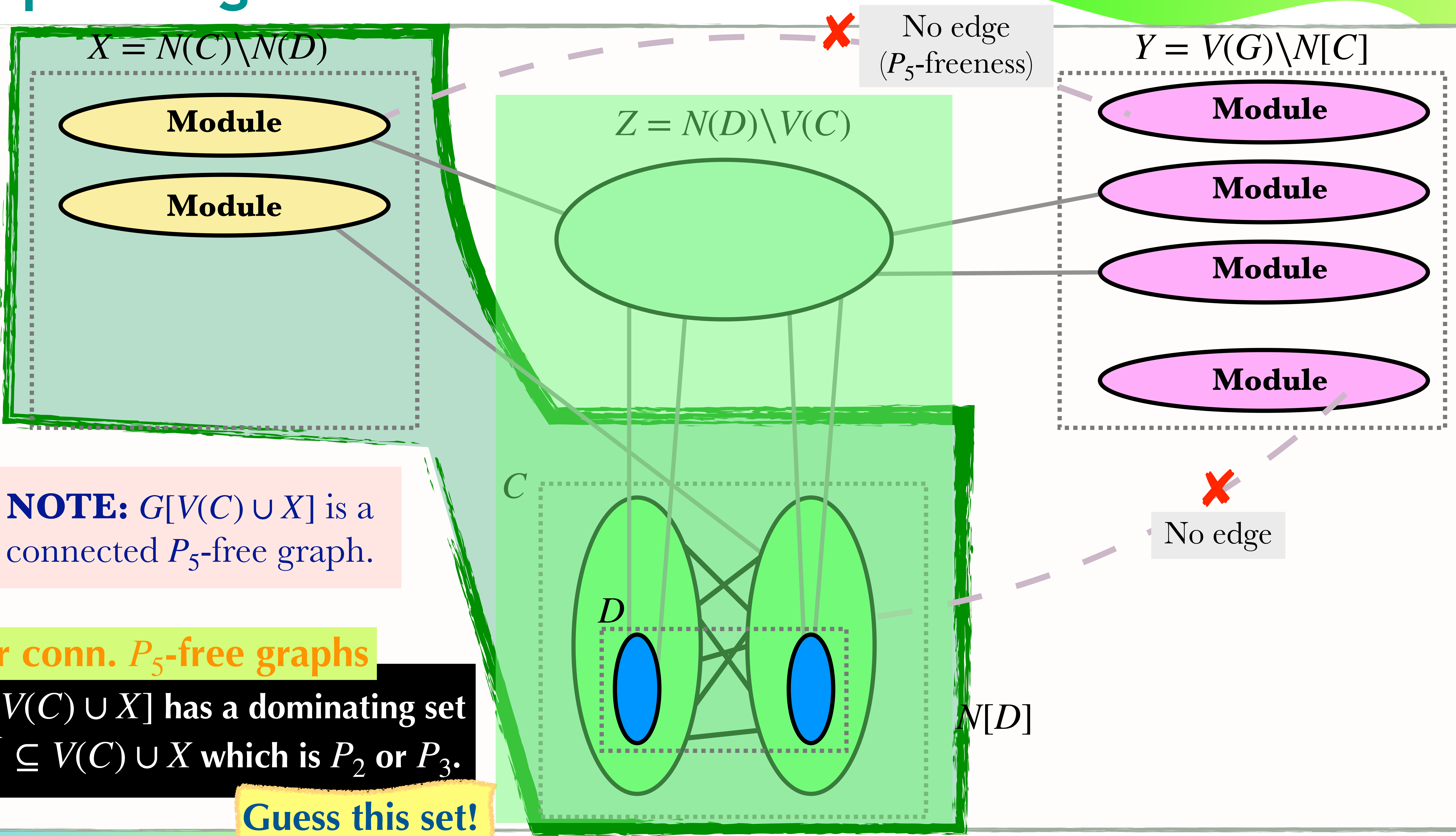
Yellow and pink sets are connected components of $G - N[D]$

Separating Y



NOTE: $G[V(C) \cup X]$ is a connected P_5-free graph.

