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Indian Institute of Technology Kharagpur  
Department of Computer Science and Engineering  
CS60086 Selected Topics in Algorithms: Mid-Semester Examination 2026

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Duration: 2 Hours

Full Marks: 60

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Answer all questions.

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1. (a) Prove or disprove: A graph cut is called an even cut if both the partitions of the graph has an even number of vertices. The following algorithm computes a minimum weight even cut in a weighted undirected graph. If the number of vertices is an odd integer, then output that there is no even cut in the graph. Otherwise, compute a Gomory-Hu tree of the graph and return the minimum of all fundamental even cuts.
- (b) Prove or disprove: A graph cut is called an odd cut if both the partitions of the graph has an odd number of vertices. The following algorithm computes a minimum weight odd cut in a weighted undirected graph. Compute a Gomory-Hu tree of the graph and return the minimum of all fundamental odd cuts.

**[10+10 = 20 Marks]**

2. A set of vertices of a graph is called a vertex cover if the set contains at least one end point of every edge. Consider the following polytope.

$$\begin{aligned} \min \quad & \sum_{v \in \mathcal{V}} w_v x_v \\ \text{s.t.} \quad & x_u + x_v \geq 1 \quad \forall \{u, v\} \in \mathcal{E}, \\ & 0 \leq x_i \leq 1 \quad \forall i \in \mathcal{V}. \end{aligned}$$

- (a) Show that the above polytope is the vertex cover polytope of a graph  $\mathcal{G}(\mathcal{V}, \mathcal{E})$  if and only if  $\mathcal{G}$  is a bipartite graph.
- (b) Show that every vertex of the above polytope is half integral.
- (c) Write its dual linear program and complementary slackness conditions.

**[8+8+(2+2) = 20 Marks]**

3. (a) Let  $\mathcal{M} = (\mathcal{S}, \mathcal{J})$  be a matroid. Write a set of linear inequalities with  $|\mathcal{S}|$  number of variables which is the same as the convex hull defined by the characteristic vectors of all the independent sets of  $\mathcal{M}$ . Prove your claim.

- (b) Prove or disprove: Let  $\mathcal{G} (\mathcal{V}, \mathcal{E})$  be an undirected graph. Let us consider the pair  $(\mathcal{V}, \mathcal{F})$  where a subset  $\mathcal{A} \subseteq \mathcal{V}$  belongs to  $\mathcal{F}$  if and only if  $\mathcal{A}$  is an independent set of  $\mathcal{G}$ . Then  $(\mathcal{V}, \mathcal{F})$  is a matroid.

**[(2+12)+6 = 20 Marks]**

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All the best