INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Selected Topics in Algorithms 2025-26: Second Class Test

Date of Examination: 27^{th} **March 2026**
Duration: 45 minutes
Subject: CS60086 Selected Topics in Algorithms
Total marks: 20

1. Define total dual integrality. Show that if a linear system of inequalities $Ax \leqslant b$ is totally dual integral, A is a rational matrix, and b is an integral vector, then the polytope $P = \{x : Ax \leqslant b\}$ is an integral polytope.

[3+4=7 Marks]

2. Define a network matrix. Show that the consecutive ones matrix is a network matrix.

[2+4=6 Marks]

3. Suppose we use the primal algorithm, discussed in the class, for computing a min-cost flow to compute a min-cost perfect matching in a bipartite graph. What would be the worst case time complexity of that algorithm to compute a min-cost perfect matching in a bipartite graph? Answer in Θ notation. Justify your answer by explaining what would be the worst-case time complexity of every step of the algorithm including the steps involved in the primal algorithm for computing a min-cost flow and the reduction from min-cost flow to transshipment.

[2+5=7 Marks]