Separating Y



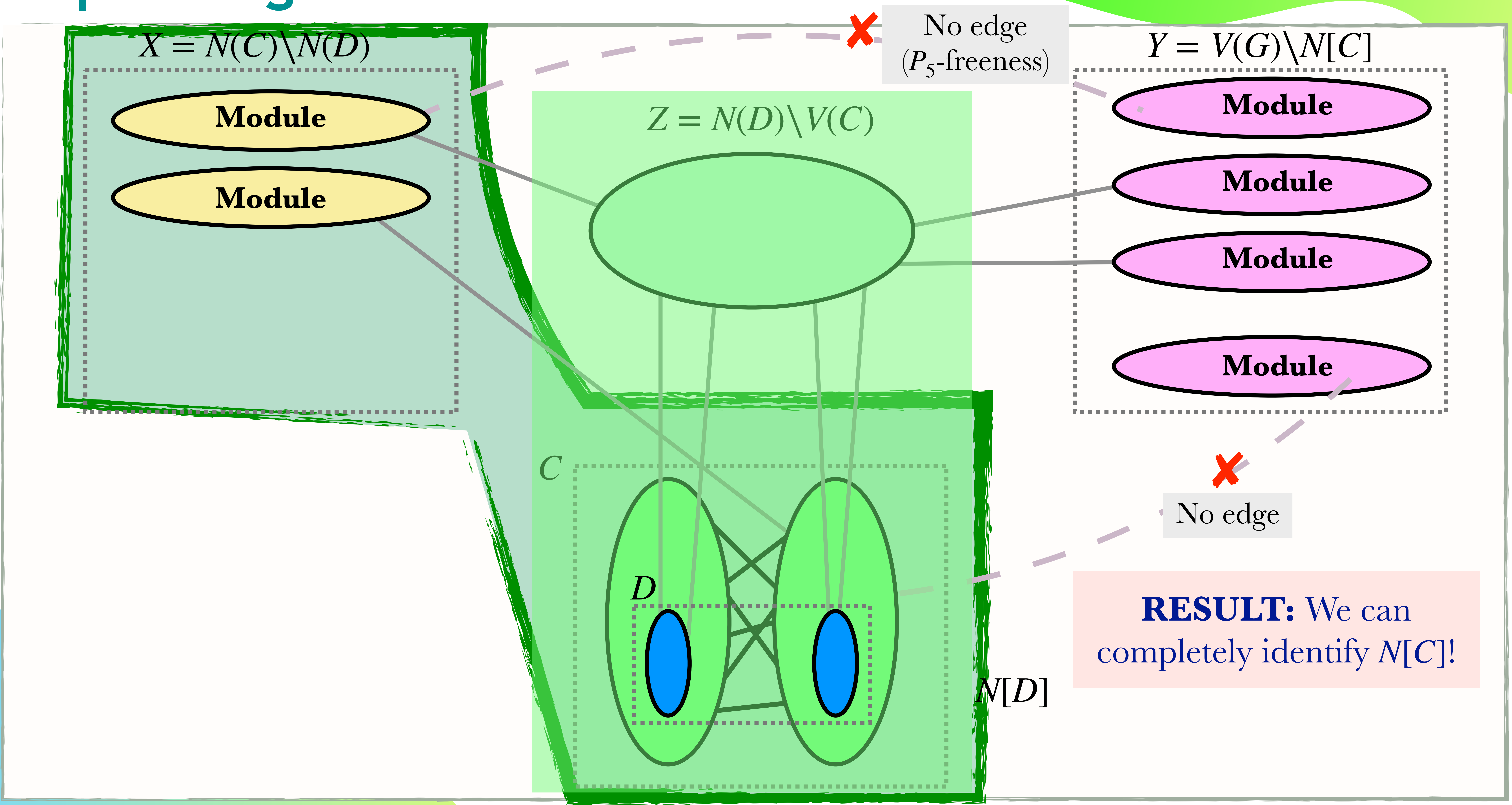
NOTE: $G[V(C) \cup X]$ is a connected P_5-free graph.

For conn. P_5-free graphs

$G[V(C) \cup X]$ has a dominating set $\widehat{D} \subseteq V(C) \cup X$ which is P_2 or P_3.

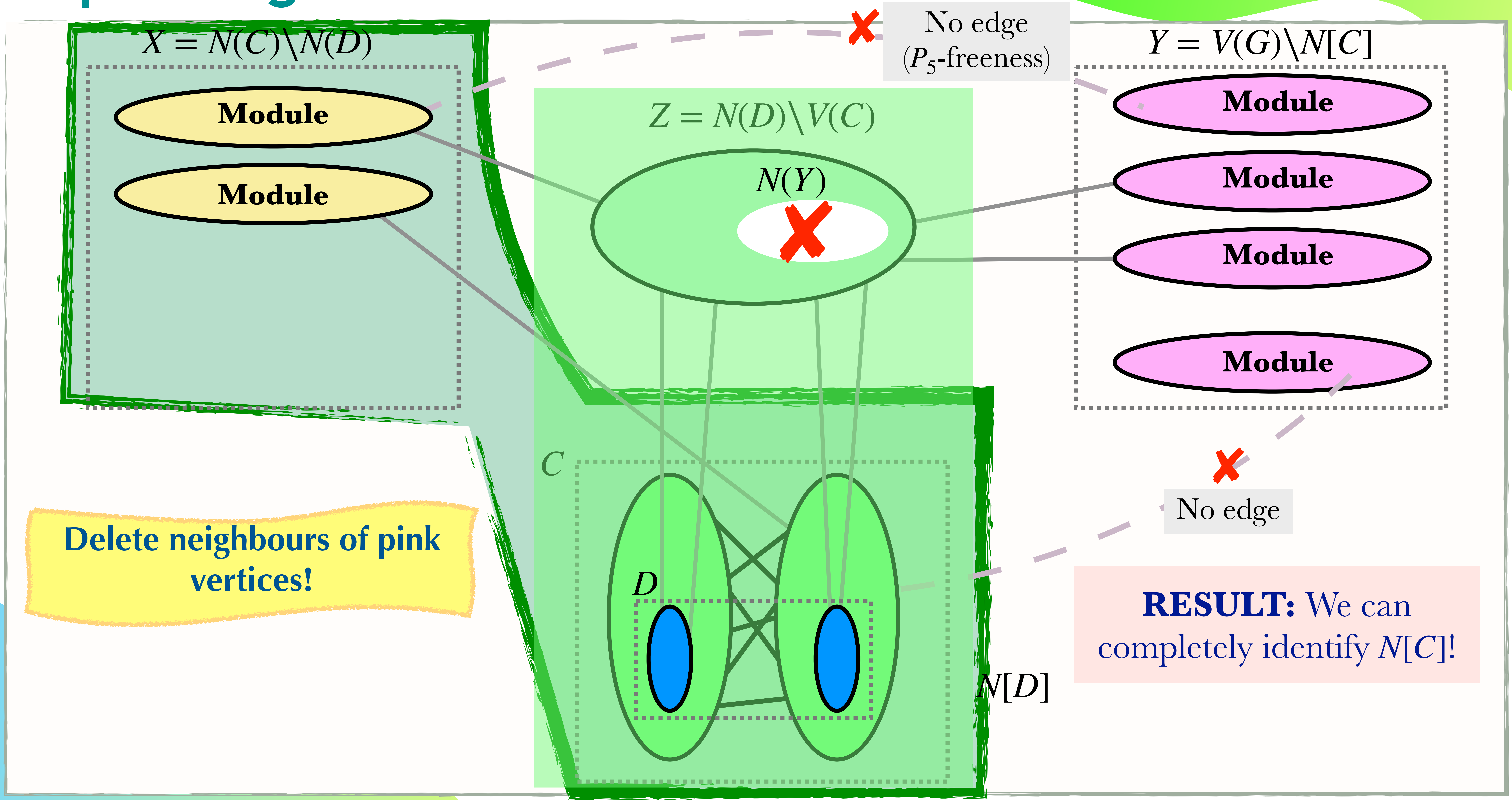
Guess this set!

