Practice Problems: Total Unimodularity

Palash Dey
Indian Institute of Technology, Kharagpur

February 6, 2025

Submit the solutions of the questions marked ($\star$) in PDF format generated using Latex by **February 13, 2025**.

---

1. ($\star$) Design a polynomial-time algorithm for the Odd Cut problem.
2. Show that the edge incidence matrix of any directed graph is a network matrix.
3. Show that the consecutive ones matrix is a network matrix.
4. Let $\mathcal{A}$ be a TU matrix. Show that duplicating a row or column preserves TU property.
5. Let $\mathcal{A}$ be a TU matrix. Show that negating a row or column preserves TU property.
6. Let $\mathcal{A}$ be a TU matrix. Show that adding a row or column with single 1 entry and all ther entries being zero, preserves TU property.
7. Show that the coefficient matrix of the following linear programing formulation of the minimum-cost flow problem is totally unimodular.

$$\begin{aligned} \text{minimize} \quad & p \cdot y \\ \text{s.t.} \quad & \forall v \in \mathcal{V}, x(\delta^+(v)) - x(\delta^-(v)) = b_v \\ & \forall e \in \mathcal{E}, x_e \leqslant c_e \\ & x \geqslant 0 \end{aligned}$$

   Can we conclude from it that there is a integral optimal flow if all the demands (b vector) are integral?
8. Give an example of a polytope $\{x : Ax \leqslant b, x \geqslant 0\}$ which is integral but the matrix A is not TU.
9. Give an example of a polytope $\{x : Ax \leqslant b, x \geqslant 0\}$ which is integral and every entry of A is either 0 or ± 1 but the matrix A is not TU.