
The SORCERY and PERILS of PACKING
STOC 2026
We packed rectangles too!
I love Algorithms
Paul Erdős
Master of Combinatorics
Terence Tao
Wizard of Analysis
OUR TOOLS
Container Packing
Dynamic Program
Linear Programming (LP)
Color Coding
L-packing
Generalized Assignment Problems
Corridor Decomposition
Fine-Grained Complexity
ANALYSIS & PROOFS
P != NP
3-SUM Conjecture
APX-HARDNESS
ETR-HARDNESS
SETH
Better Approximations
Efficient Algorithms
New Techniques
FROM THEORY TO PRACTICE (ONE RECTANGLE AT A TIME)
WE STUDY
✔ Rectangle Packing
✔ Strip Packing
✔ Bin Packing
and more!
WE USE MAGIC
✔ LP relaxations
✔ Dynamic Programming
✔ Decomposition
✔ Color Coding
WE ACHIEVE
✔ Better Approximations
✔ New Algorithms
✔ Tight Hardness Results
✔ Fine-Grained Bounds
PACKING IS MAGIC!

Celebrating a Sapphire Milestone:
65 Years!

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Diffuse reflection diameter and radius for convex-quadrilateralizable polygons

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ABSTRACT

In this paper we study the *diffuse reflection diameter* and *diffuse reflection radius* problems for *convex-quadrilateralizable polygons*. In the *usual model* of diffuse reflection, a light ray incident at a point on the reflecting surface is reflected from that point in all possible inward directions. A ray reflected from a polygonal edge may graze that reflecting edge but an incident ray cannot graze the reflecting edge. The *diffuse reflection diameter* of a simple polygon P is the minimum number of diffuse reflections that may be needed in the worst case to illuminate any target point t from any point light source s inside P. We show that the diameter is upper bounded by $\frac{3n-10}{4}$ in our *usual model* of diffuse reflection for *convex-quadrilateralizable polygons*. To the best of our knowledge, this is the first upper bound on diffuse reflection diameter within a fraction of n for such a class of polygons. We also show that the *diffuse reflection radius* of a *convex-quadrilateralizable simple polygon* with n vertices is at most $\frac{3n-10}{8}$ under the *usual model* of diffuse reflection. In other words, there exists a point inside such a polygon from which $\frac{3n-10}{8}$ *usual diffuse reflections* are *always* sufficient to illuminate the entire polygon. In order to establish these bounds for the usual model, we first show that the diameter and radius are $\frac{n-4}{2}$ and $\lfloor\frac{n-4}{4}\rfloor$ respectively, for the same class of polygons for a *relaxed model* of diffuse reflections; in the *relaxed model* an incident ray is permitted to graze a reflecting edge before turning and reflecting off the same edge at any interior point on that edge. We also show that the worst-case diameter and radius lower bounds of $\frac{n-4}{2}$ and $\lfloor\frac{n-4}{4}\rfloor$ respectively, are sometimes attained in the *usual model*, as well as in the *relaxed model* of diffuse reflection.

Packing Maximum Number of Rectangles into a Square

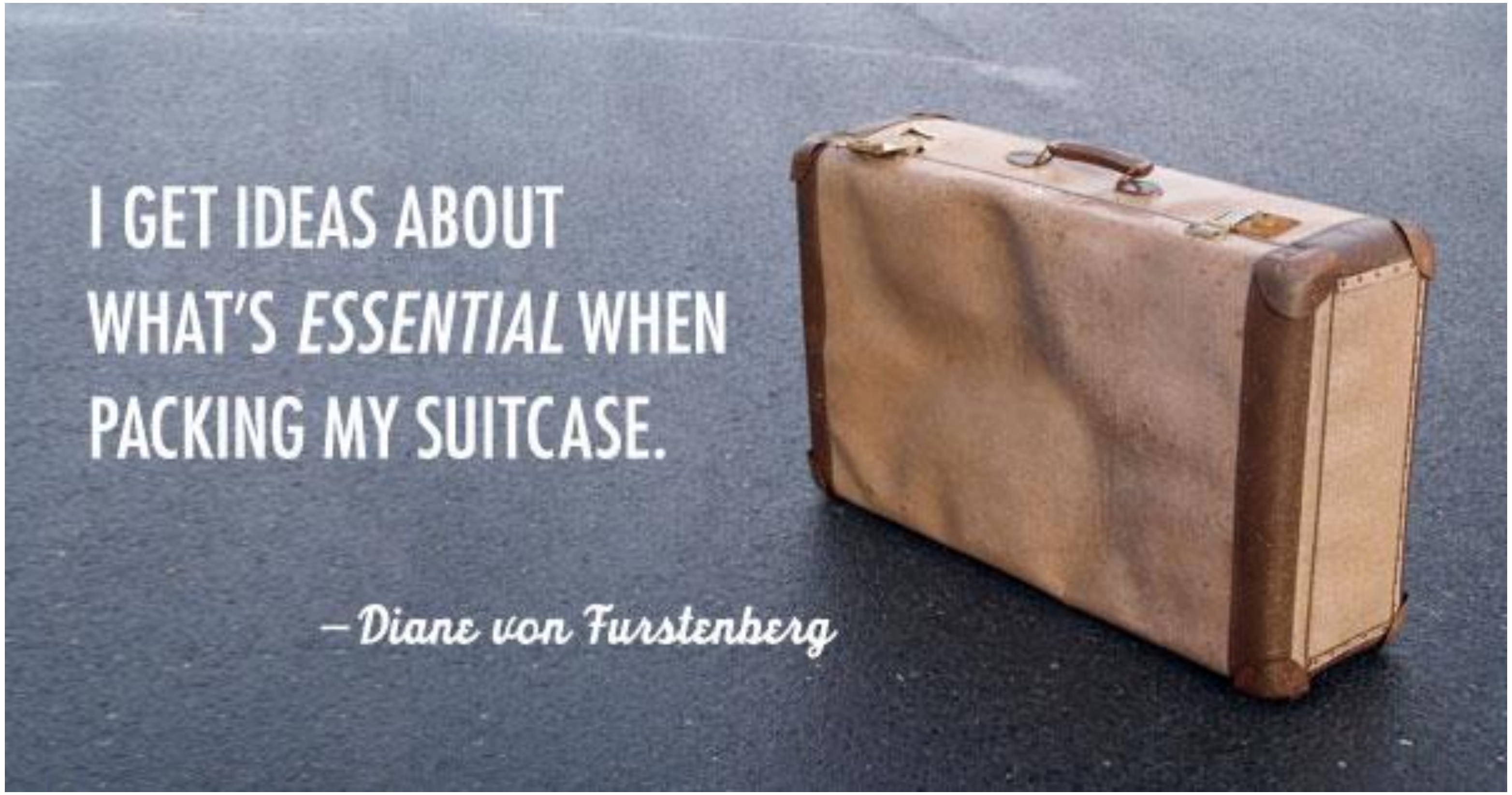
Arindam Khan

Joint work with

Debajyoti Kar,
IISc Bangalore

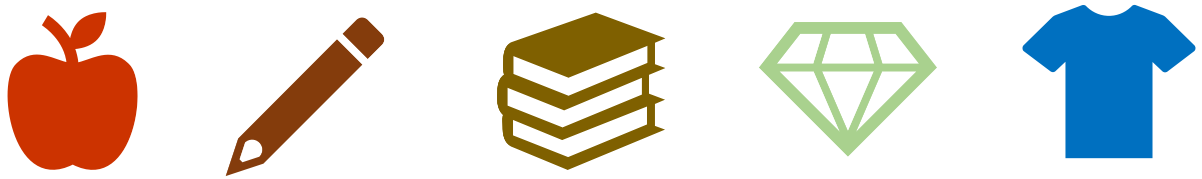
Andreas Wiese,
TU Munich

*To appear in **Symposium on Theory of Computing (STOC), 2026***


I GET IDEAS ABOUT
WHAT'S *ESSENTIAL* WHEN
PACKING MY SUITCASE.
—Diane von Furstenberg

The Knapsack Problem

- Items each with a size and profit
- Knapsack of capacity K



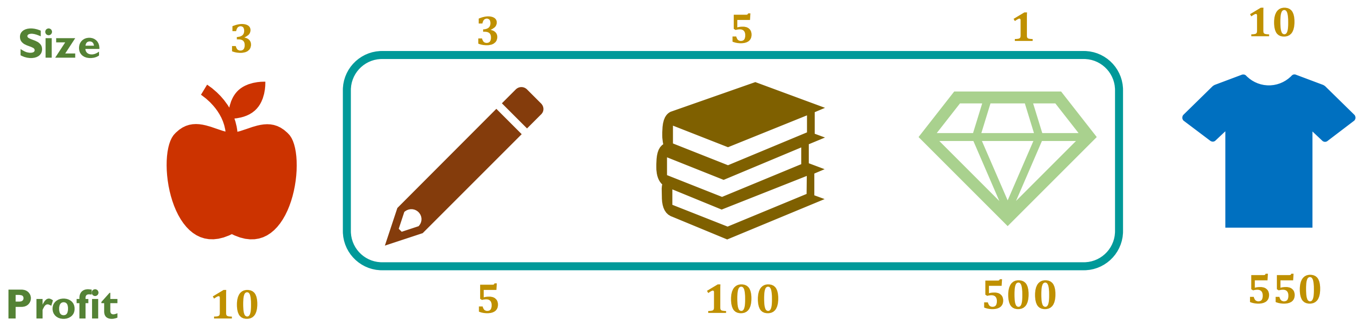
Size	3	3	5	1	10
Profit	10	5	100	500	550

Goal: Pack maximum profit subset of total size at most K

$K = 10$

The Knapsack Problem

- Items each with a size and profit
- Knapsack of capacity K

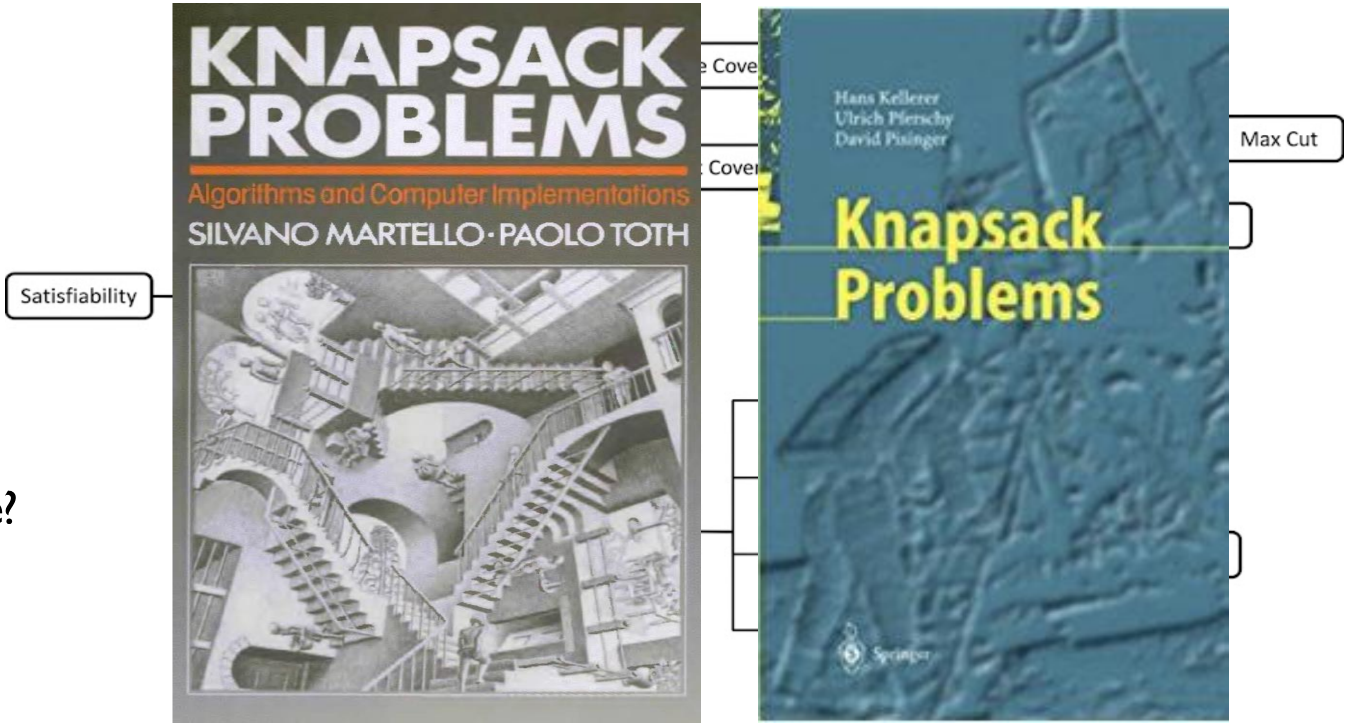


Goal: Pack maximum profit subset of total size at most K

$K = 10$

The Knapsack Problem

- One of Karp's 21 NP-complete problems
- Weakly NP-hard:
  Can be solved in pseudopolynomial time (when all numeric data are polynomial in number of items).
- Cannot be solved exactly in polynomial time
- Can we find near-optimal solution in polynomial time?
- **Approximation algorithms**
- **FPTAS exists!**



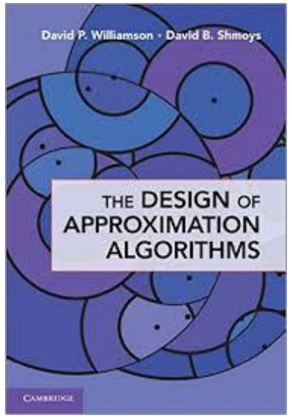
Approximation Algorithms

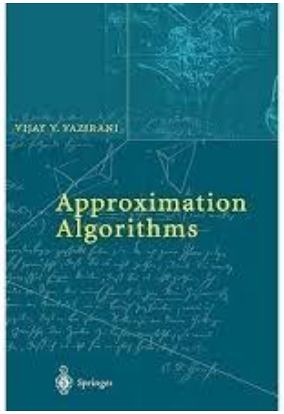
Approximation Algorithms

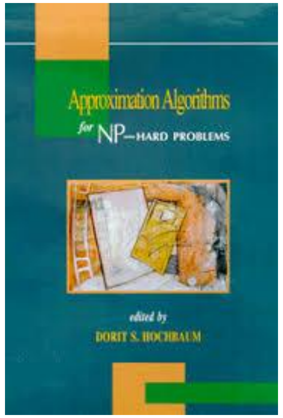
- Approximation algorithms are efficient algorithms that find near-optimal solution.
- For a minimization problem, an algorithm A is α-(absolute) approximation (α>1) if $ALG(I) \le \alpha\, OPT(I)$ for all input instances I .

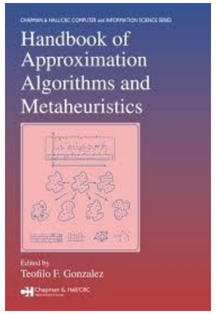
➢ For a maximization problem α-approximation ($\alpha > 1$):

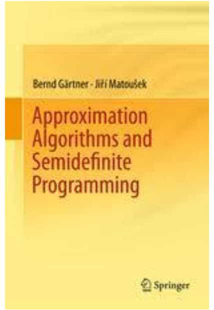
$$OPT \ge ALG \ge \frac{1}{\alpha} \cdot OPT$$

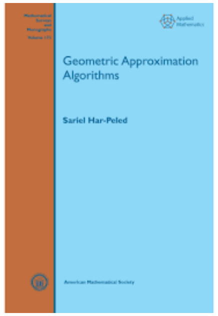


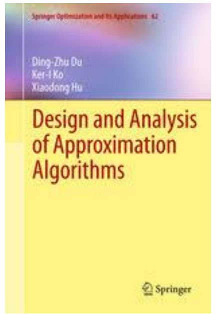












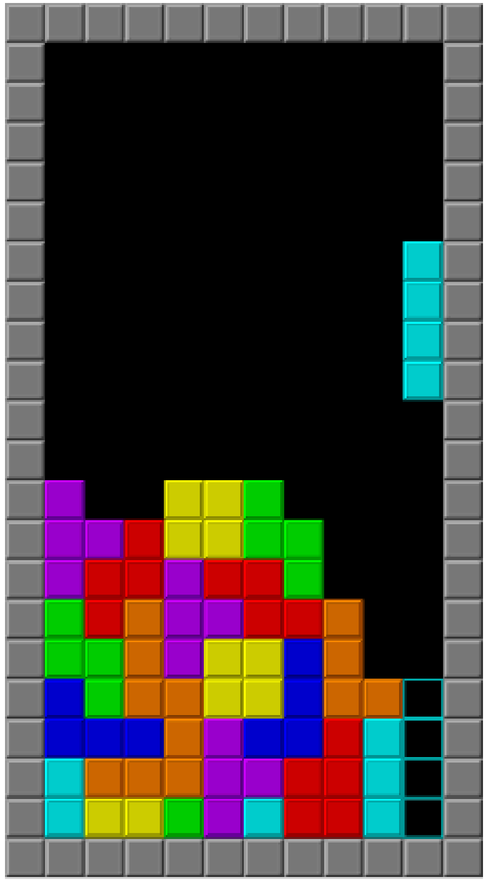
PTAS

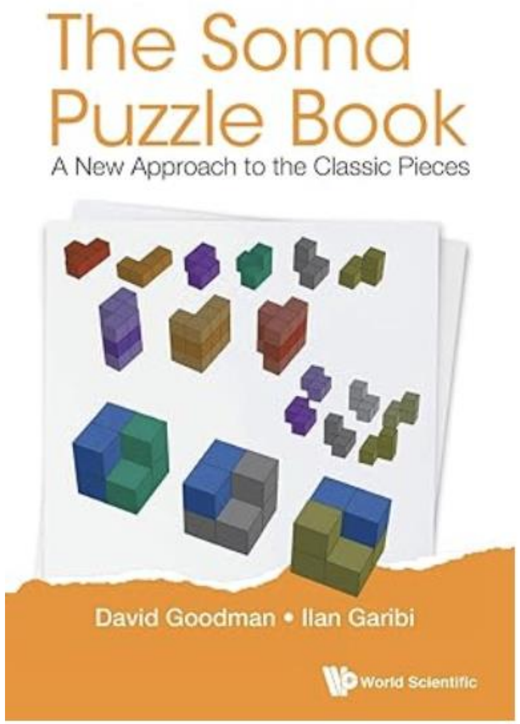
- Polynomial Time Approximation Schemes (PTAS):
  If for every constant $\varepsilon > 0$, there exists a polynomial-time ($O(n^{f(\varepsilon)})$-time) algorithm A_ε such that $A_\varepsilon(I) \le (1+\varepsilon)\, OPT(I)$.
- Efficient PTAS (EPTAS): if running time is $O(f(\varepsilon).n^c)$.
- Fully PTAS (FPTAS): if running time is $O((n/\varepsilon)^c)$.