Separating Y



RESULT: We can completely identify $N[C]$!

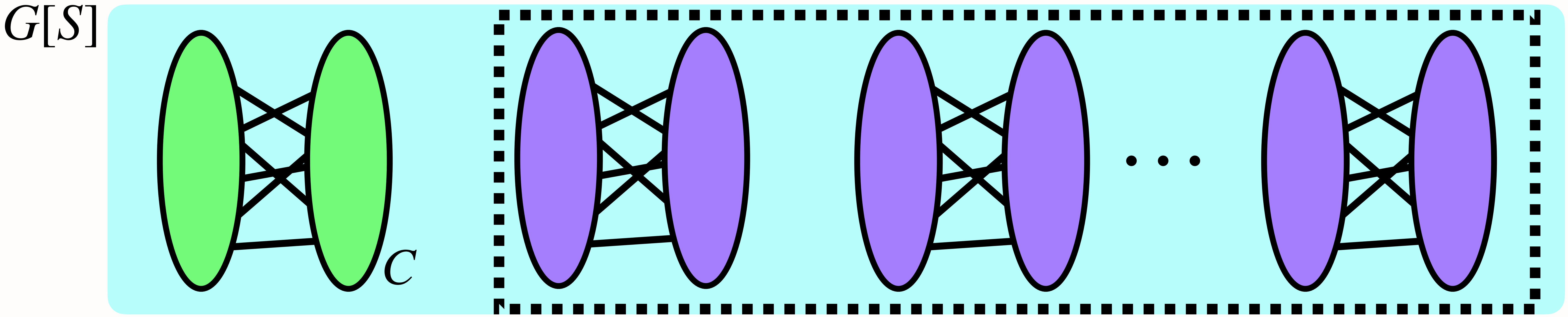
Separating Y



Delete neighbours of pink vertices!

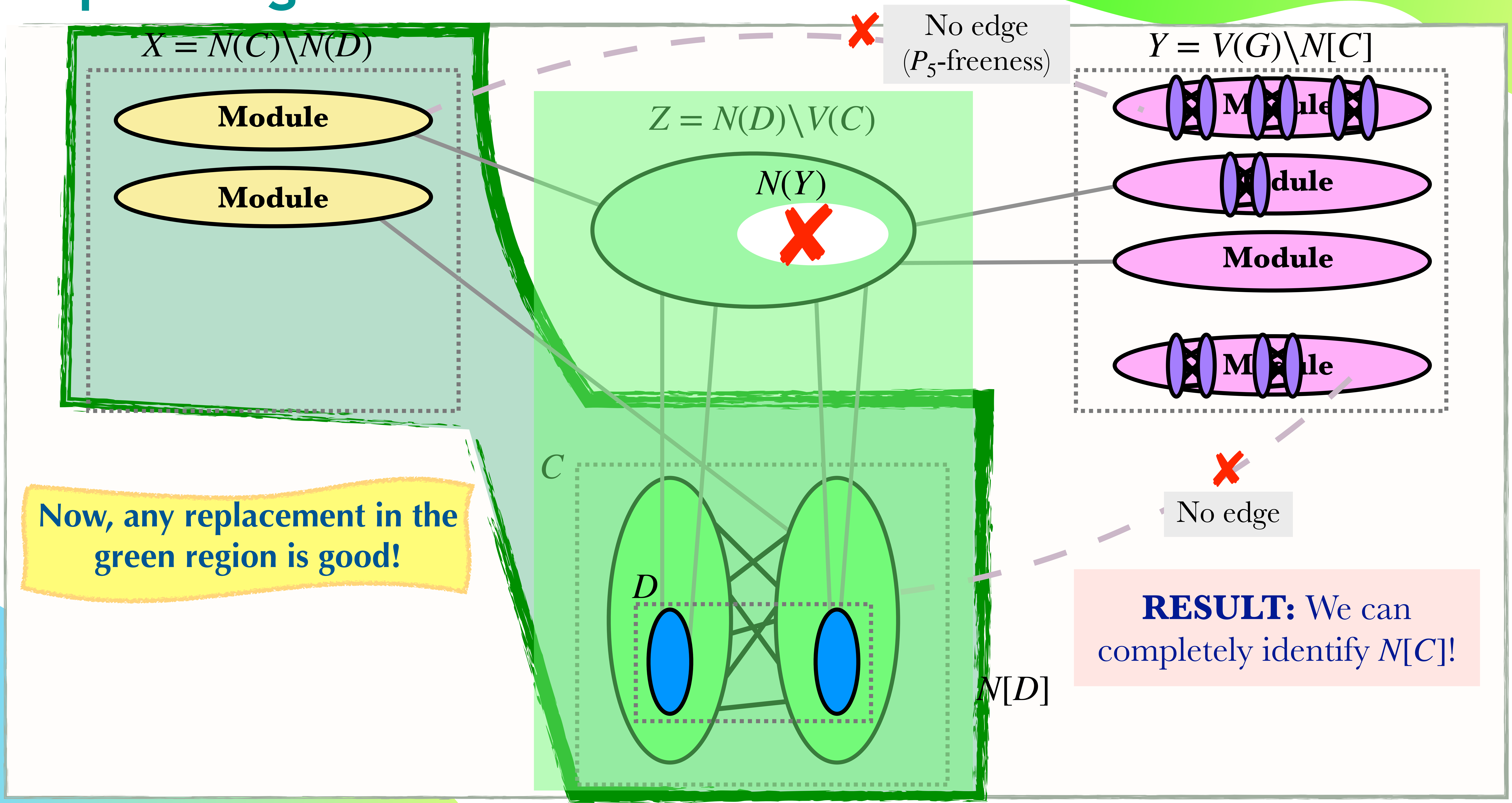
RESULT: We can completely identify $N[C]$!

