# Practice Problems: Edmond's Blossom Algorithm, Matroid

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Submit the solutions of the questions marked ($\star$) in PDF format generated using Latex by **January 22, 2025**.

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## 1 Edmond's Blossom Algorithm

1. **($\star$) Gallai-Edmond Decomposition:** The vertex set $V$ of every graph $G = (V, E)$ can be partitioned into three sets, $A$, $B$, and $D$. The set $D$ consists of all vertices which are left unmatched in at least one maximum cardinality matching. They are called inessential vertices. The other vertices, i.e. the vertices in $V \setminus D$ are called essential vertices since they are matched in every maximum matching of $G$. The set $A$ consists of all essential vertices which are neighbor of at least one vertex in $D$. The remaining set of essential vertices is called $B$. Design an algorithm to construct the Gallai-Edmond decomposition of any given graph. (Hint: modify Edmond's blossom algorithm)

## 2 Matroids

Let $M = (S, I)$ be any matroid.

2. Let $B \subset S$ be any subset of $S$ and $J$ a basis of $B$. We define another matroid $M' = (S', I')$ where $S' = S \setminus B$ and $I' = \{J' \subseteq S' : J' \cup J \in I\}$. Prove that $M'$ is a matroid that does not depend of $J$ and the rank of any $A \subseteq S'$ in $M'$ is the rank of $A \cup B$ in $M$ minus the rank of $B$ in $M$.

3. Let $J \in I$ and $e \in S$ be any. Then $J \cup \{e\}$ contains at most one circuit.

4. A *branching* of a directed graph is a forest (of the underlying undirected graph) in which each edge has in-degree at most one. Suppose the edges of $G$ are weighted. We want to compute a maximum weight branching of $G$. Model this problem as a weighted matroid intersection problem.

5. Let $G$ be a bipartite graph with $(L, R)$ be a bipartition of the vertices. A transversal matroid $M$ is defined on the set $L$ where a subset $X \subseteq L$ is an independent set if and only if $X$ can be perfectly matched to a subset of $R$. Show that the independence system defined above is a matroid.

6. Show that every uniform matroid is a transversal matroid.

7. Let $M = (S, I)$ be any matroid. Define the dual $M^\star = (S, I^\star)$ of $M$ as follows. The ground set of $M^\star$ is the ground set of $M$. A subset $A$ of $S$ is an independent set of $M^\star$ if and only if $S \setminus A$ contains a basis of $M$. Show that $M^\star$ is a matroid.

8. Let $(X_1, \ldots, X_k)$ be a partition of a set $S$ and $d_i \geqslant 0$ for every $i \in [k]$. We say a set $A \subseteq S$ is independent if $|A \cap X_i| \leqslant d_i$ for every $i \in [k]$. Prove that this independence system is a matroid.

9. **($\star$)** Consider the following orientation problem. Given an undirected graph $G = (V, E)$ and integers $d_v \geqslant 0$ for every $v \in V$, compute if it is possible to orient the edges of $G$ such that the out-degree of every vertex $v \in V$ is at most $d_v$ in the resulting directed graph. Design a polynomial-time algorithm for this problem.

10. Suppose we are given an undirected graph $\mathcal{G}$ whose every edge has a (not necessarily distinct) color. Consider the problem of computing if the graph has a spanning tree whose all edges have different colors. Design a polynomial-time algorithm for this problem by reducing it to the matroid intersection problem.

11. Given a matrix whose columns are colored, design a polynomial-time algorithm to select a maximum set of linearly independent columns using at most one per color.