Tutorial 2

Approximation and Online Algorithms

Submission Deadline - 23rd August 2026

All problems marked (*) need to be submitted by the submission deadline.

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1. Prove that the sort and then list schedule algorithm outputs an optimal schedule when the processing time of the shortest job is more than $OPT/3$.

2. Prove that the job scheduling problem (as discussed in the class) does not admit an FPTAS using the fact that the problem is strongly NP-complete.

3. (*) (Problem made by TA)

   (a) Give an $\mathcal{O}(n \log n)$ time algorithm to solve bin packing problem exactly when bin size is 1 and all items have sizes $s_i \geq \frac{1}{3}$.

   (b) Prove that if bin size is 1 and all items have sizes $s_i \geq \frac{1}{k}$ for some constant $k \geq 4$, then the problem is NP-complete. (Hint: Reduce from 3-partition problem)

   (c) Prove that if bin size is 1 and all items have sizes $s_i \geq \frac{1}{k}$ for some constant $k \geq 4$, then we can get a polynomial time approximation algorithm with better than 1.5 approximation guarantee. (Hint: First Fit Decreasing (FFD) algorithm is a polynomial time algorithm for bin packing and it gives an approximation bound of $ALG_{FFD} \leq \frac{11}{9} * OPT + \frac{6}{9}$)

4. (*) (Problem made by TA) Consider a restricted Bin Packing problem where all items have size $s_i \leq 1/k$. A proposed algorithm performs a binary search on the number of bins B, using a Job Scheduling EPTAS as a blackbox to find a makespan $\leq 1 + 1/k$ with minimum number of bins, and then repacks the overflowing items into new bins.

   (a) Prove that this algorithm yields a $(1 + \frac{2}{k-1})$ approximation ratio

   (b) Explain why this algorithm is inefficient in practice and provide a simpler algorithm that achieves a strictly better approximation bound.