Solution Covering Family



✱ **[Handling $V(C) \geq 2$]**

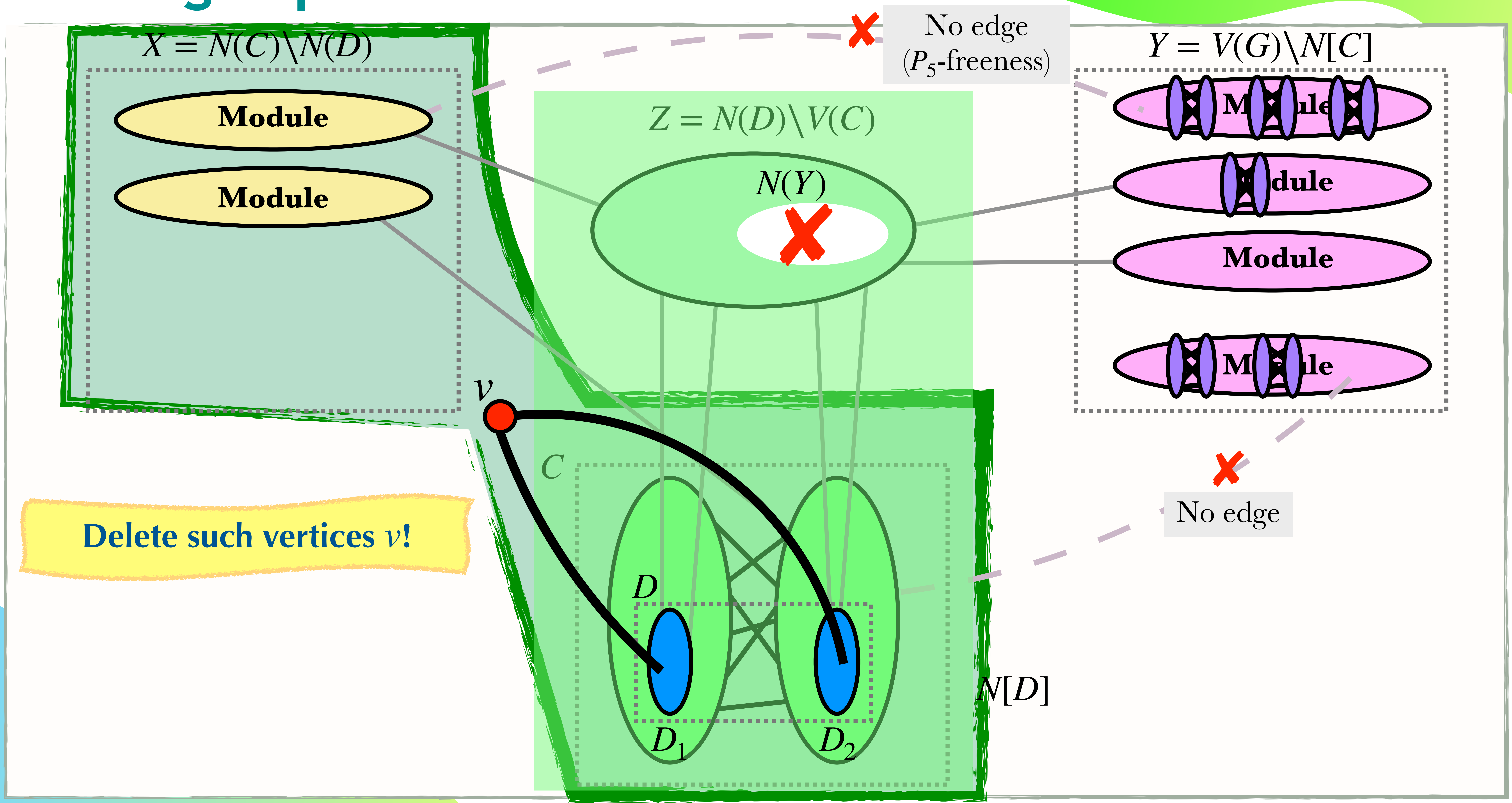
Separating Y



Now, any replacement in the green region is good!

RESULT: We can completely identify $N[C]$!

