Practice Problems: Fast Subset Convolution

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1. In the Steiner Tree problem, we are given an undirected graph $\mathcal{G}$, a set $\mathcal{K} \subseteq \mathcal{V}[\mathcal{G}]$ of terminals. The goal is to find a tree $\mathcal{T}$ with the minimum number of edges that includes every vertex in $\mathcal{K}$. Design an inclusion-exclusion based algorithm for the Steiner Tree problem that runs in time $\mathcal{O}^\star\left(2^{|\mathcal{K}|}\right)$ and polynomial space.

2. The domatic number of a graph $\mathcal{G}$ is the minimum integer k such that $\mathcal{V}[\mathcal{G}]$ can be partitioned into k sets $\mathcal{V}_1, \ldots, \mathcal{V}_k$ such that each $\mathcal{V}_i$ is a dominating set of $\mathcal{G}$. Design an inclusion-exclusion based algorithm for computing the domatic number of any graph in $\mathcal{O}^\star(2^n)$ time and polynomial space, where n is the number of vertices of $\mathcal{G}$.

3. Design a $\mathcal{O}^\star(2^n)$ time algorithm for counting the number of spanning forests of an undirected graph using fast subset convolution.

4. Given two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$ where $\mathcal{U} = [n]$, show how we can compute the following functions for every subset $S \subseteq \mathcal{U}$ together using fast subset convolution in $\mathcal{O}^\star(2^n)$ time.

$$\max_{X \subseteq S} f(X) + g(S \setminus X)$$

$$\min_{X \subseteq S} f(X) + g(S \setminus X)$$

5. Design a $\mathcal{O}^\star(2^n)$ time algorithm for computing the domantic number of an undirected graph using fast subset convolution.

6. The covering product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$, denoted by $f \star_c g$ is defined for all $S \subseteq \mathcal{U}$ as

$$(f \star_c g)(S) = \sum_{X,Y \subset S : X \cup Y = S} f(X)g(Y)$$

The packing product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$, denoted by $f \star_p g$ is defined for all $S \subseteq \mathcal{U}$ as

$$(f \star_p g)(S) = \sum_{X,Y \subset S : X \cap Y = \emptyset} f(X)g(Y)$$

The intersecting covering product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$, denoted by $f \star_{ic} g$ is defined for all $S \subseteq \mathcal{U}$ as

$$(f \star_{ic} g)(S) = \sum_{X,Y \subset S : X \cup Y = S, X \cap Y \neq \emptyset} f(X)g(Y)$$

Design a $\mathcal{O}^\star(2^n)$ time algorithm for computing the covering product, packing product, and intersecting covering product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$.