INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Selected Topics in Algorithms: First Class Test

Date of Examination: 30^{th} **January 2026**
Duration: 45 minutes
Subject: CS60086 Selected Topics in Algorithms
Total marks: 20

1. Prove or disprove: There exists an algorithm that, given an unweighted graph $\mathcal{G}$, its Gomory-Hu tree $\mathcal{T}$, and an edge e in $\mathcal{G}$, outputs a Gomory-Hu tree of $\mathcal{G} \setminus \{e\}$ in $o(n)$ time.

[6 Marks]

Solution Sketch. Let $\mathcal{G}$ be a cycle on n vertices. A star graph is a Gomory-Hu tree of $\mathcal{G}$. If we delete any edge e from $\mathcal{G}$, then since $\mathcal{G} \setminus \{e\}$ is a path, assuming $n \geqslant 5$, the unique Gomory-Hu tree of $\mathcal{G} \setminus \{e\}$ is $\mathcal{G} \setminus \{e\}$ itself which is $\Omega(n)$ edges addition/deletion away from any star on the vertex set of $\mathcal{G}$. □

2. Design an algorithm that takes an undirected unweighted graph $\mathcal{G}$ as input and outputs a vertex that belongs to every maximum cardinality matching of $\mathcal{G}$.

[7 Marks]

Solution Sketch. Compute a maximum matching M. Compute the alternative BFS tree. Every vertex marked "odd" must be matched in every maximum cardinality matching of $\mathcal{G}$ thanks to Tutte's theorem. □

3. Prove or disprove: Let $\mathcal{M} = (S, \mathcal{I})$ be a matroid, $J \in \mathcal{I}$, $e_1, e_2 \in S$. Then $J \cup \{e_1, e_2\}$ contain at most two circuits.

[7 Marks]

Solution Sketch. Counter example: Let $S = \{x, y, z\}$ and $\mathcal{M}$ is a uniform matroid of rank 1 over S. That is every subset of S of cardinality at most 1 is an independent set in $\mathcal{M}$. Let us take $J = \{x\}$. Then $J \cup \{y, z\}$ has three circuits, namely $\{x, y\}, \{y, z\}, \{z, x\}$. □