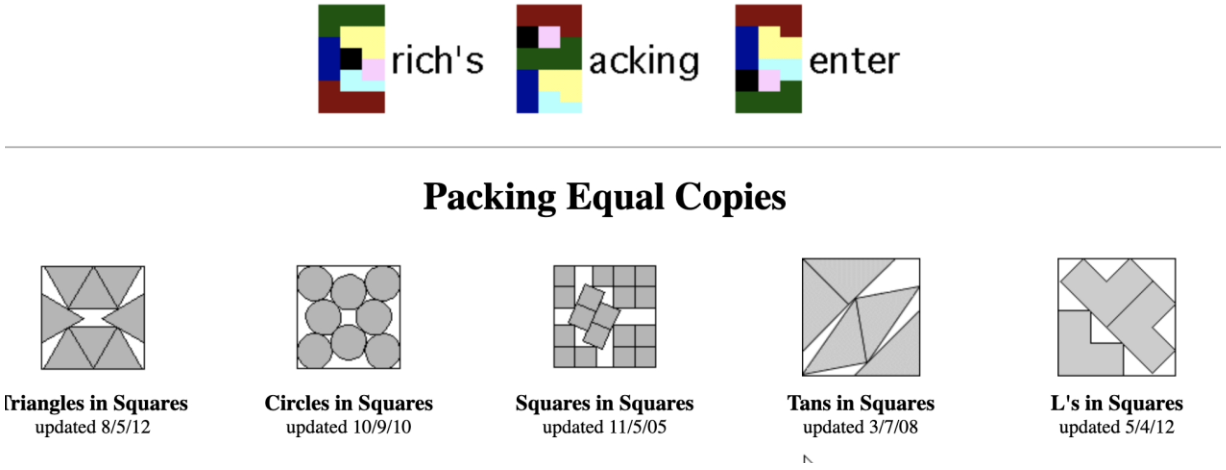
- APX-hardness implies no PTAS.
- W[1]-hardness implies no EPTAS.
- Strong NP-hardness implies no FPTAS.

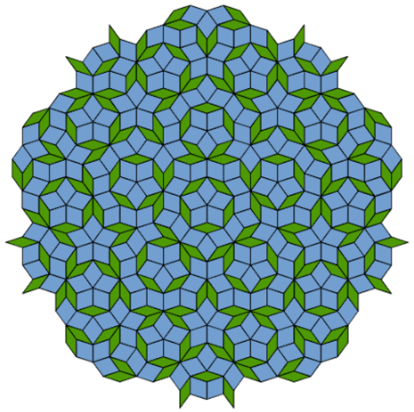
Packing is hard!

Packing Problems: Placement of objects nonoverlappingly under some constraints









Packing Problems: Placement of objects nonoverlappingly under some constraints

On Packing Squares with Equal Squares

P. Erdös

Stanford University and The Hungarian Academy of Sciences

AND

R. L. Graham

Bell Laboratories, Murray Hill, New Jersey

Communicated by the Managing Editors

Received November 11, 1974

PERFECTLY PACKING A SQUARE BY SQUARES OF NEARLY HARMONIC SIDELENGTH

TERENCE TAO

Abstract. A well known open problem of Meir and Moser asks if the squares of sidelength $1/n$ for $n \geq 1$ can be packed perfectly into a square of area $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. In this paper we show that for any $1/2 < t < 1$, and any n_0 that is sufficiently large depending on t, the squares of sidelength n^{-t} for $n \geq n_0$ can be packed perfectly into a square of area $\sum_{n=n_0}^{\infty} \frac{1}{n^{2t}}$. This was previously known for $1/2 < t < 2/3$ (in which case one can take $n_0 = 1$).

"I think packing problems are appealing to mathematicians and computer scientists because they seem very simple – just place these items into the container. Yet they tend to be extremely complicated to actually solve."
-- Erik Demaine (MIT).

This talk: Rectangle Packing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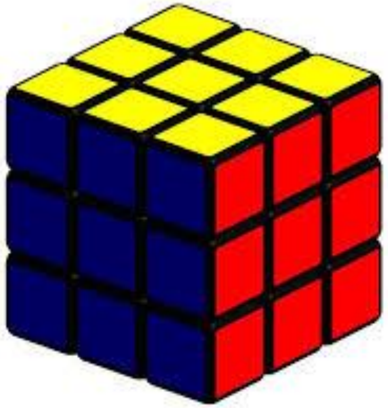
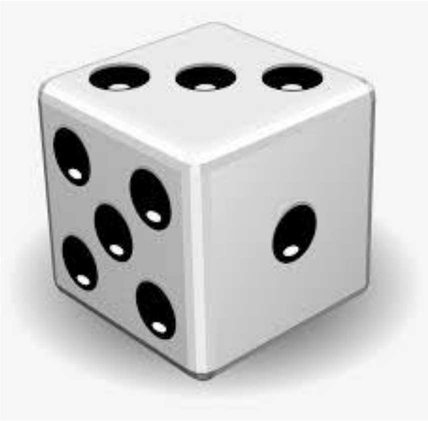
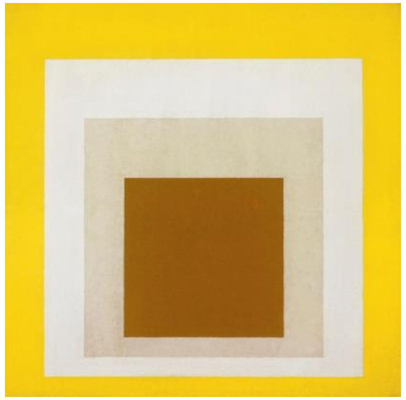
500
LeftoverCurrency.com

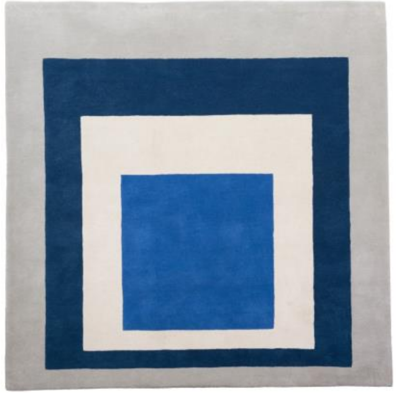

alamy stock photo



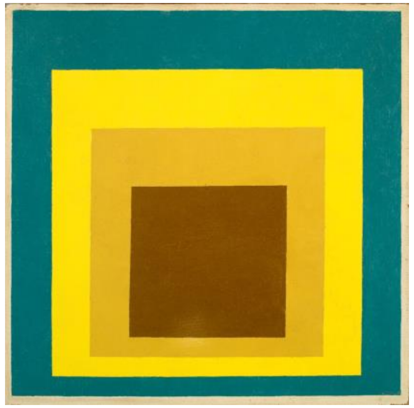


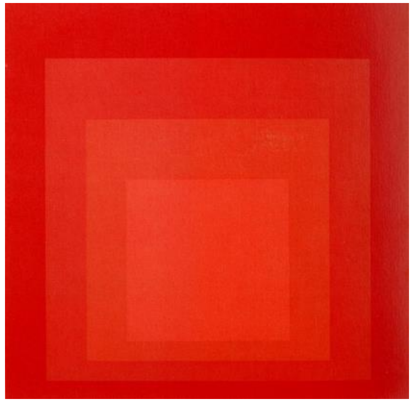

1 2 3 4
5 6 7 8
9 10 11 12
13 15 14



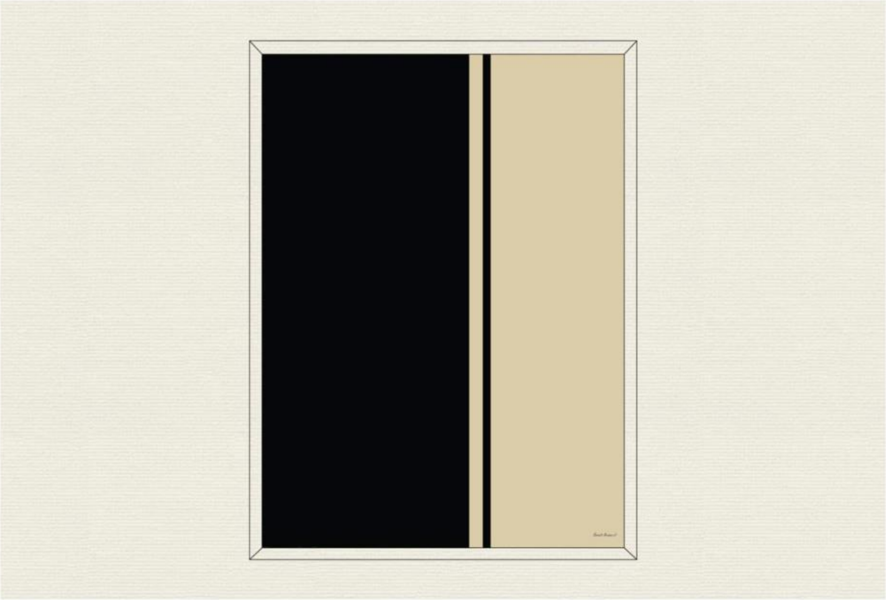


"I still like to believe that the square is a human invention. And that tickles me. So when I have a preference for it then I can only say excuse me."
– Josef Albers (on *Homage to the square*)





28. *Black Fire I*, 1961, by Barnett Newman

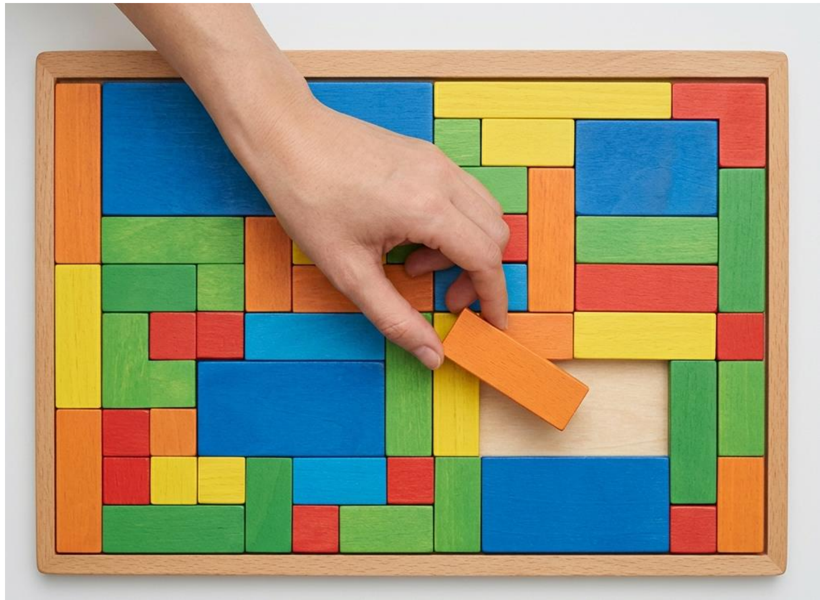


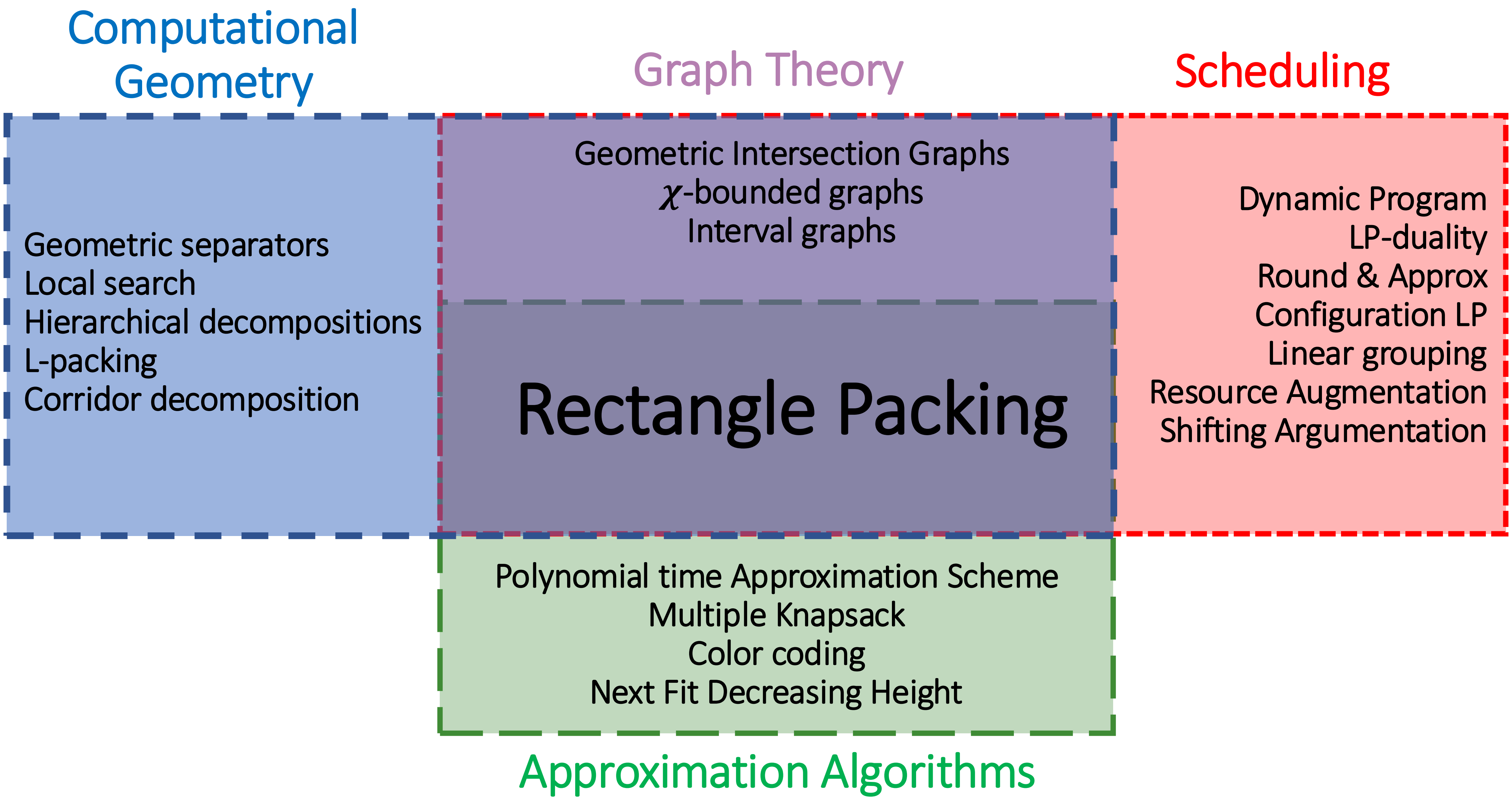
Sold for: $84.2 million

26. *Suprematist Composition*, 1916, by Kazimir Malevich

Sold for: $85.8 million

- Introduced by Gilmore & Gomory in the 1960s, in the context of **Cutting Stock problem** in Operations Research
- [Coffman-Garey-Johnson-Tarjan ’80] – First algorithmic analysis
- [Christensen-K.-Pokutta-Tetali ’17] – Survey on approximation algorithms for multidimensional packing
- [Ali-Ramos-Carravilla-Oliveira ’22] – Comprehensive survey on Geometric Packing with >200 research articles