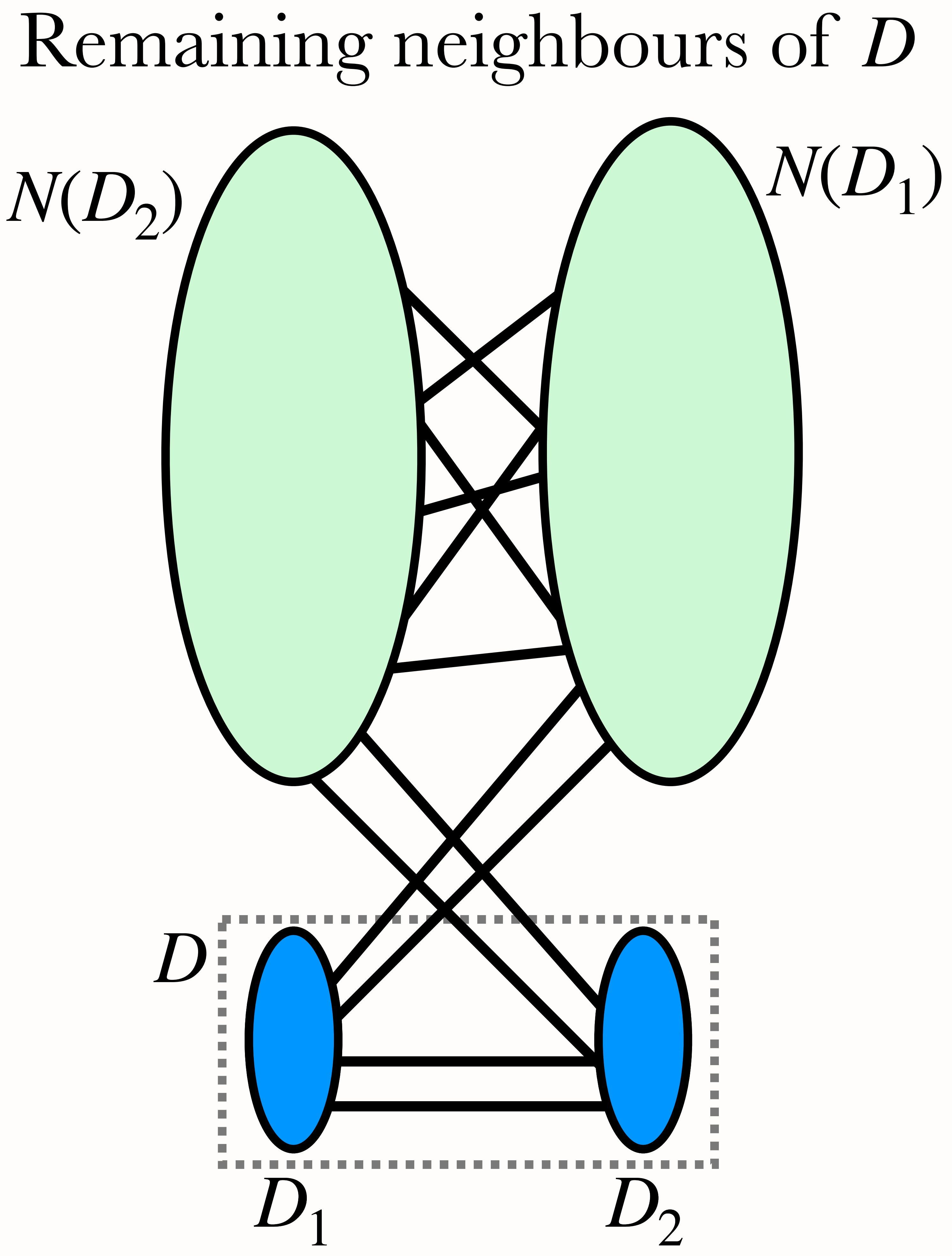
Finding Replacement For C

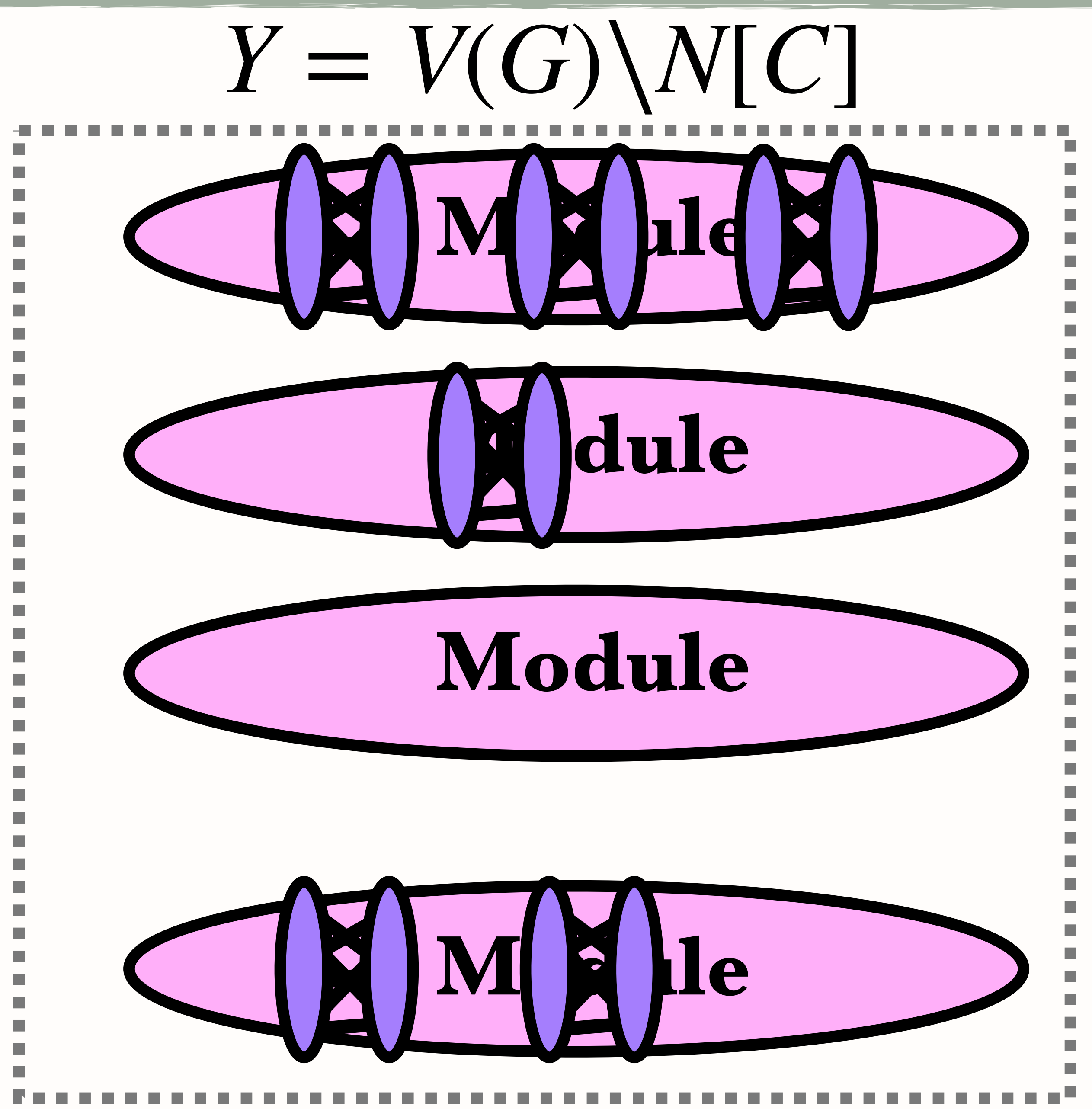


Delete such vertices v!

