Problems: Fibonacci Heaps

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1. Analyse the running time of the Dijkstra's algorithm for single source shortest path and Prim's algorithm for computing a minimum spanning tree in a graph when the priority queue being used in a Fibonacci Heap instead of a Binary Heap. Assume that the number of vertices in the graph is n and the number of edges m.

2. Suppose in the Fibonacci Heap, we allowed an operation IncreaseKey, which takes as input the pointer to a node in a Fibonacci Heap H and a new key value k, and changes $x.\text{key}$ to k. Note that k could be larger than, equal to, or less than the current key value. Design an efficient algorithm to perform this operation so that the amortised time complexity of this operation is $\mathcal{O}(\log n)$ without changing the amortised complexity of other heap operations.

3. [based on CLRS book] Suppose that we generalise the cascading-cut rule to cut a node x from its parent as soon as it loses its k^{th} child, for some integer constant k. For what value of k, we have $D(n) = O(\log n)$? What are the running times of the standard operations on Fibonacci heaps with this new definition? Give an argument with the accounting method for the amortized costs of each operation.

4. Show that the height of a tree in an n-node Fibonacci heap could be $n - 1$.

5. What is the maximum possible degree of an n-node Fibonacci heap? Given any natural number n, provide an example of a sequence of operations so that the Fibonacci heap has n nodes after this sequence of operations and there is a node of the maximum possible degree of an n-node Fibonacci heap. Prove your claim.

6. Prove that every node of degree k has at least F_{k+2} nodes in the subtree rooted at that node. Here, F_n denotes the n-th Fibonacci number ($F_0 = 0, F_1 = 1$).

7. What is the maximum number of cascading cuts possible in a single decrease key operation on any n-node Fibonacci heap? Given any natural number n, provide an example of a sequence of operations with the last operation of the sequence being a decrease key operation and that decrease key operation performs the maximum number of cascading cuts possible on any n-node Fibonacci heap. Prove your claim.

8. What is the maximum number of root nodes possible in an n-node Fibonacci heap at any point in time? What is the maximum number of root nodes possible in an n-node Fibonacci heap immediately after the Extract-Min operation? Justify your answers.