Indian Institute of Technology Kharagpur
CS60023 Approximation and Online Algorithms: First Class Test

Date of Examination: 17^{th} **August 2026**
Duration: 45 minutes
Total marks: 20

1. Prove or disprove:

   (a) There sort in non-increasing order and then schedule using the list scheduling algorithm achieves a $\frac{4}{3}$ factor approximation algorithm for the problem of scheduling n jobs into m identical machines with minimum makespan.

   (b) For every $\varepsilon > 0$, provide an instance where the makespan of the schedule output by this algorithm on that instance is at least $(\frac{4}{3} - \varepsilon)\text{OPT}$.

[4+8=12 Marks]

Solution Sketch.

Tight Example for Sort-and-List Scheduling

Consider m identical machines and the following jobs:

$$\underbrace{2m-1, 2m-1}_{\text{2 jobs}}, \underbrace{2m-2, 2m-2}_{\text{2 jobs}}, \ldots, \underbrace{m+1, m+1}_{\text{2 jobs}}, \underbrace{m, m, m}_{\text{3 jobs}}.$$

The jobs are already sorted in non-increasing order.

Under the Sort-and-List (LPT) algorithm, the resulting schedule is

Machine	Load
M_1	$(2m-1) + m + m = 4m - 1$
M_2	$(2m-1) + m = 3m - 1$
M_3	$(2m-2) + (m+1) = 3m - 1$
M_4	$(2m-2) + (m+1) = 3m - 1$
$\vdots$	$\vdots$
M_m	$(m+1) + (m-1) = 2m$

Thus,

$$C_{LPT} = 4m - 1.$$

On the other hand, an optimal schedule is

Machine	Load
M_1	$m + m + m = 3m$
M_2	$(2m-1) + (m+1) = 3m$
M_3	$(2m-1) + (m+1) = 3m$
M_4	$(2m-2) + (m+2) = 3m$
$\vdots$	$\vdots$
M_m	$(m+1) + (2m-2) = 3m$

Therefore,

$$\mathrm{OPT} = 3m.$$

Hence, the approximation ratio is

$$\frac{C_{\mathrm{LPT}}}{\mathrm{OPT}} = \frac{4m-1}{3m} = \frac{4}{3} - \frac{1}{3m}.$$

$\square$

2. Define FPTAS. Prove or disprove: if there exists an FPTAS for the minimum vertex cover problem on unweighted graphs, then P=NP.

[2+6=8 Marks]