Finding Replacement For C

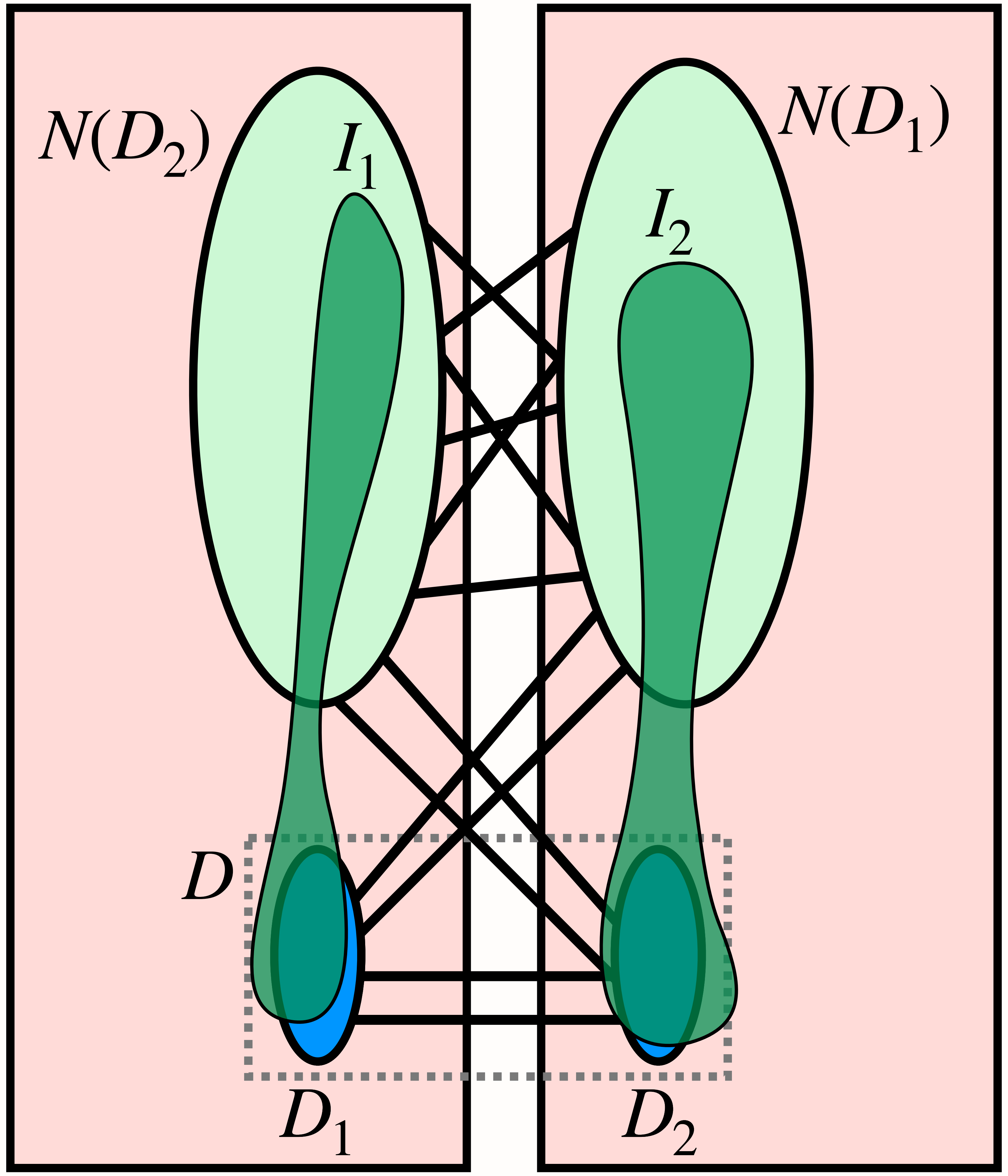
- Partition ***remaining*** $N[D]$ into the two disjoint sets, $N(D_1)$ and $N(D_2)$.