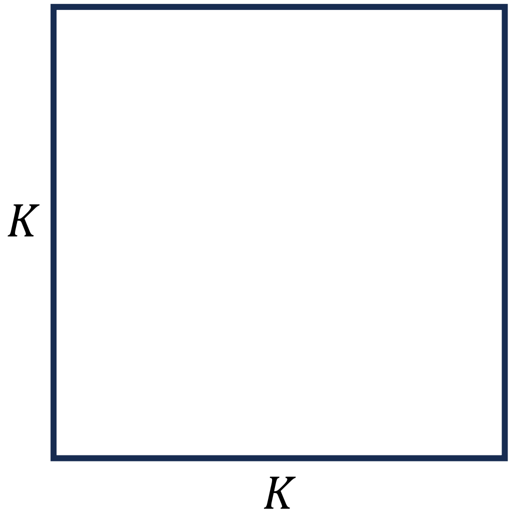


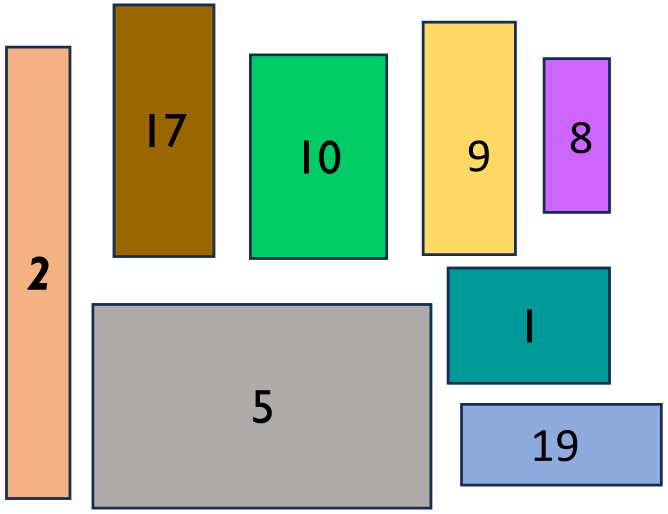

Computational Geometry
Graph Theory
Scheduling
Geometric separators
Local search
Hierarchical decompositions
L-packing
Corridor decomposition
Geometric Intersection Graphs
χ-bounded graphs
Interval graphs
Rectangle Packing
Dynamic Program
LP-duality
Round & Approx
Configuration LP
Linear grouping
Resource Augmentation
Shifting Argumentation
Polynomial time Approximation Scheme
Multiple Knapsack
Color coding
Next Fit Decreasing Height
Approximation Algorithms

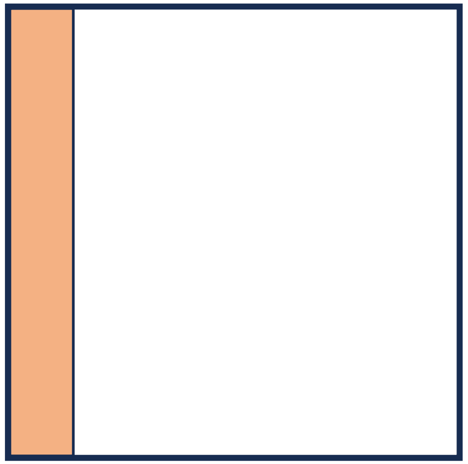
Formal Setup

Input:

- Square $K \times K$ knapsack
- Items – Axis-aligned rectangles with profits
- Side lengths in $\{1, 2, \dots, K\}$
- Can be rotated by $90°$







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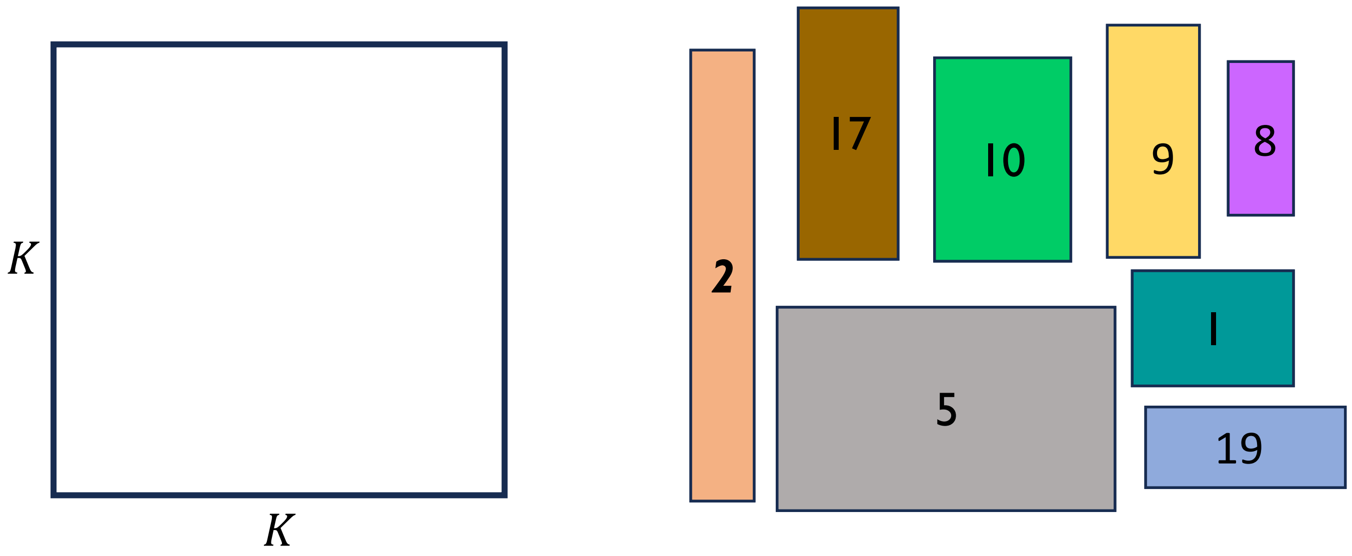

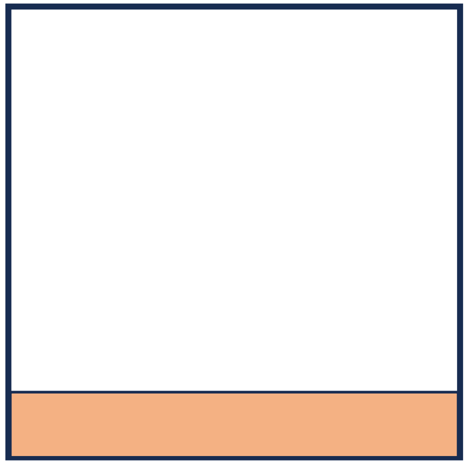

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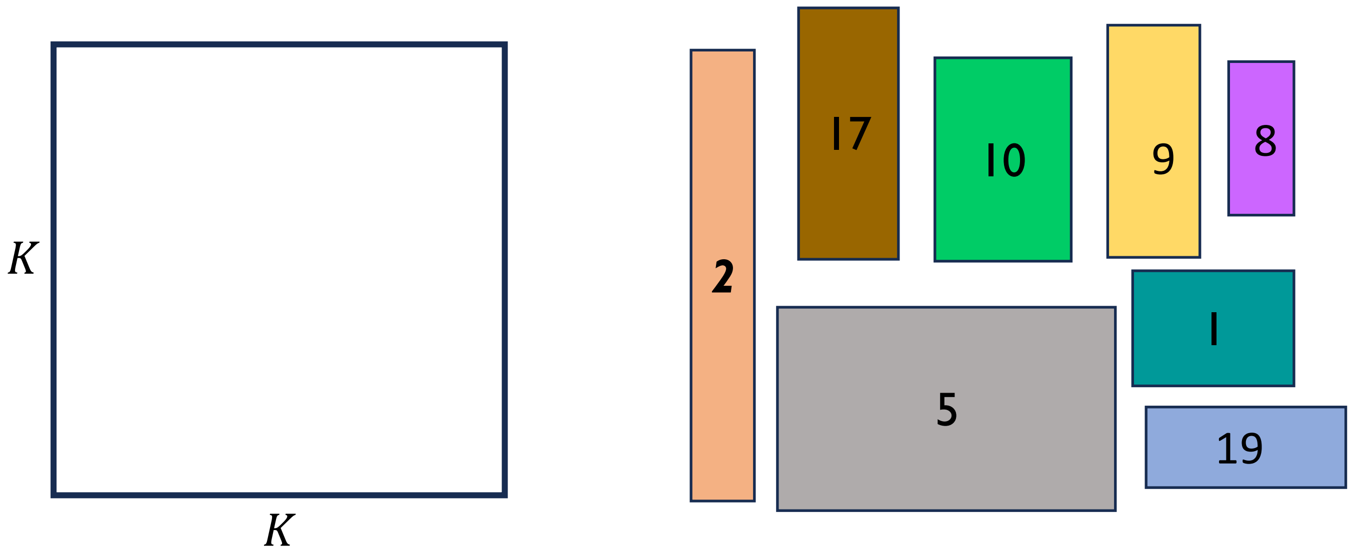
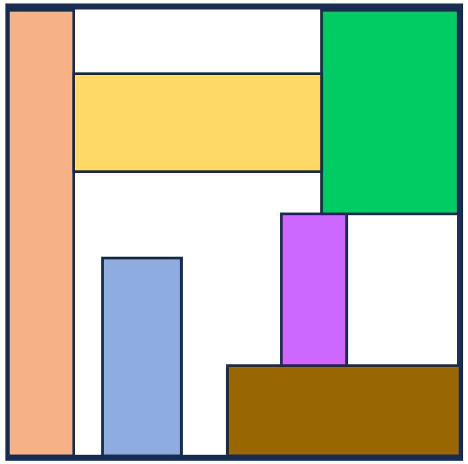
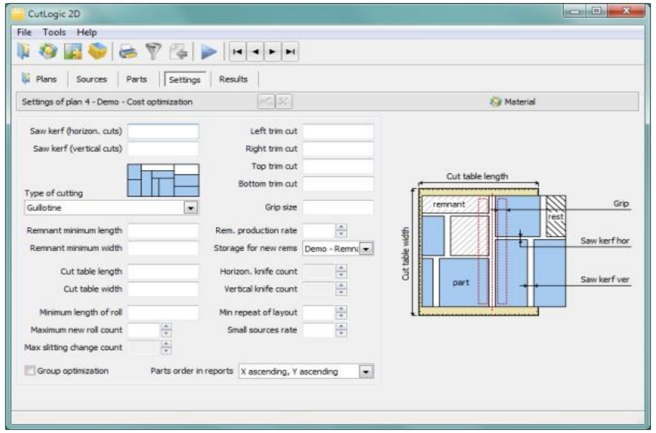


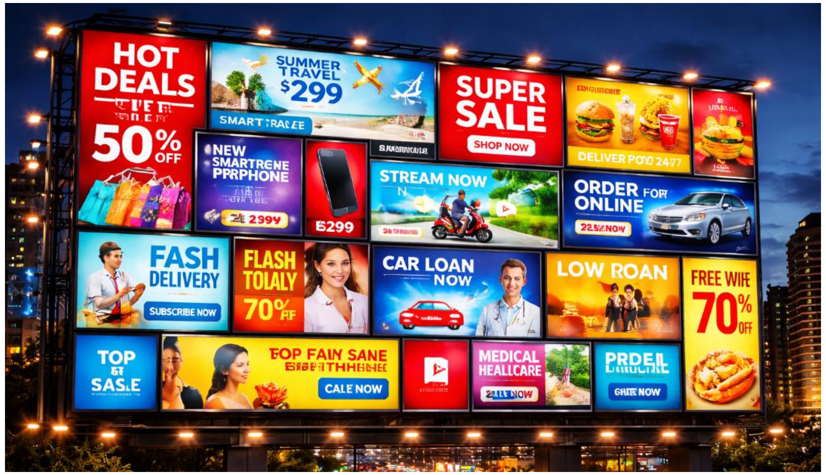
Goal: Pack maximum profit subset of rectangles non-overlappingly



Applications

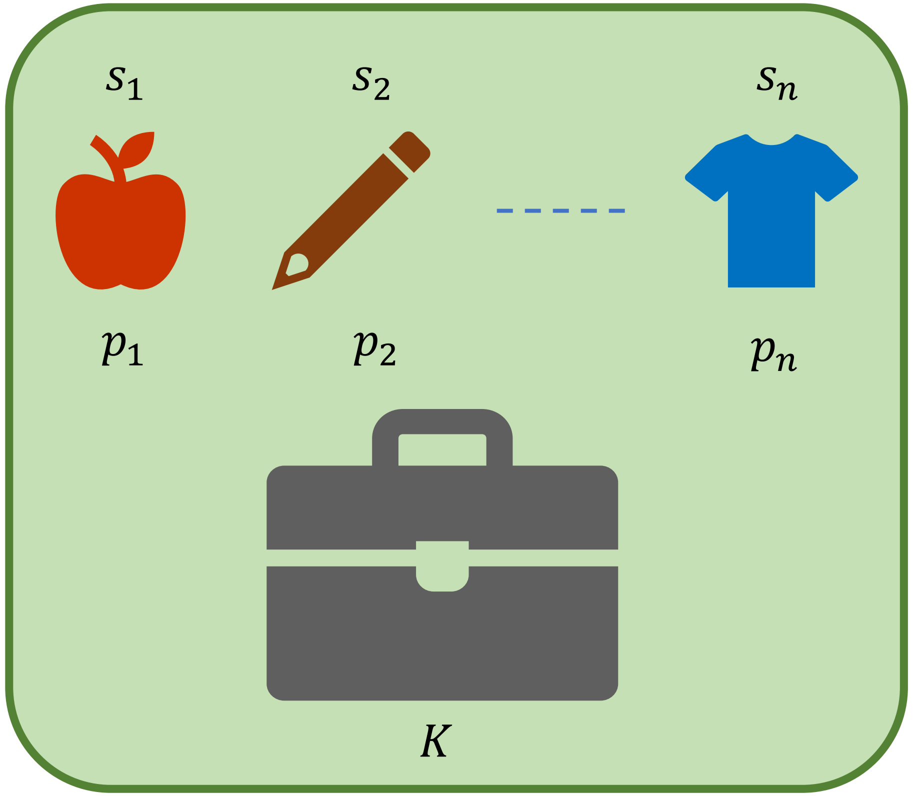
- VLSI design
- Cutting Stock
- Ad-placements
- Truck loading
- Robotics

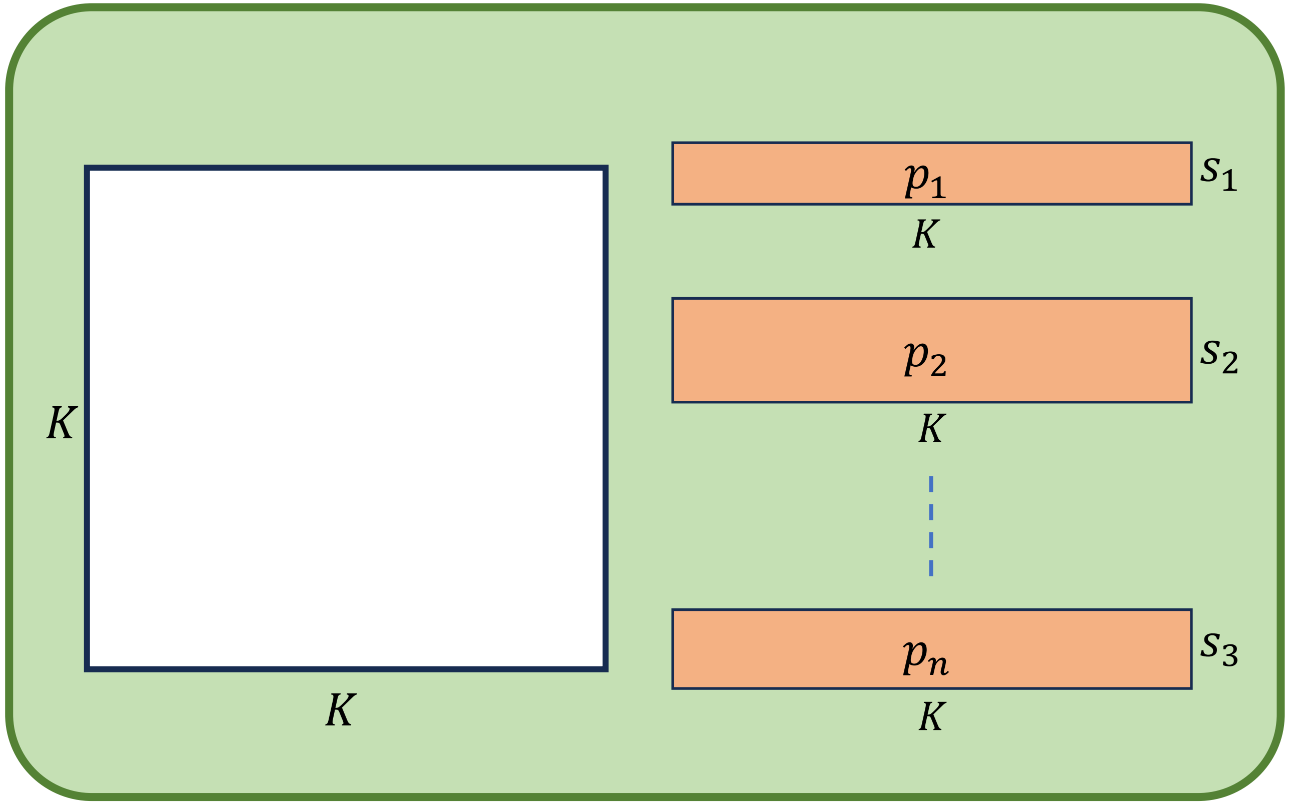




Is the problem NP-hard?

Reduction from classical Knapsack





Knapsack packs profit $P \iff$ There exists Rectangle Packing of profit P

What if all rectangles have equal profit?

