Practice Problems: Fast Fourier Transform and Dynamic Programming

Palash Dey
Indian Institute of Technology, Kharagpur

April 3, 2026

Submit the solutions of the questions marked $(\star)$ in PDF format generated using Latex by **April 11, 2026**.

---

1. In the inverse DFT problem, we are given evaluation of a polynomial of degree at most n at every n-th root of unity. The goal is to find the coefficient representation of the polynomial. Design a $\mathcal{O}(n \log n)$ time algorithm for the problem.

2. **$(\star)$** Design a $\mathcal{O}(n \log n)$ time algorithm to multiply two n bit integers.

3. *[CLRS]* Consider two sets A and B, each having n integers in the range from 0 to $10n$. The Cartesian sum of A and B is defined by $C = \{x + y : x \in A, y \in B\}$. The integers in C lie in the range from 0 to $20n$. Show how, in $\mathcal{O}(n \log n)$ time, to find the elements of C and the number of times each element of C is realized as a sum of elements in A and B.

4. A set of arcs $\mathcal{F}$ in a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ is called a directed feedback arc set of $\mathcal{G}$ if $\mathcal{G}[\mathcal{V}, \mathcal{A} \setminus \mathcal{F}]$ is acyclic. Show that we can compute a minimum cardinality directed arc set of any directed graph in $\mathcal{O}^\star(2^n)$ time, where n is the number of vertices of $\mathcal{G}$.

5. **$(\star)$** The cut-width of a vertex ordering π of a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is

$$\max_{v \in \mathcal{V}} \left|\{\{w, x\} \in \mathcal{E} : \pi(w) \leqslant \pi(v) < \pi(x)\}\right| .$$

   The cut-width of a graph is the minimum cut-width taken over all orderings of its vertices. Show that we can compute the cut-width of a graph in time $\mathcal{O}^\star(2^n)$ time, where n is the number of vertices of $\mathcal{G}$.

6. The domatic number of a graph $\mathcal{G}$ is the minimum integer k such that $\mathcal{V}[\mathcal{G}]$ can be partitioned into k sets $\mathcal{V}_1, \ldots, \mathcal{V}_k$ such that each $\mathcal{V}_i$ is a dominating set of $\mathcal{G}$. Show how we can compute the domatic number of any graph in $\mathcal{O}^\star(3^n)$ time, where n is the number of vertices of $\mathcal{G}$.

7. In the EXACT SAT problem, we are given a CNF formula. The task is to check if there exists an assignment of its variables so that every clause has exactly one literal set to TRUE. Show that there is a $\mathcal{O}^\star(2^m)$ time algorithm for EXACT SAT, where m is the number of clauses in the formula.