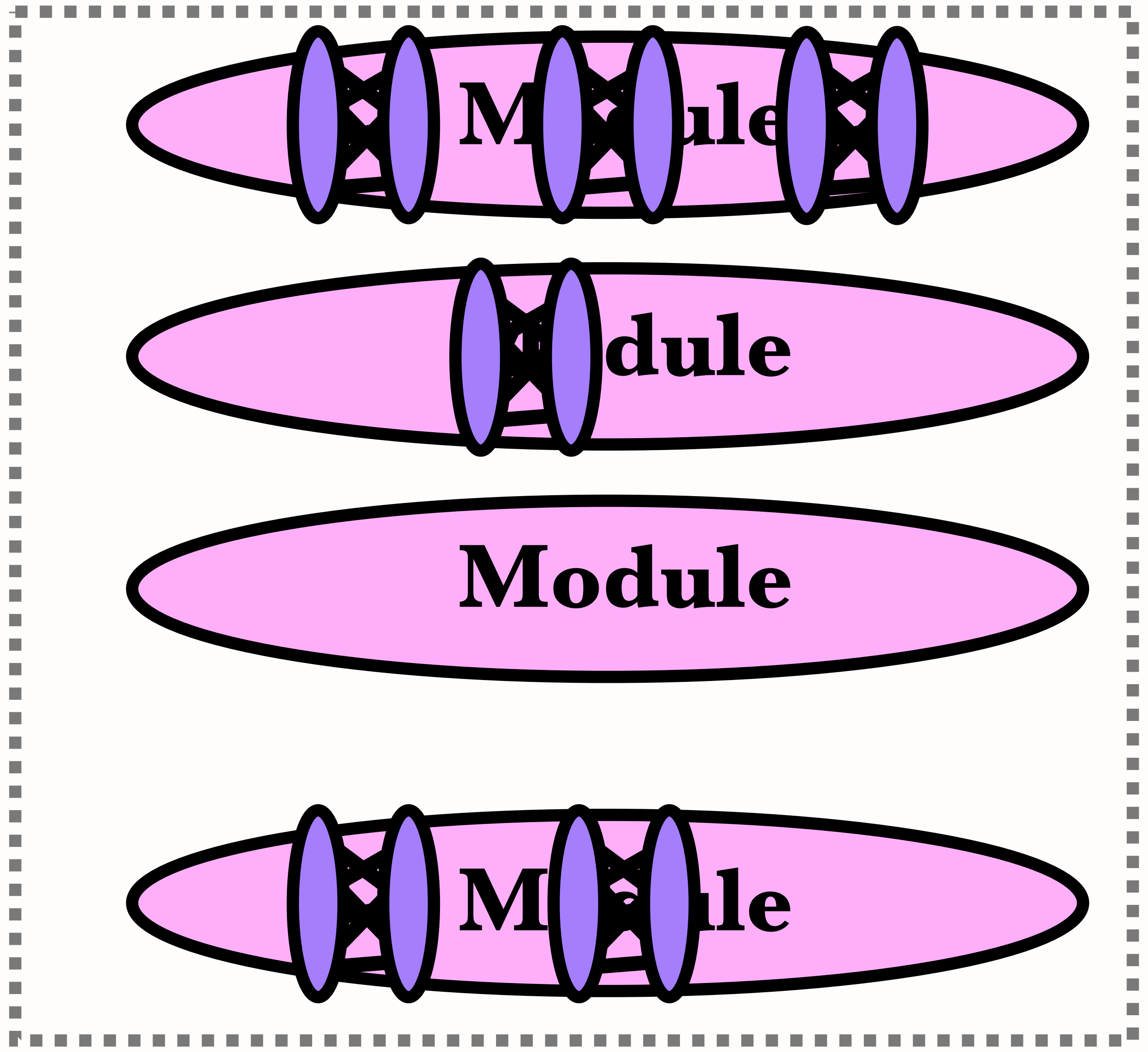


Finding Replacement For C

- Partition ***remaining*** $N[D]$ into the two disjoint sets, $N(D_1)$ and $N(D_2)$.
- Find independent sets I_1 and I_2 of maximum weights in $N(D_2) \cup D_1$ and $N(D_1) \cup D_2$, resp.