Goal: Pack maximum **number** of rectangles

Focus of this talk

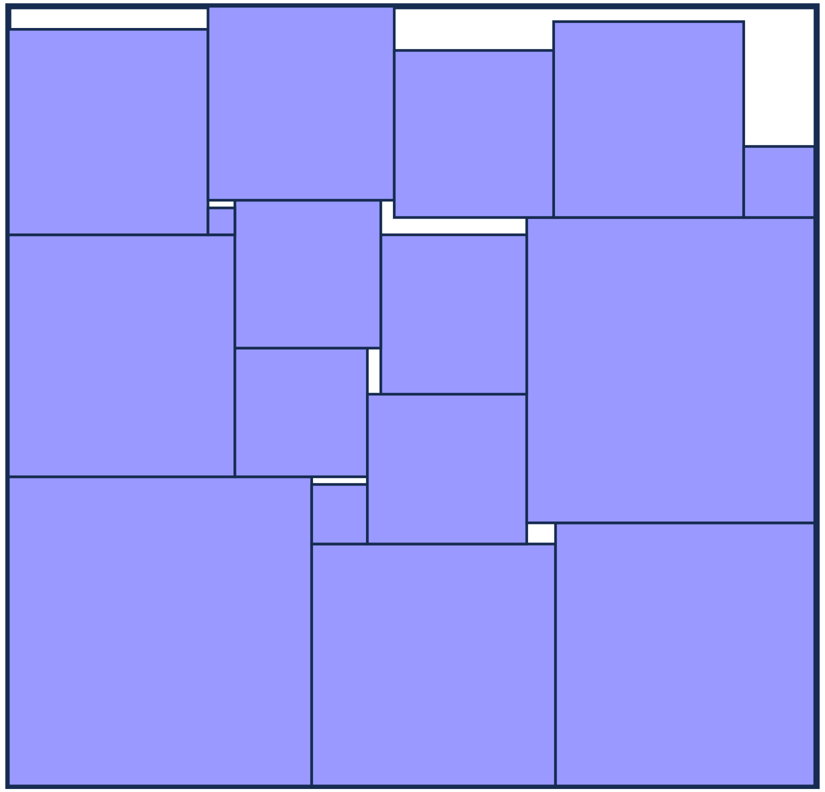
Is this special case NP-hard?

Previous reduction doesn't work.
O(n log n) time exact algorithm for 1D.

[Leung-Tam-Wong-Young-Chin '90] – Strongly NP-hard even for squares

No FPTAS

Look for approximations – can we get PTAS?

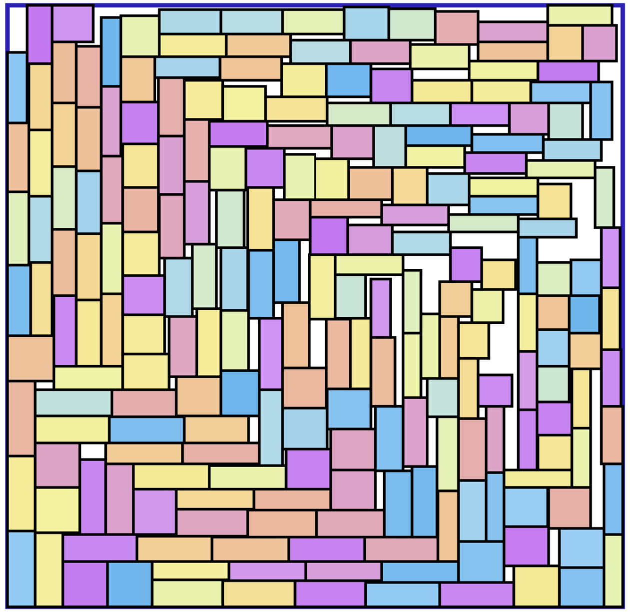


α-approximation $(\alpha > 1)$ – Packs at least $\frac{1}{\alpha} \cdot |OPT|$ rectangles

Is it easy to get an $O(1)$-approximation?

Intuitive strategy? – extend ideas from 1D?

Pick rectangles of minimum area



Lemma [Steinberg '97]

Any set of rectangles I with

- area$(I) \leq \frac{1}{2} K^2$,
- each rectangle has area $\leq \frac{1}{4} K^2$,

can be packed into a $K \times K$ square

Lemma [Steinberg '97]

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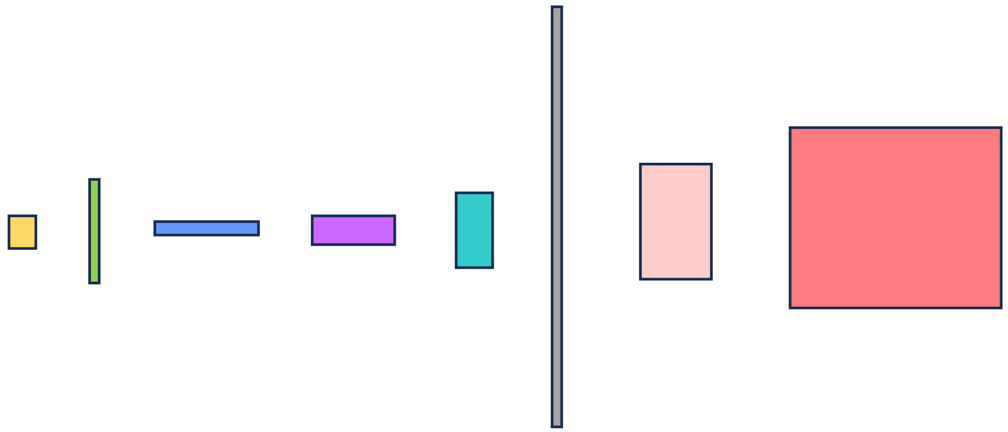

Naïve algorithm:

- Greedily select rectangles (in increasing area) as long as area does not exceed $\frac{1}{2}K^2$
- Discard rectangles of area $> \frac{1}{4}K^2$

Lemma [Steinberg '97]

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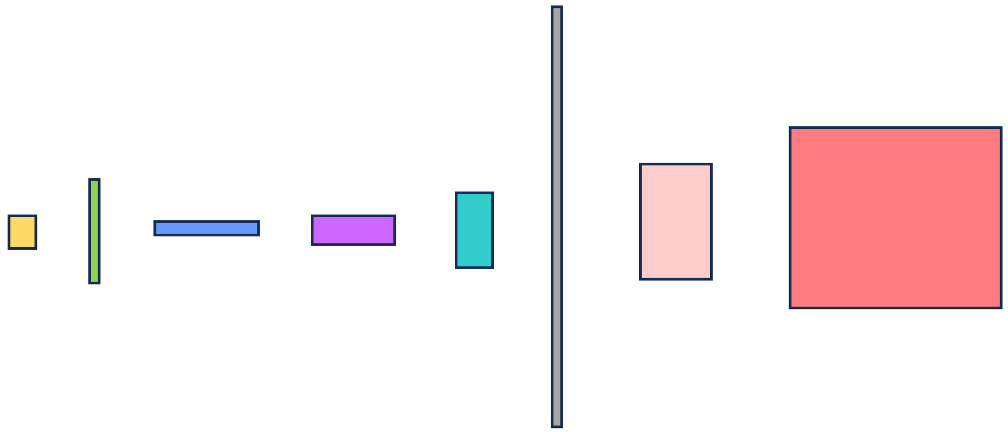


Naïve algorithm:

- Greedily select rectangles (in increasing area) as long as area does not exceed $\frac{1}{2}K^2$
- Discard rectangles of area $> \frac{1}{4}K^2$
- Pack all remaining rectangles (or only one rectangle if none remains)

$|ALG| \geq \frac{1}{\alpha} \cdot |OPT|$

α – Approximation ratio

Prior work:

Best-known in polynomial time

- [Jansen-Zhang, SODA '04] – $(2+\epsilon)$-approximation
- [Gálvez-Grandoni-Heydrich-Ingala-K.-Wiese, FOCS '17] – $\left(\frac{4}{3}+\epsilon\right)$-approximation
- [Adamaszek-Wiese, SODA '15] – $(1+\epsilon)$-approximation in $n^{(\log n)^{O_\epsilon(1)}}$ time, assuming $K = n^{(\log n)^{O(1)}}$
- [Gálvez-Grandoni-K.-Romero-Wiese, SoCG '21] – $\left(\frac{5}{4}+\epsilon\right)$-approximation in $(nK)^{O_\epsilon(1)}$ time
- [Jansen-SolisOba, MFCS '07, Heydrich-Wiese, SODA'16, Jansen-K.-Lira-Sreenivas, ICALP'22, Buchem-Deuker-Wiese, SoCG '24, Acharya-Bhore-Gupta-K.-Mondal-Wiese, ICALP'24] – Efficient PTAS when all items are squares (or fat objects like cubes, spheres or d-dimensional regular polytopes, etc.)
- [Bansal et al., ISAAC '09] – PTAS when profit of an item is equal to its area
- [Fishkin et al., MFCS '05] – PTAS when all rectangles are small
- [K.-Maiti-Sharma-Wiese, SoCG '21] – PTAS when the packing is guillotine separable and $K = n^{O(1)}$.

$|ALG| \geq \frac{1}{\alpha} \cdot |OPT|$

α – Approximation ratio

[Grandoni-Kratsch-Wiese, ESA '19] – Parameterized approximation scheme

Assuming $|OPT| = k$, returns a solution of size $\geq \frac{k}{1+\epsilon}$, in time $k^{O(k/\epsilon)}\, n^{O_\epsilon(1)}$ time.

PTAS as long as $k = O\left(\frac{\log n}{\log \log n}\right)$, also W[1]-hard -- rules out EPTAS (No PTAS in $f(\epsilon)n^{O(1)}$ time).

Big open question: Does there exist a PTAS (or even in quasi-poly or pseudo-poly time) in general?

One of the top 10 open problems in bin packing and knapsack survey by [Christensen-K.-Pokutta-Tetali '17]

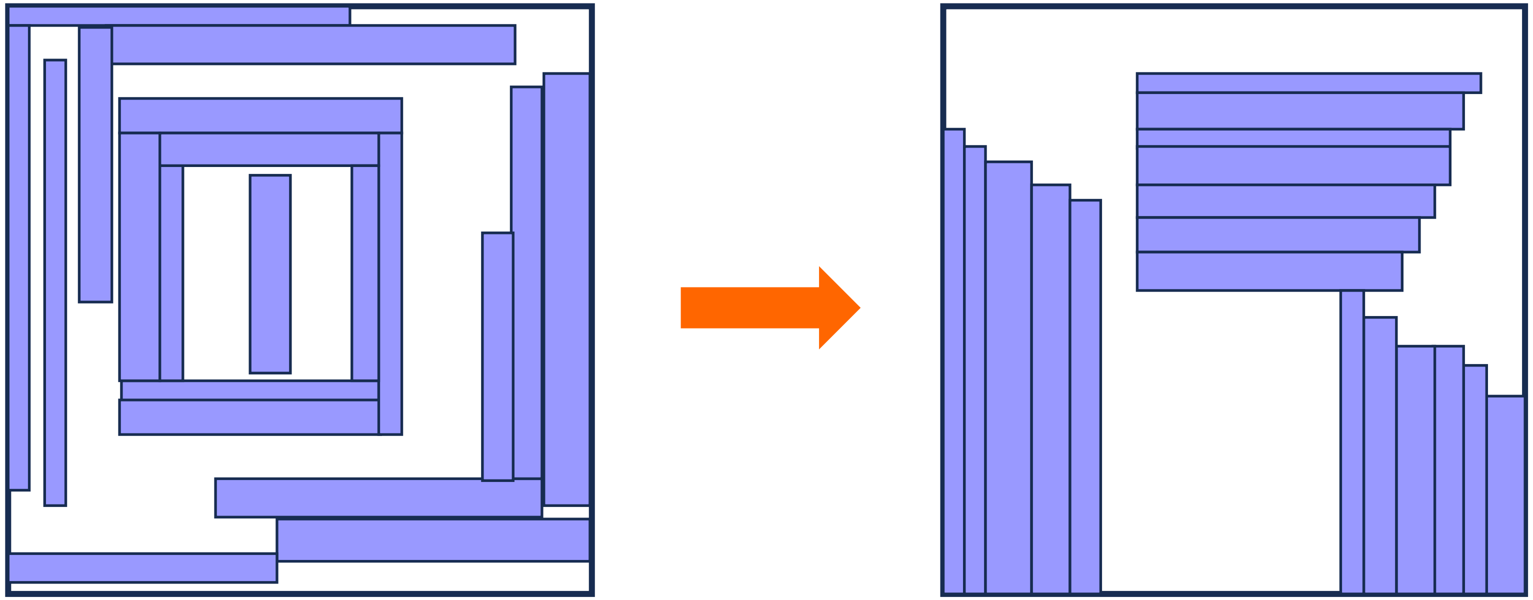
Our Result

There is a PTAS!

Techniques

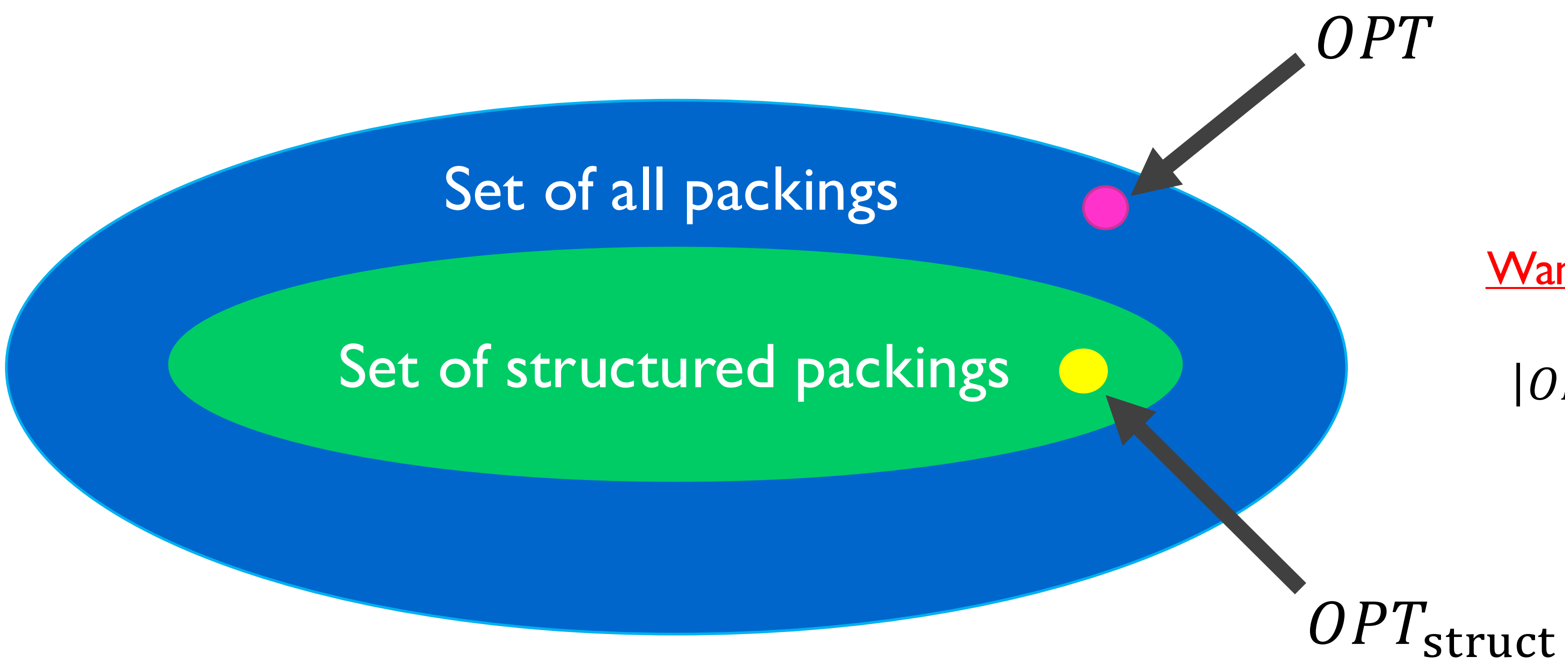
- Show the existence of "structured" packings

- **Algorithm:** Compute the best structured packing using dynamic programming



- Show the existence of "structured" packings

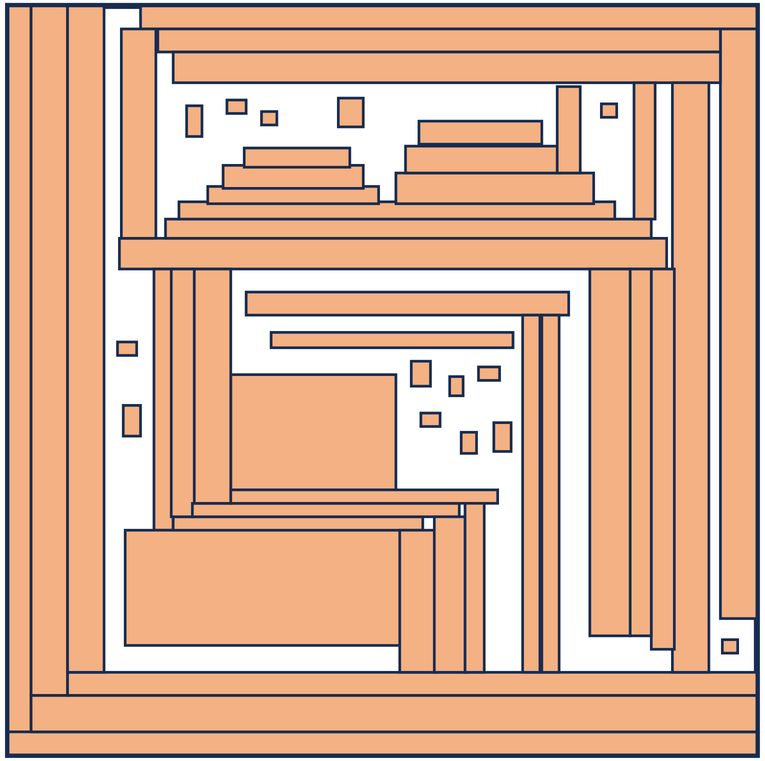
- **Algorithm:** Compute the best structured packing using dynamic programming



