

Problems: Karger and Karger-Stein's Min-Cut Algorithms

Introduction to P, NP, and Reductions

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1. Prove that the worst-case time complexity of one run of Karger's min-cut algorithm is $\mathcal{O}(n^2)$. Assume that we can pick an edge uniformly randomly in $\mathcal{O}(\log m)$ time.
2. Extend Karger's Min-Cut algorithm for weighted undirected graphs with positive edge weights without changing its worst-case time complexity.
3. **[CLRS]** Show that if an algorithm makes at most a constant number of calls to polynomial-time subroutines and performs an additional amount of work that also takes polynomial time, then it runs in polynomial time. Also show that a polynomial number of calls to polynomial-time subroutines may result in an exponential-time algorithm.
4. Define decision and optimization versions of the following computational problems. Prove that if there exists a polynomial-time algorithm for the decision version of the problem, then there exists a polynomial-time algorithm for the search version of the problem.
 - (a) Vertex Cover.
 - (b) Hamiltonian Cycle.
 - (c) Hamiltonian Path.
 - (d) Longest Path.
 - (e) Subset Sum.
5. Prove that if $\text{NP} \neq \text{co-NP}$, then $\text{P} \neq \text{NP}$.
6. Show that many-to-one reducibility relation is transitive.