$Y = V(G) \setminus N[C]$



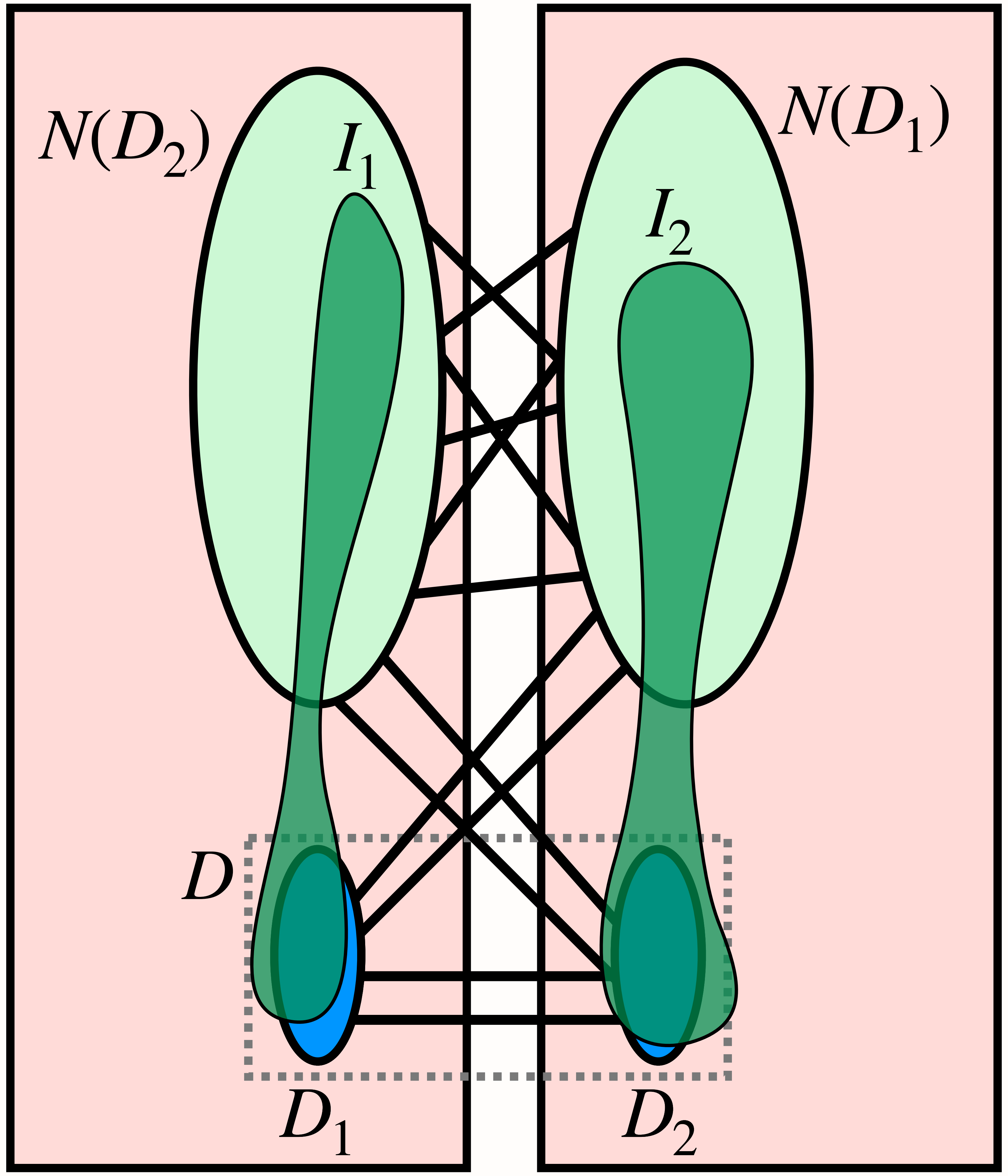
RECALL

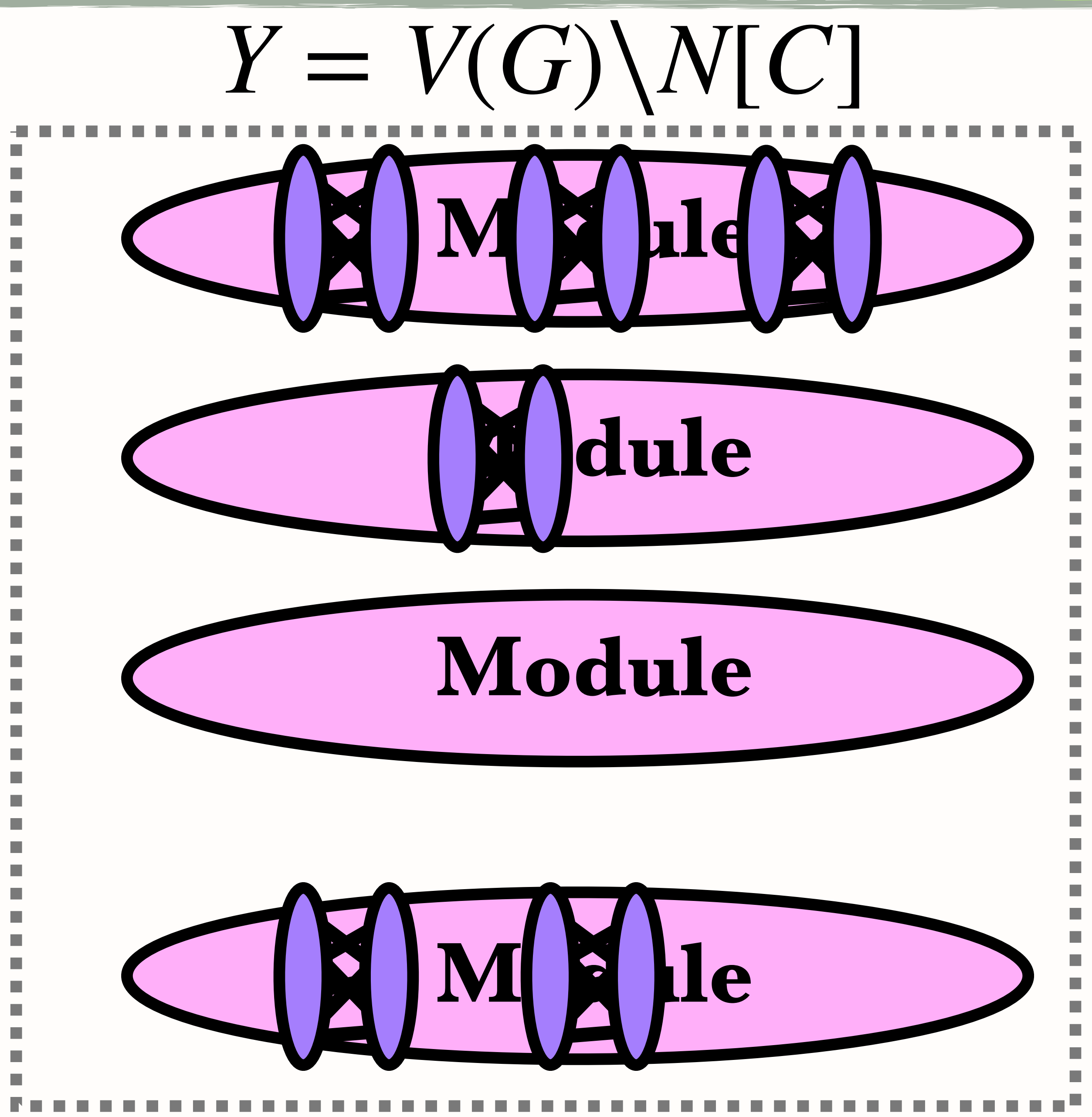
Max. Wt. Independent Set on P_5-free graphs has a polynomial time algorithm (Lokshtanov et al.)

Finding Replacement For C

- Partition ***remaining*** $N[D]$ into the two disjoint sets, $N(D_1)$ and $N(D_2)$.
- Find independent sets I_1 and I_2 of maximum weights in $N(D_2) \cup D_1$ and $N(D_1) \cup D_2$, resp.

$I_1 \cup I_2$ is a replacement for C!





Solution Covering Family

For a graph G and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, a family of vertex subsets $\mathscr{C} \subseteq 2^{V(G)}$ is a ***solution covering family*** if for any $S \subseteq V(G)$ of maximum total weight where $G[S]$ is bipartite, there is a sub-family $\mathscr{C}' \subseteq \mathscr{C}$ such that:

- $S = \cup_{X \in \mathscr{C}'} X$
- Sets in $\mathscr{C}'$ are pairwise disjoint
- No edge between different sets in $\mathscr{C}'$, i.e., for distinct $X, Y \in \mathscr{C}'$, $E(X, Y) = \emptyset$

LEMMA

Given a P_5-free graph G on n vertices and a weight function $\mathsf{w} : V(G) \to \mathbb{Q}$, there is a polynomial-time algorithm that outputs a solution covering family of size $O(n^6)$.

Two Ingredients

THEOREM

Odd Cycle Transversal admits a polynomial time algorithm on P_5-free graphs.

Ingredient 1: A polynomial-sized ***solution covering family***.

Ingredient 2: Translating solution to finding ***independent sets on a P_5-free auxiliary graph*** over a solution covering family.

Conclusion & Open Problems

- Independent Set admits a polynomial time algorithm on P_i-free graphs, which also works for the counting version. Can we obtain such an algorithm for Odd Cycle Transversal?

Conclusion & Open Problems

- Independent Set admits a polynomial time algorithm on P_i-free graphs, which also works for the counting version. Can we obtain such an algorithm for Odd Cycle Transversal?

- The complete complexity classification Odd Cycle Transversal on H-free graph where H is connected is fully known. What about disconnected Hs?

Conclusion & Open Problems

- Independent Set admits a polynomial time algorithm on P_i-free graphs, which also works for the counting version. Can we obtain such an algorithm for Odd Cycle Transversal?
- The complete complexity classification Odd Cycle Transversal on H-free graph where H is connected is fully known. What about disconnected Hs?
- Complexity status of other classical graph problems on such graph classes?

Conclusion & Open Problems

- Independent Set admits a polynomial time algorithm on P_i-free graphs, which also works for the counting version. Can we obtain such an algorithm for Odd Cycle Transversal?
- The complete complexity classification Odd Cycle Transversal on H-free graph where H is connected is fully known. What about disconnected Hs?
- Complexity status of other classical graph problems on such graph classes?

Thanks!