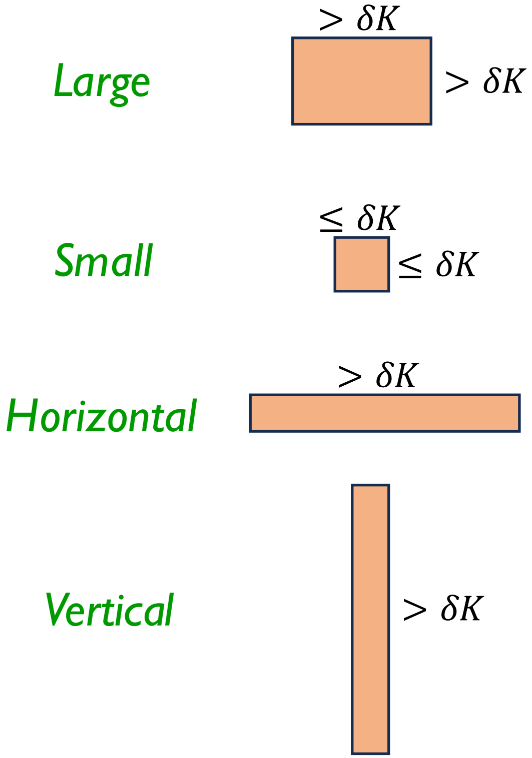
Want:

$$|OPT_{\text{struct}}(I)| \geq (1-\epsilon)|OPT(I)|$$

Restructure OPT to a "nice" packing



Item Classification:



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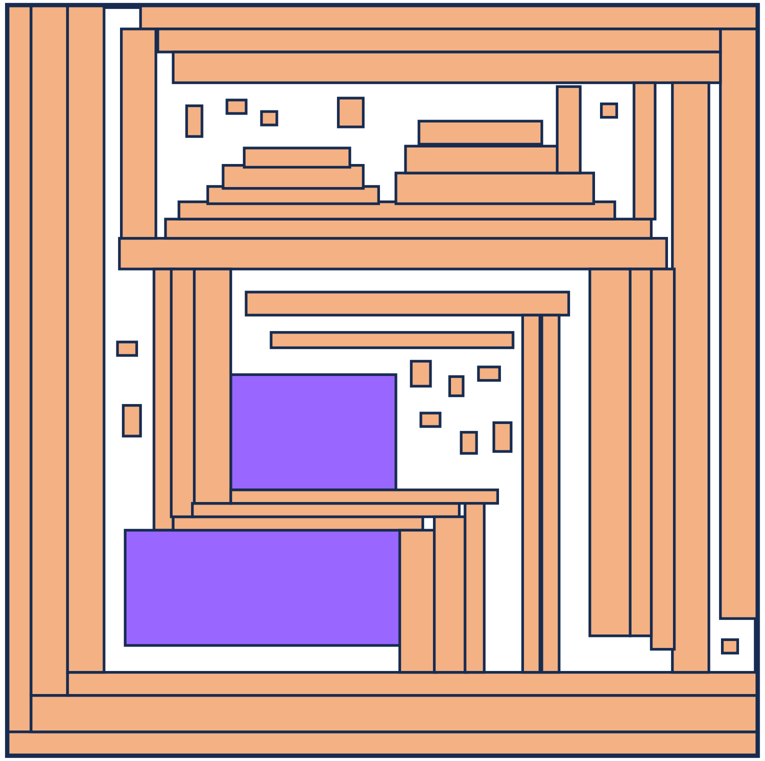

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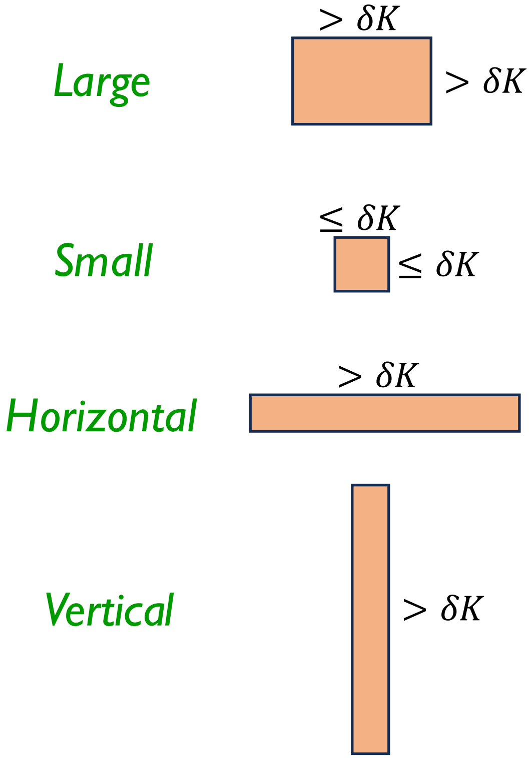
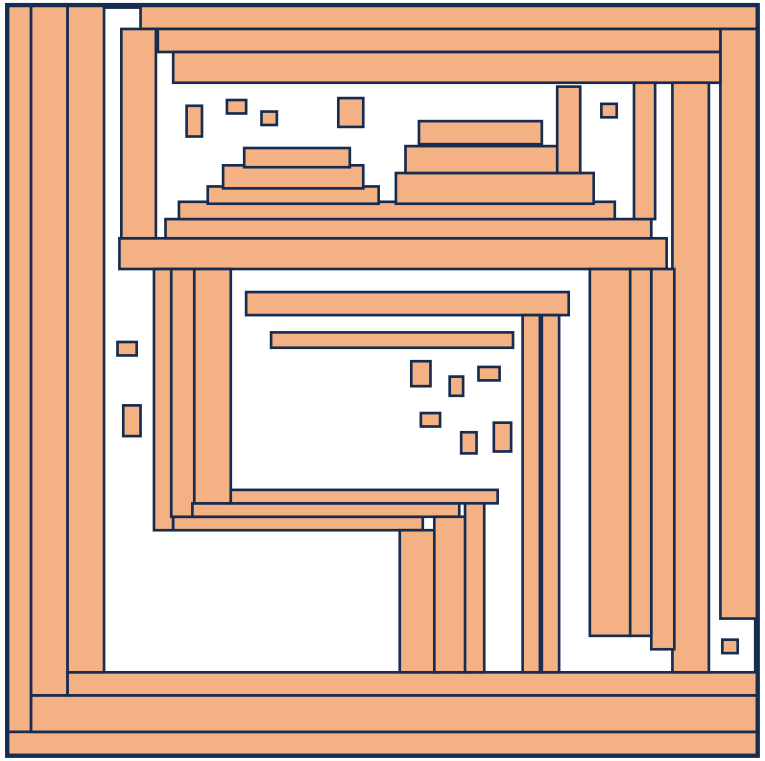


If $|OPT| \leq \frac{1}{\epsilon\delta^2}$, compute OPT by brute-force enumeration.

Else, number of large items in OPT is at most $\epsilon \cdot |OPT|$

Restructure OPT to a "nice" packing



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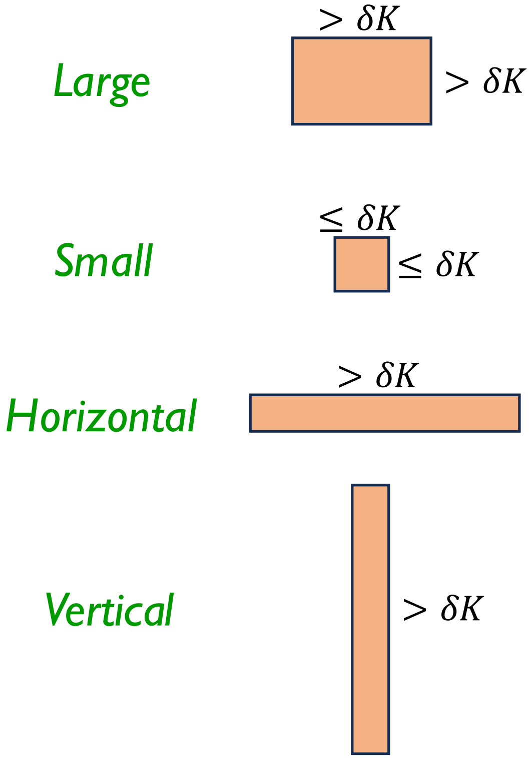

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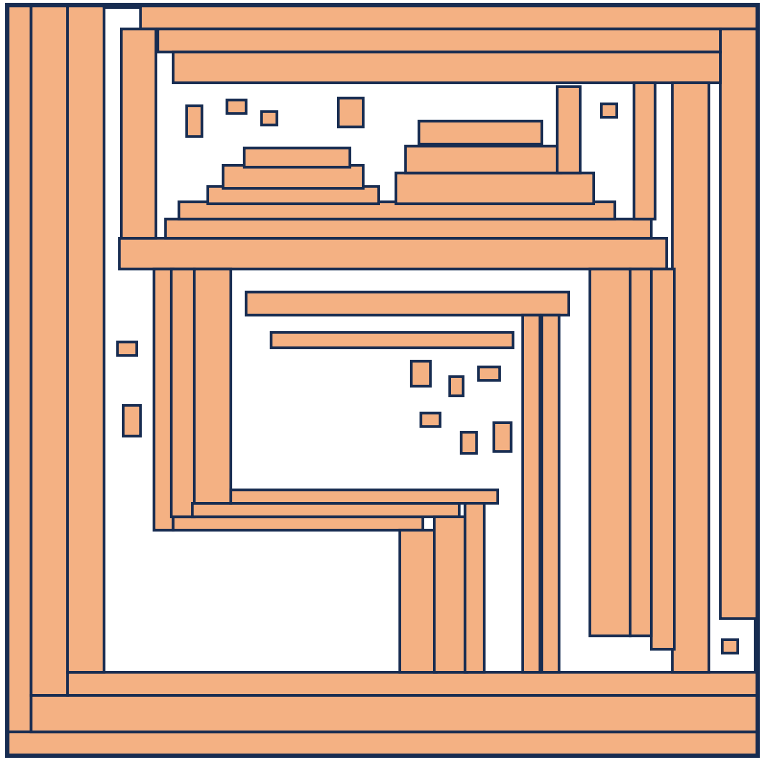

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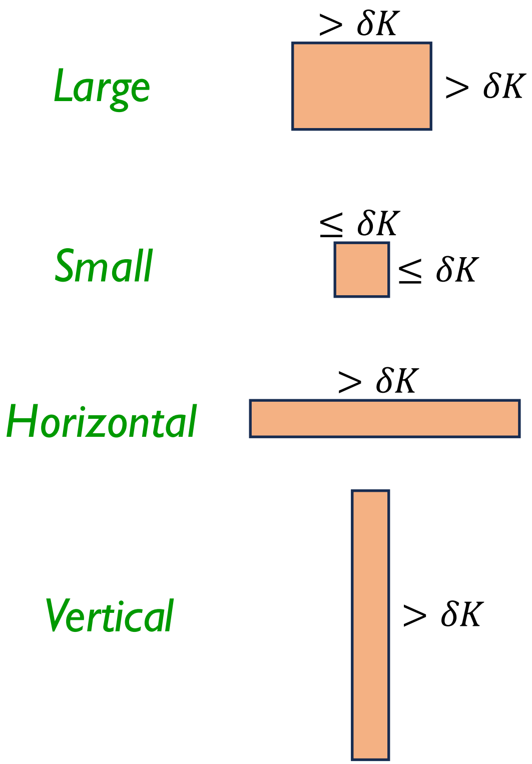

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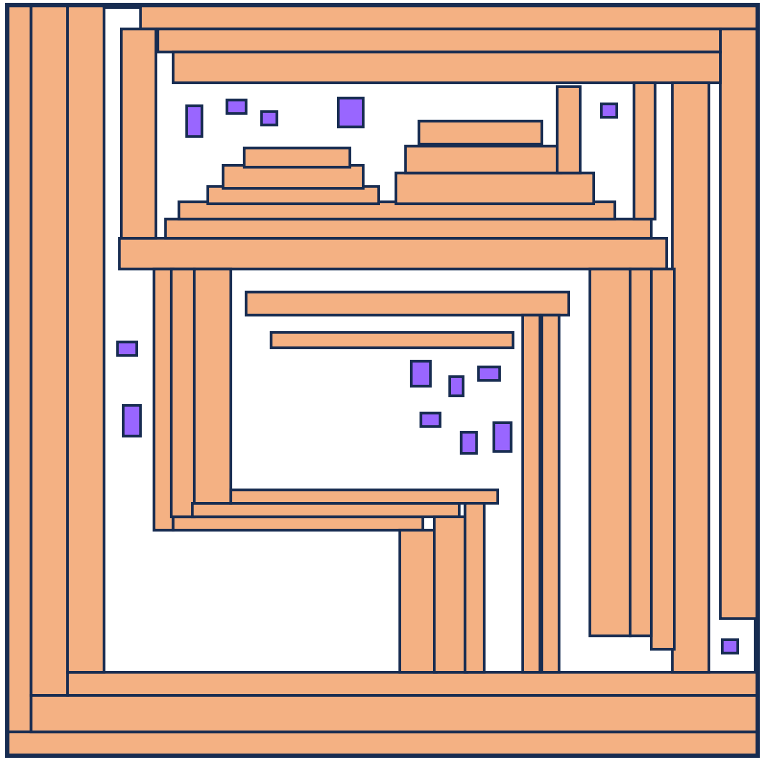
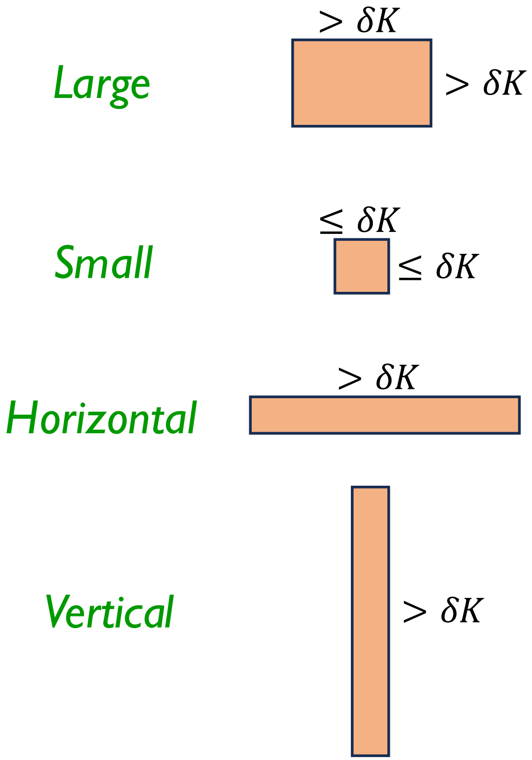


Small items can be repacked later greedily.

