Practice Problems: Primal-Dual Method for Approximation Algorithm

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Submit the solutions of the questions marked ($\star$) in PDF format generated using Latex by **March 30, 2026**.

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1. For a given graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ and a subset $S \subseteq \mathcal{V}$ of vertices, we define $\delta(S) = \{e \in \mathcal{E} : |e \cap S| = 1\}$. That is, $\delta(S)$ is the set of edges whose exactly one end-point belongs to S. Show that two vertices s and t are connected in a graph $\mathcal{G}$ if and only if $\delta(S)$ is non-empty for every subset $S \subset V[\mathcal{G}]$ such that $|S \cap \{s, t\}| = 1$.

2. Design a primal-dual method based approximation algorithm for the minimum weight vertex cover problem.

3. **($\star$)** Design a primal-dual based 2 factor approximation algorithm for the minimum weight $0-1$ Knapsack problem. Design a polynomial-time separation oracle for the linear programming relaxation that you have used for your 2 factor approximation algorithm.

4. Design a polynomial-time separation oracle for the linear programming relaxation of the feedback vertex set problem discussed in the class.

5. *[7.4 in WS book]* In the primal-dual method based 2-approximation algorithm for the generalized steiner forest problem discussed in the class, we first added certain edges, and then removed unnecessary edges in the order opposite of the order the edges were added. Prove that, even if one removes unnecessary edges in any order, the approximation ratio of the modified algorithm remains at least 2. Write the complete pseudocode of the modified algorithm and provide a complete proof of its approximation guarantee.