Item Classification:



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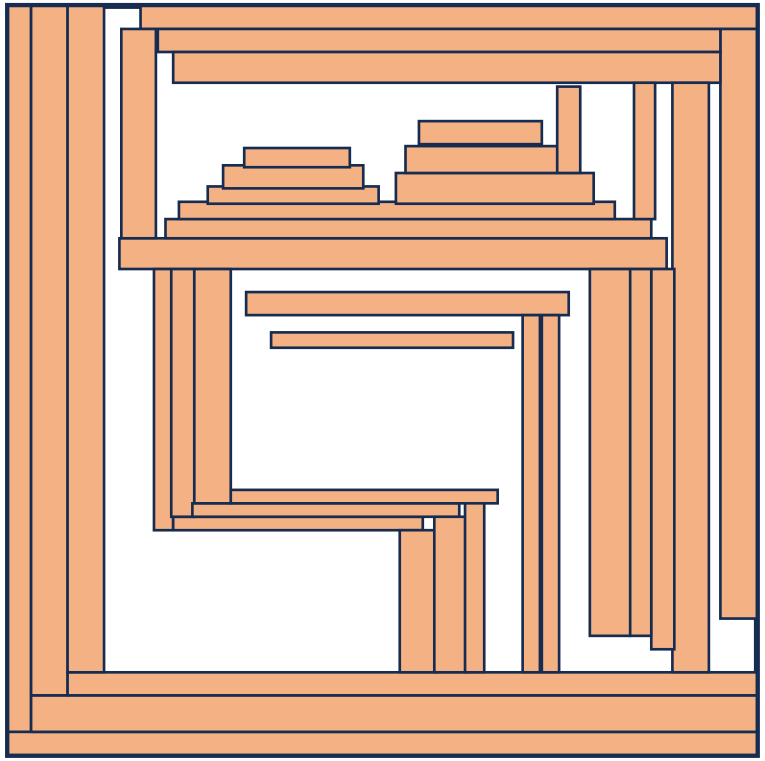

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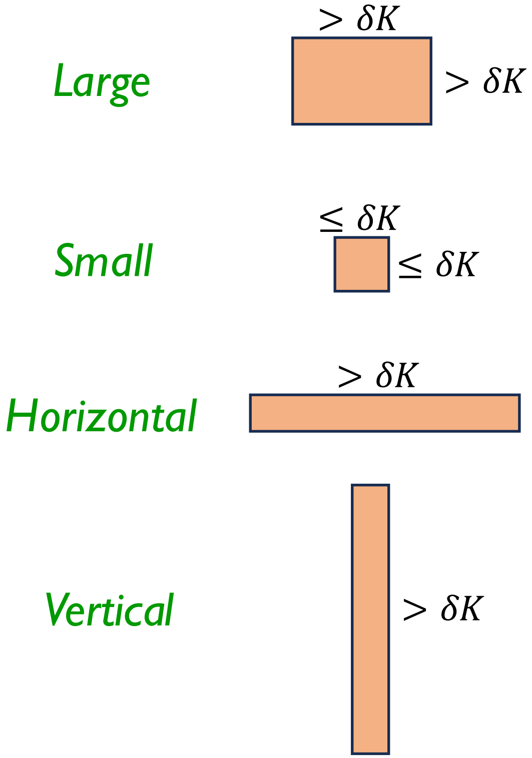

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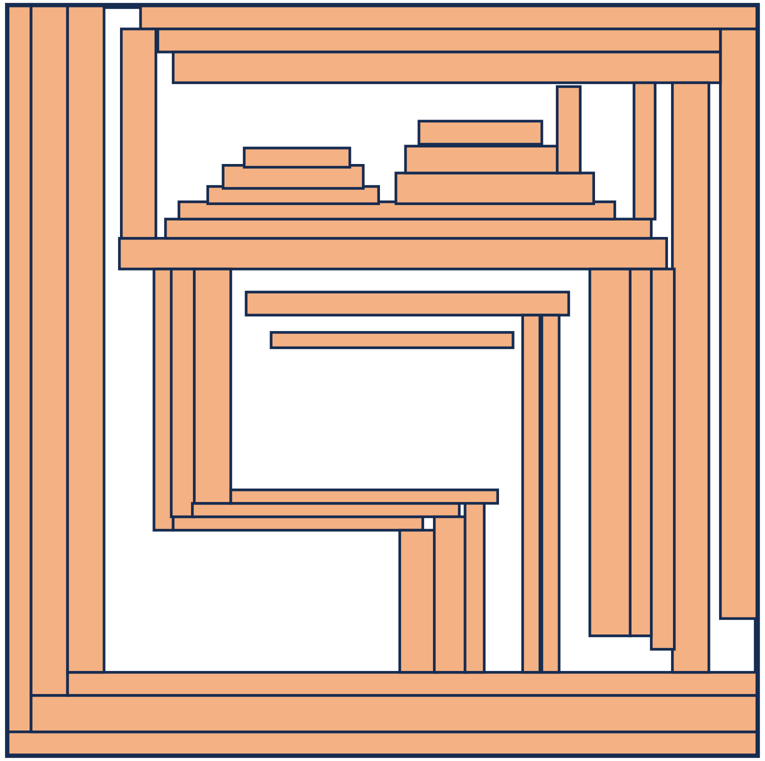

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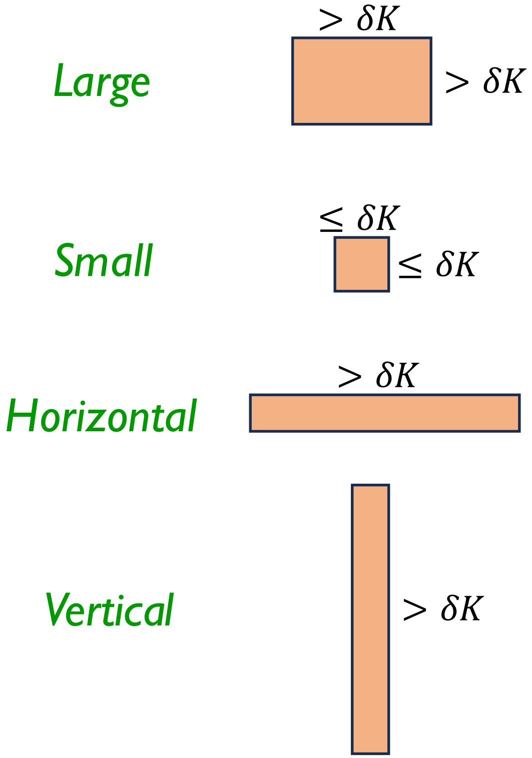

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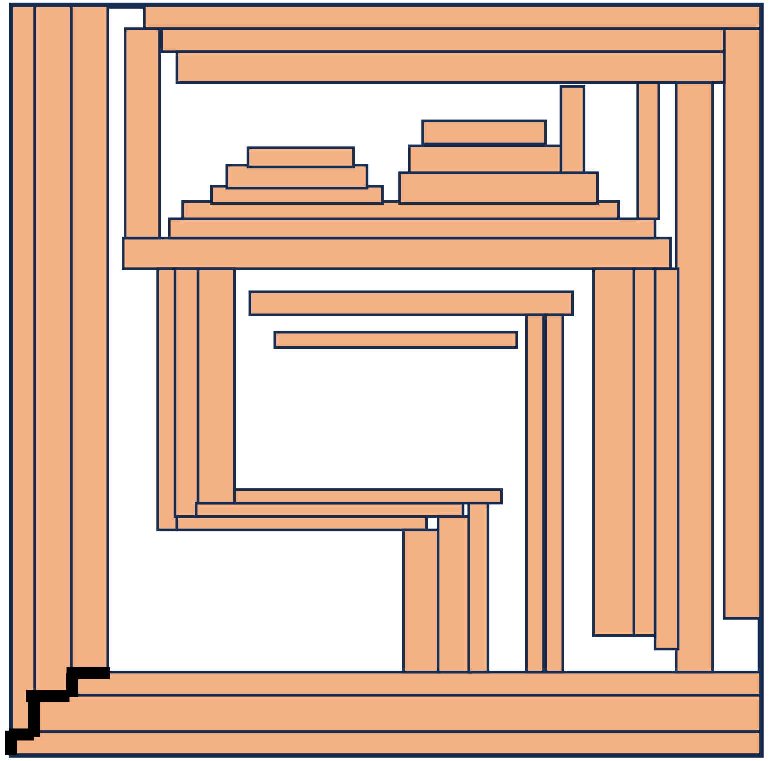

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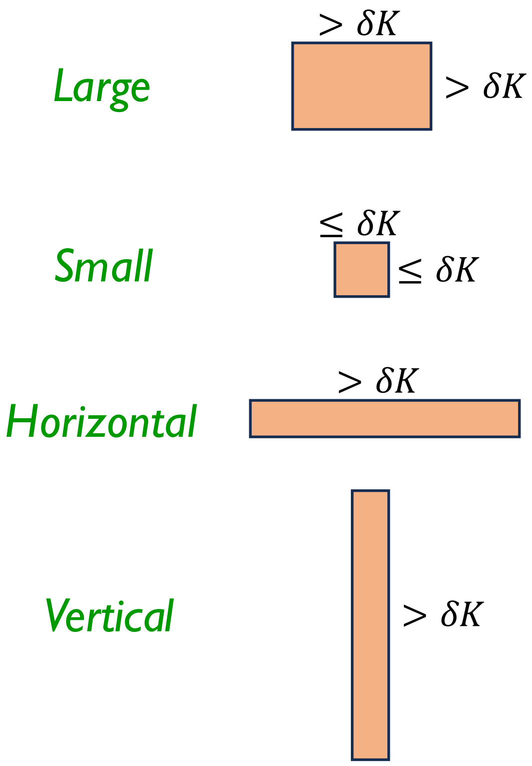
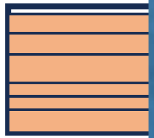
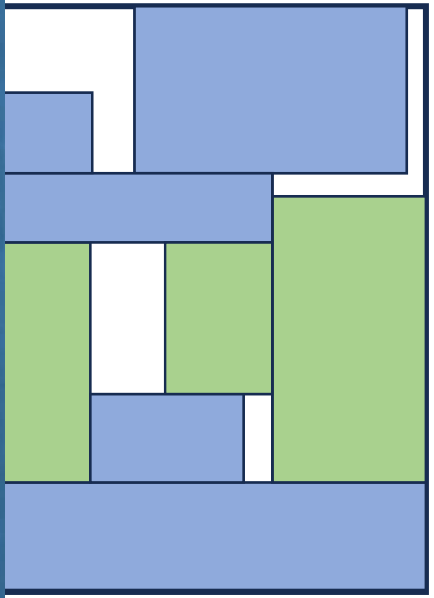


Container Packing

Knapsack partitio

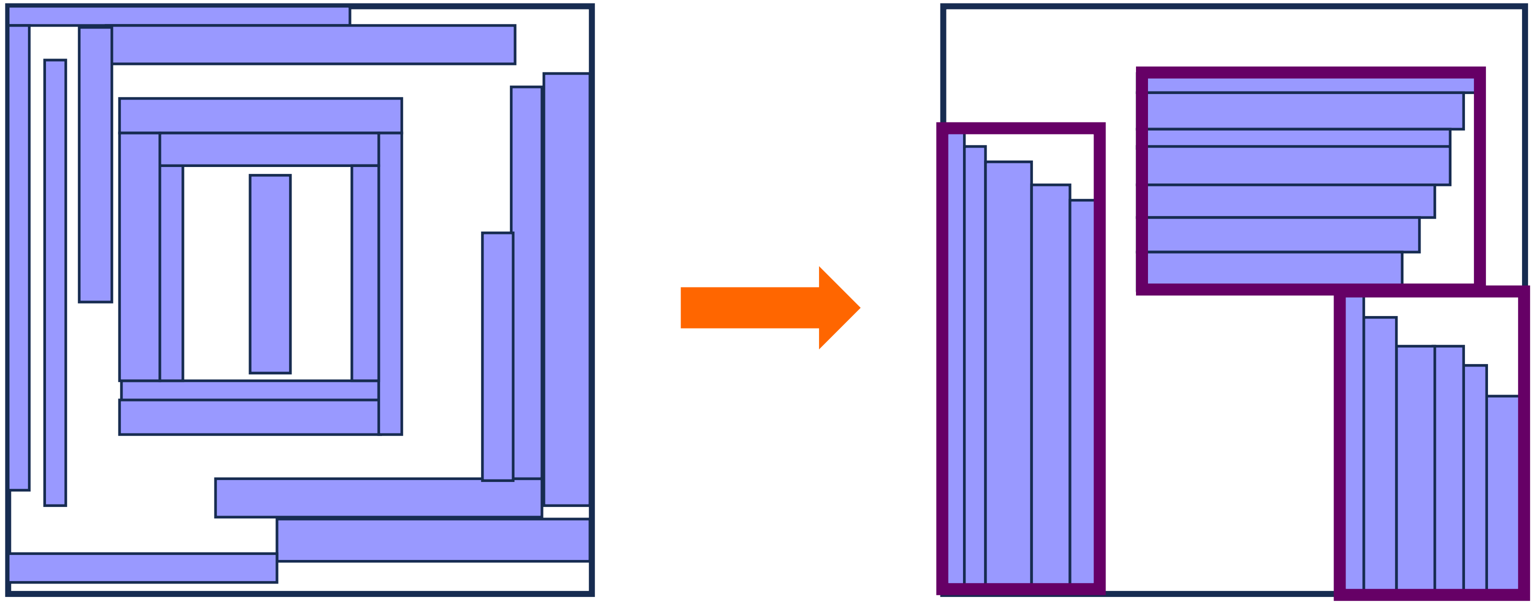




- Show the existence of "structured" packings

Container packing

- **Algorithm:** Compute the best structured packing using dynamic programming



How to compute the best container packing?

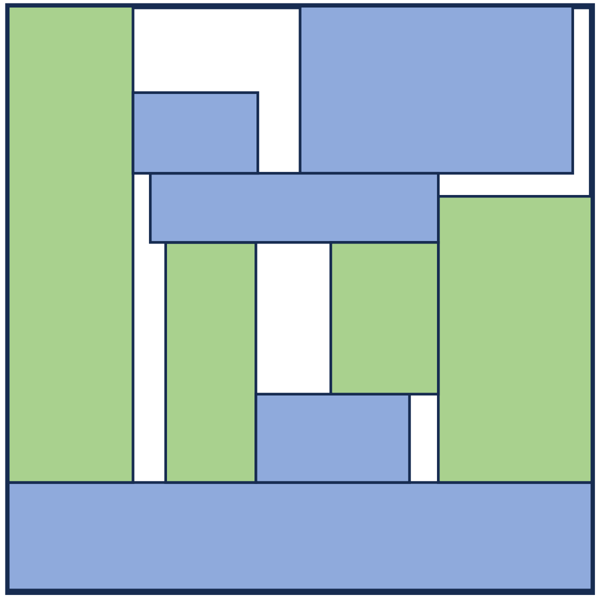
Enumerate all possible **configurations** of placing $O_\epsilon(1)$ containers.

How to place maximum number of rectangles into the containers?

Reduces to classical Knapsack problem, but with $O_\epsilon(1)$ knapsacks.

Multiple Knapsack Problem

PTAS exists!



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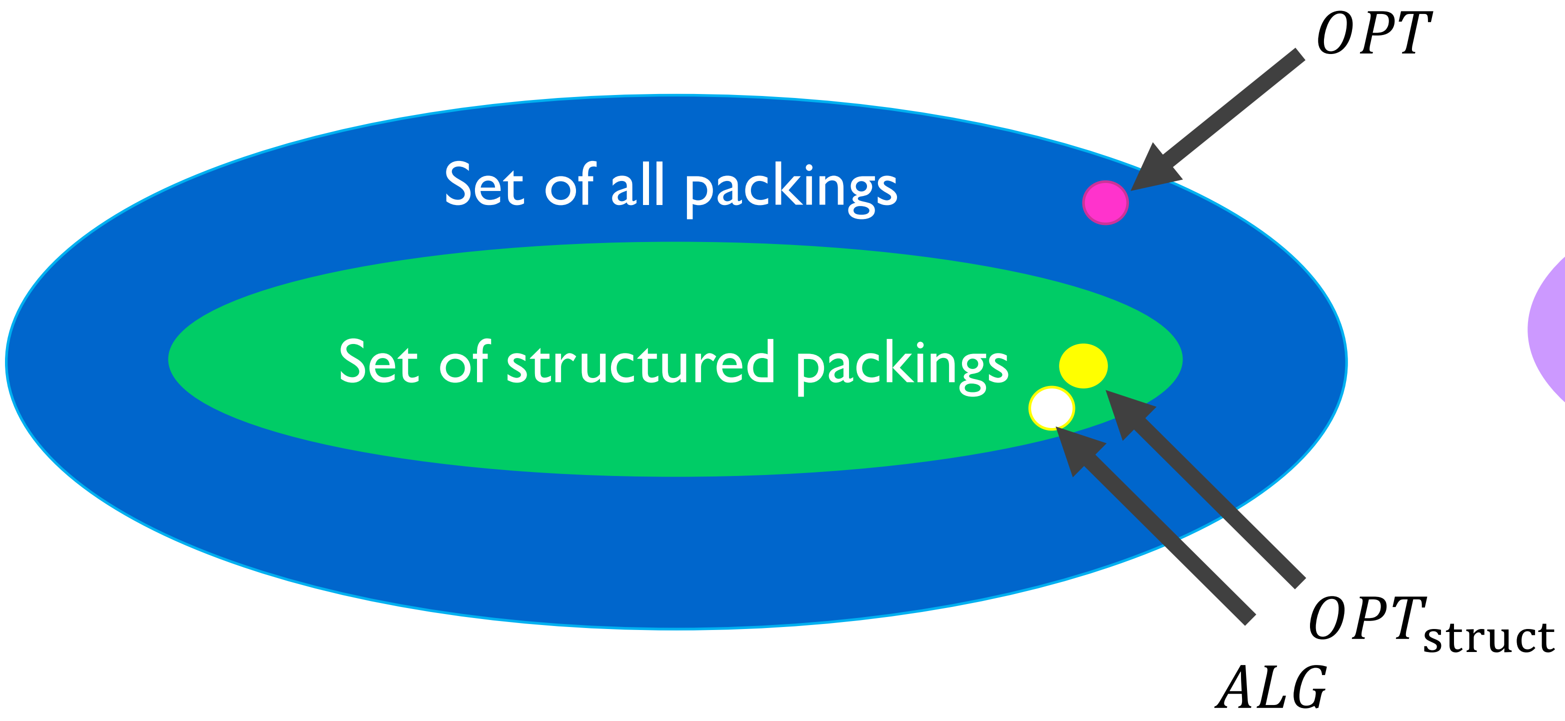


Want:

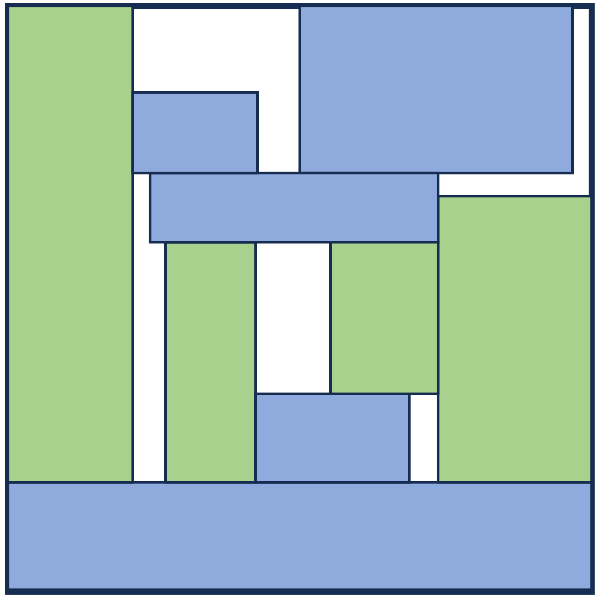
$$|OPT_{\text{struct}}| \geq (1-\epsilon)|OPT|$$

Structural question: How many rectangles are present in the "best" container packing?

Let OPT_{cont} – Best container packing

Then $|OPT| \geq |OPT_{\text{cont}}|$.

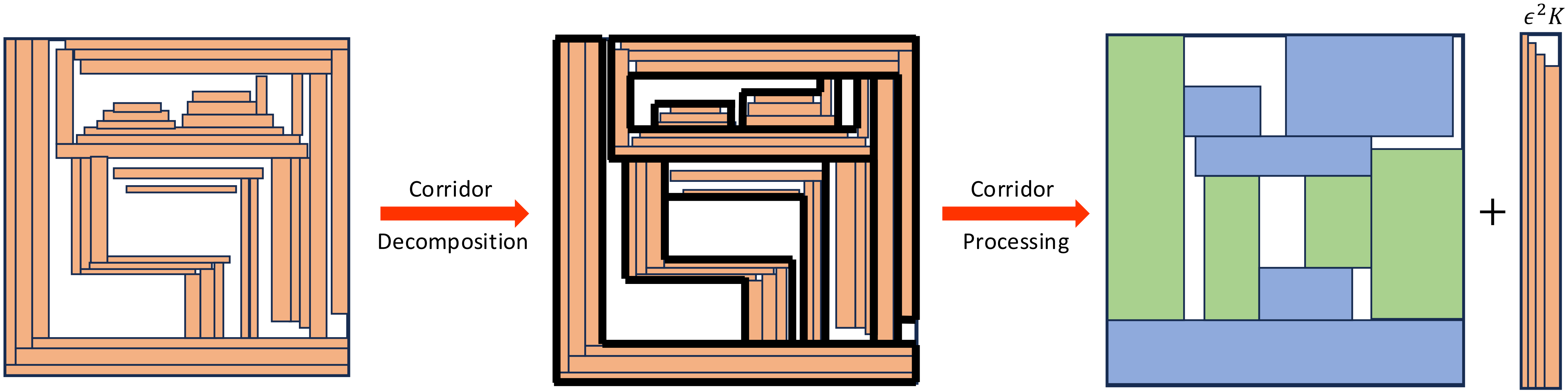
Question: What is the smallest α (> 1) such that $|OPT_{\text{cont}}| \geq \frac{1}{\alpha} \cdot |OPT|$



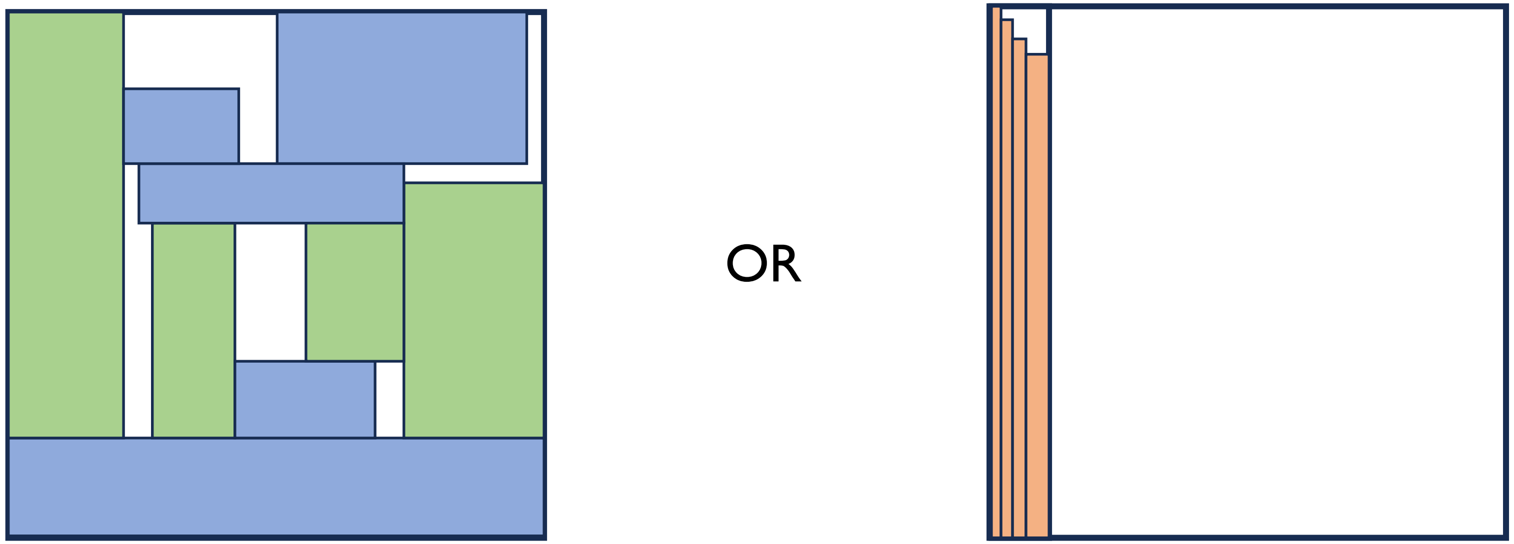
- [Jansen-Zhang, SODA '04] – $\alpha \leq 2 + \epsilon$
- [Gálvez-Grandoni-Heydrich-Ingala-K.-Wiese, FOCS '17] – $\alpha \leq \frac{4}{3} + \epsilon$

Our work

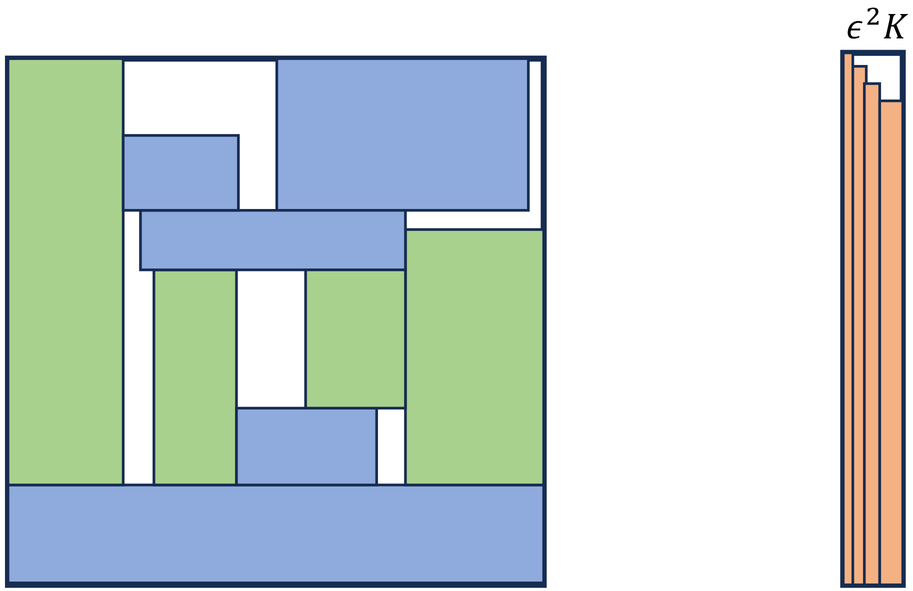
$\alpha \leq 1 + \epsilon$



Easy 2-approximation: Take the better of two container packings



Idea behind $\left(\frac{4}{3}+\epsilon\right)$-approximation of [Gálvez-Grandoni-Heydrich-Ingala-K.-Wiese, FOCS '17]



Resource contraction lemma:

How many items need to be discarded from a container packing to empty an $\epsilon^2 K$-width strip?

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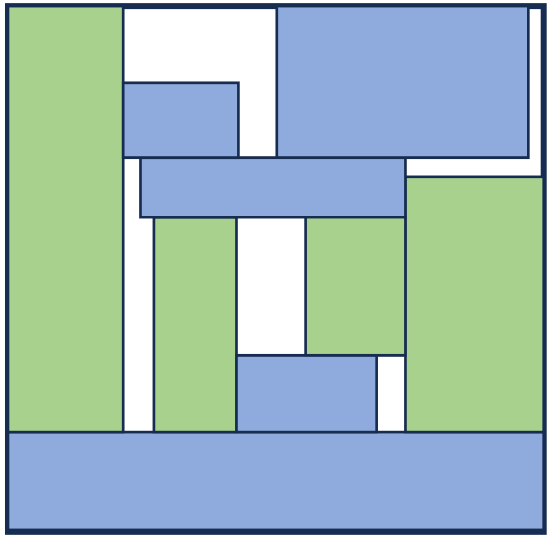

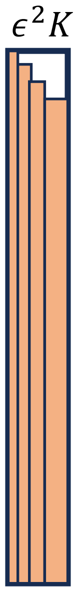

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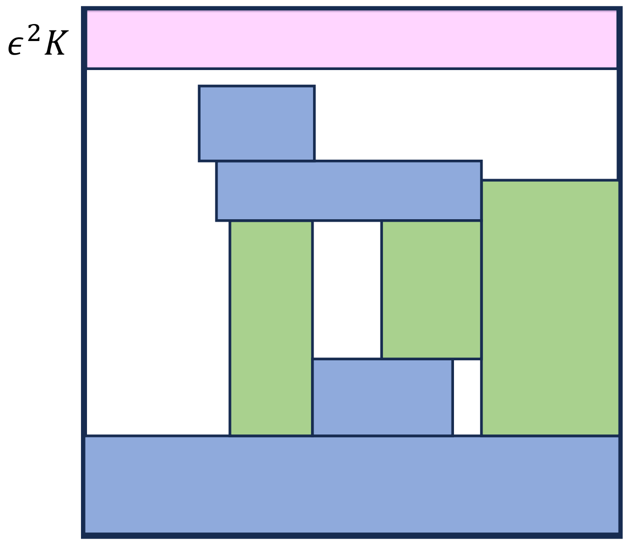

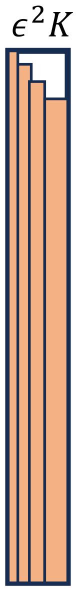

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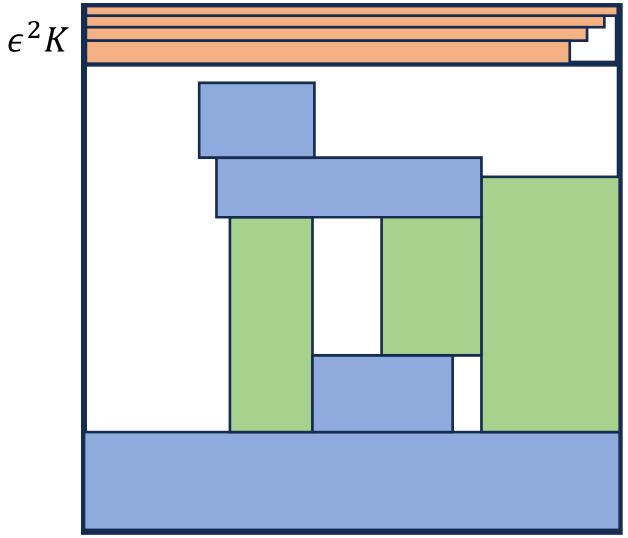

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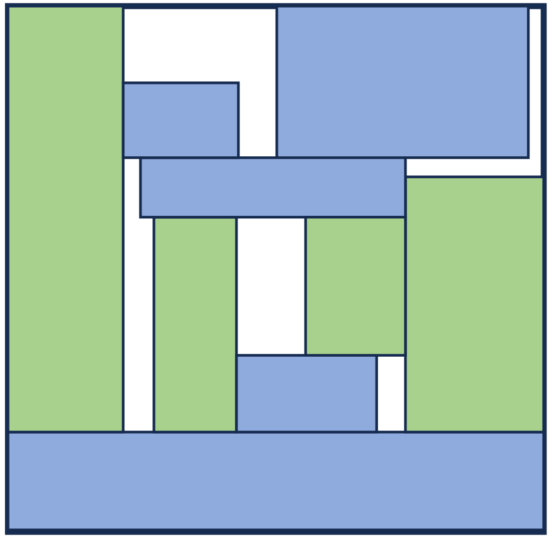
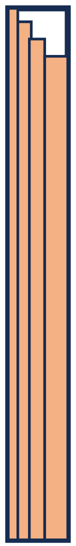


$\mathcal{A}$



$\mathcal{B}$

- $|\mathcal{A}| + |\mathcal{B}| = |OPT|$
- $|OPT_{\text{cont}}| \geq |\mathcal{A}|$
- $|OPT_{\text{cont}}| \geq \frac{2}{3}|\mathcal{A}| + |\mathcal{B}|$

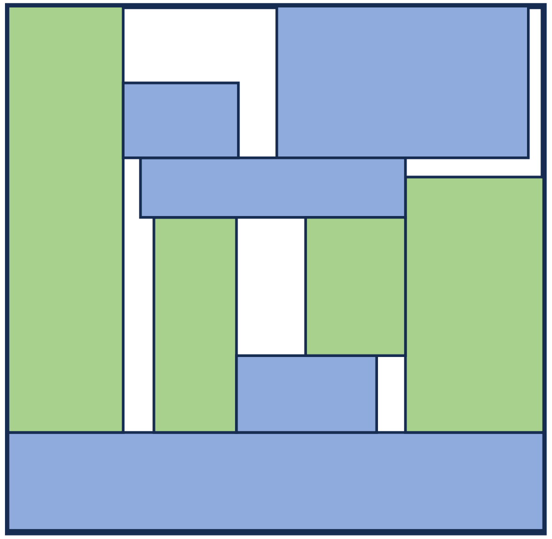
$\Longrightarrow |OPT_{\text{cont}}| \geq \frac{3}{4}|OPT|$

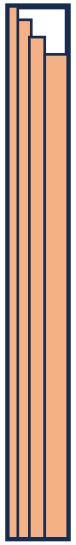
Resource contraction lemma:

How many items need to be discarded from a container packing to empty an $\epsilon^2 K$-width strip?

[Gálvez-Grandoni-Heydrich-Ingala-Khan-Wiese, FOCS '17] –
Suffices to discard $\leq \frac{1}{3}|\mathcal{A}|$rectangles

Idea behind $\left(\frac{4}{3}+\epsilon\right)$-approximation of [Gálvez-Grandoni-Heydrich-Ingala-K.-Wiese, FOCS '17]





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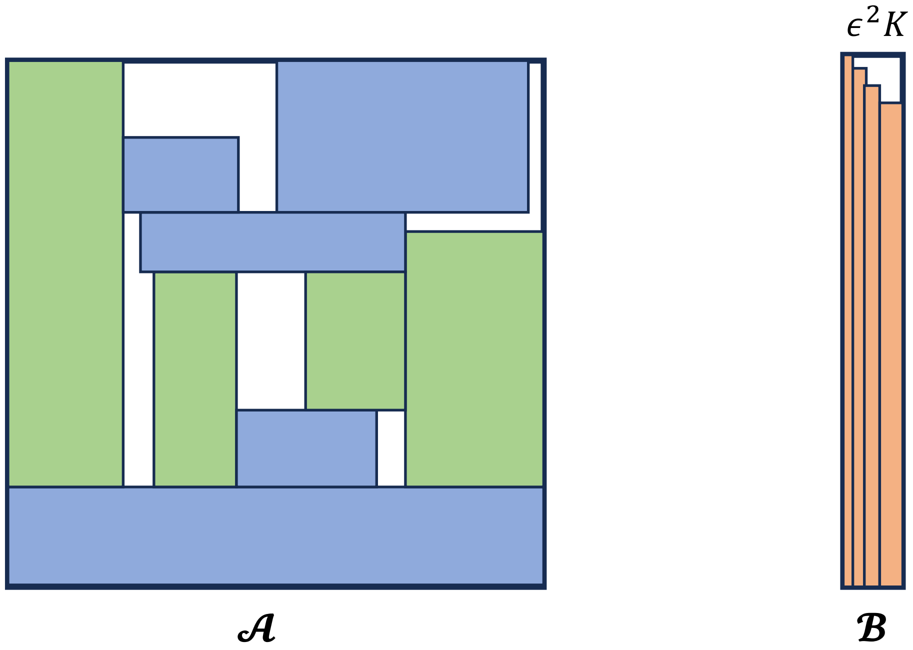
Resource contraction lemma:

How many items need to be discarded from a container packing to empty an $\epsilon^2 K$-width strip?

Our new Resource contraction lemma

Suffices to discard $\leq \epsilon|\mathcal{A}|$ rectangles only!

Idea behind $\left(\frac{4}{3}+\epsilon\right)$-approximation of [Gálvez-Grandoni-Heydrich-Ingala-K.-Wiese, FOCS '17]



- $|\mathcal{A}| + |\mathcal{B}| = |OPT|$
- $|OPT_{\text{cont}}| \geq (1-\epsilon)|\mathcal{A}| + |\mathcal{B}|$

  $\geq (1-\epsilon)|OPT|$

Resource contraction lemma:

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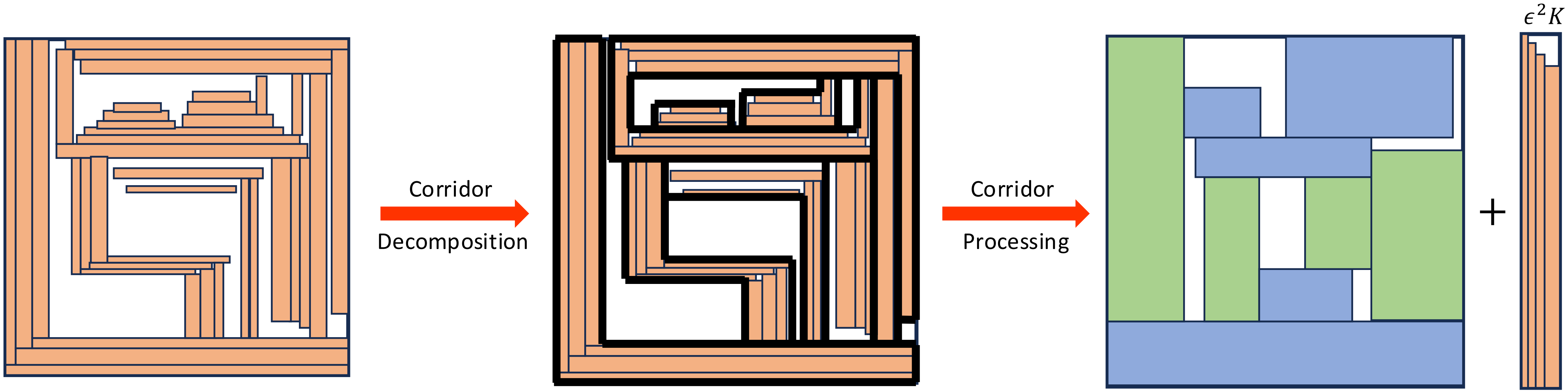
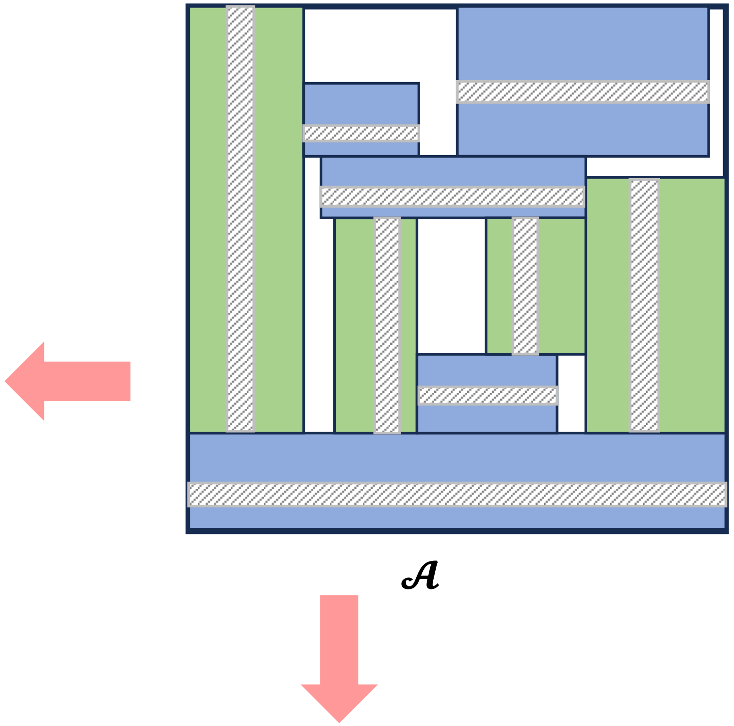
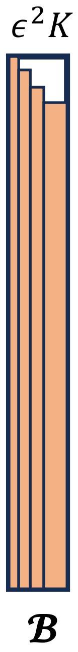
Suffices to discard $\leq \epsilon|\mathcal{A}|$ rectangles only!



We show a new corridor processing scheme that reduces number of containers from $\left(\frac{1}{\epsilon}\right)^{O(1)}$ to $\left(\log\frac{1}{\epsilon}\right)^{O(1)}$

- Most containers will be sufficiently "thick"
- For simplicity, say all containers have thickness $> \epsilon K$
- Consider random strips of thickness $\epsilon^2 K$ inside containers, and discard rectangles intersecting the strips
  $\Longrightarrow$ Expected number of rectangles remaining $\geq (1-\epsilon)|\mathcal{A}|$
- Push towards left and bottom.





Claim
This creates an empty strip of thickness $\epsilon^2 K$ at top or right knapsack boundary.

Repack $\mathcal{B}$ in the strip!

We obtain a PTAS!

But…. what is the running time?

$$n^{O_\epsilon(1)}$$

Polynomial. But for all practical purposes…..

Can we do better?

[Buchem-Deuker-Wiese, SoCG '24] – PTAS for squares in $n(\log n)^{O_\epsilon(1)}$ time

Theorem

Assuming the ***k*-SUM Conjecture**, any $(1+\epsilon)$-approximation must take $n^{\Omega(1/\epsilon)}$ time.

k-SUM problem

-3	-7	6	5	1	-2	-8	9

Given array $\mathcal{A}$ of n integers, and an integer $k \geq 2$, does there exist k values $a_1, a_2, \ldots, a_k \in \mathcal{A}$ such that

$$\boldsymbol{a_1 + a_2 + \cdots + a_k = 0}$$

-3	-7	6	5	1	-2	-8	9

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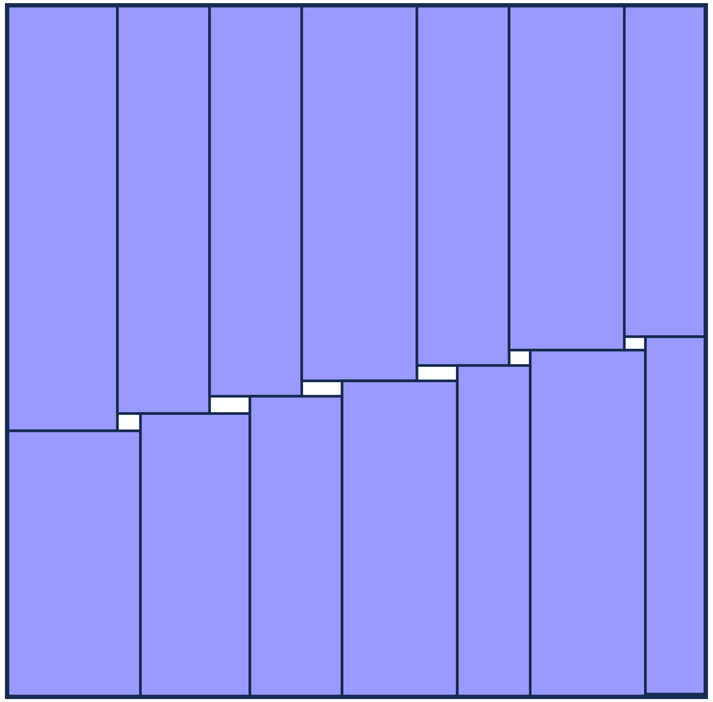
$$\boldsymbol{a_1 + a_2 + \cdots + a_k = 0}$$

- Brute force – Try all possible size-k subsets → $O(n^k)$
- Slightly intelligent → $O(n^{k-1})$
- Meet-in-the-middle algorithm → $O(n^{\lceil k/2 \rceil})$

k-SUM Conjecture

For every $k \geq 2$ and $\delta > 0$, k-SUM cannot be solved in $O(n^{\lceil k/2 \rceil - \delta})$ time.

Reduction from k-SUM to Rectangle Packing



If $\exists\, a_1, a_2, \dots, a_k \in \boldsymbol{\mathcal{A}}$ such that

$$a_1 + a_2 + \dots + a_k = 0$$

$\Longrightarrow |OPT| = 2k$

Otherwise, $|OPT| \leq 2k - 1$

An $(1+\epsilon)$-approximation with $\epsilon = \frac{1}{10k}$ can distinguish between the above.

Running time $\geq \Omega\left(n^{\lceil k/2 \rceil - \delta}\right) \geq n^{\Omega(1/\epsilon)}$

Weighted case

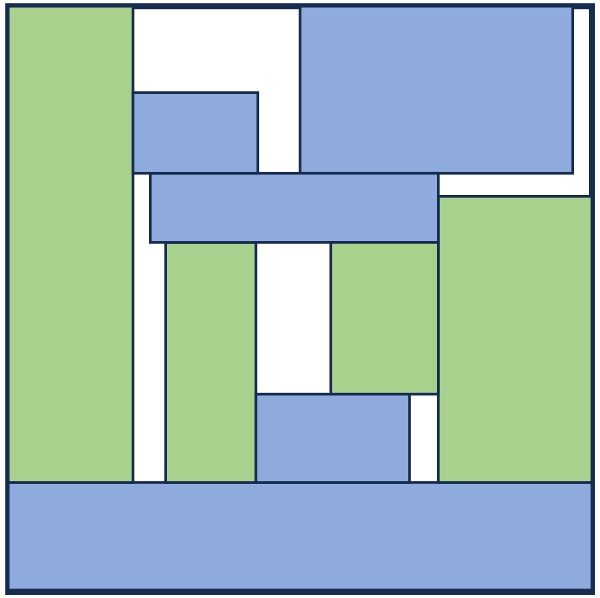
What happens when rectangles have weights?

Can container packings still give a PTAS?

OPT_{cont} – Best container packing

Then $p(OPT) \geq p(OPT_{\text{cont}})$.

Question: What is the smallest α (> 1) such that $p(OPT_{\text{cont}}) \geq \frac{1}{\alpha} \cdot p(OPT)$



➢ [Gálvez-Grandoni-Heydrich-Ingala-K.-Wiese, FOCS '17] : $\alpha \leq \frac{3}{2} + \epsilon$

Our work

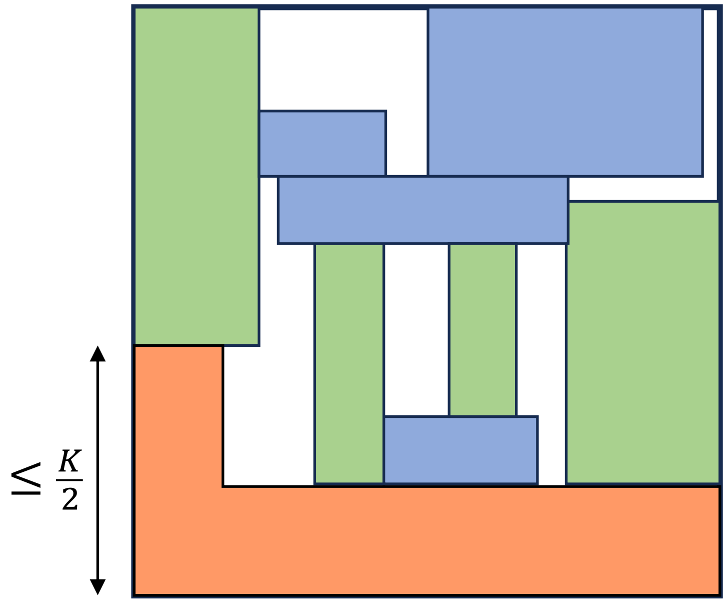
$\alpha \geq {}^{3}/_{2} - \epsilon$

☹

Theorem

There is a $\frac{190}{127} + \epsilon < 1.497$-approximation for the weighted case.

- Restructure OPT into an "L-region" and $O_\epsilon(1)$ containers.
- Best such packing can be found using a more involved DP.



Summary

- We provide a PTAS for packing maximum number of rectangles.

- Complete understanding of the power of container packings.

- Reduction from k-SUM to Rectangle Packing → any $(1+\epsilon)$-approximation must take $n^{\Omega(1/\epsilon)}$ time.

- A < 1.5-approximation for the weighted case.

Open Problems

- Does there exist a PTAS for the weighted case?

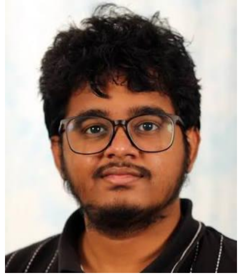
- Does there exist a PTAS for the unweighted case without rotation?

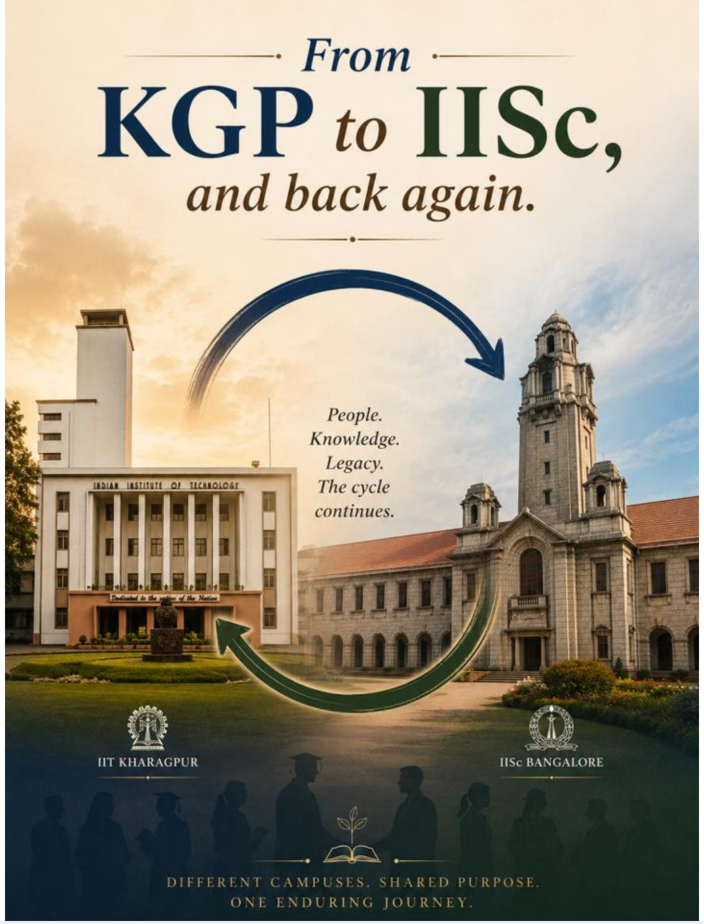
  Current best ratio: $\frac{558}{325} + \epsilon < 1.72$ [Gálvez-Grandoni-Heydrich-Ingala-K.-Wiese, FOCS '17]

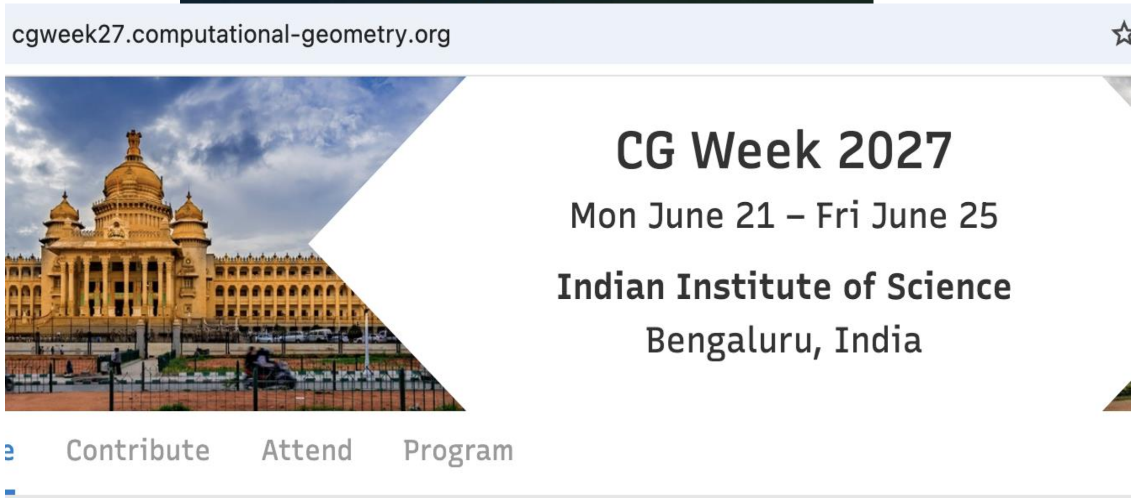
- What happens in 3D? Packing cuboids into a cube?

  Current best ratio: $\frac{24}{7} + \epsilon < 3.43$ [Jansen-Kar-K.-Sreenivas-Tutas, SoCG '25]

(axis-aligned nonoverlapping placement)	Bin-Packing Type	Knapsack Type
Rectangles movement	Pack all rectangles into minimum number of unit square bins	Pack maximum profit subset of rectangles into a unit square knapsack.
Vertically and Horizontally	**Geometric Bin Packing** 2D: 1.405 -Bansal-**K.** [SODA'14] 3D: 2.53 - Kar-**K.**-Rau [ICALP'25]	**Geometric Knapsack** 2D: This talk! 3D: 4.79, 4.28 Jansen-Kar-**K.**-Sreenivas-Tutas [SoCG'25]
Vertically	(uniform) **round-SAP** (uniform) **round-UFP** 2 – Kar-**K.**-Wiese [ESA'22]	**Storage allocation Problem (SAP), UFP: Unsplittable Flow on a Path (slicing)** PTAS – Grandoni-Momke-Wiese [STOC'24]
Not allowed	**Rectangle Coloring** O(log n) – Chalermsook-Walczak [SODA'21]	**Maximum Independent Set of Rectangles (MISR)** 3 → 2 [Galvez-**K.**-Mari-Momke-Reddy-Wiese, SODA'22, TALG'26]




From
KGP to IISc,
and back again.
People.
Knowledge.
Legacy.
The cycle
continues.
IIT KHARAGPUR
IISc BANGALORE
DIFFERENT CAMPUSES. SHARED PURPOSE.
ONE ENDURING JOURNEY.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