# Practice Problems: Fast Subset Convolution

Palash Dey
Indian Institute of Technology, Kharagpur

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Submit the solutions of the questions marked ($\star$) in PDF format generated using Latex by **April 11, 2025**.

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1. Design a $\mathcal{O}^\star(2^n)$ time algorithm for counting the number of spanning forests of an undirected graph using fast subset convolution.

2. Given two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$ where $\mathcal{U} = [n]$, show how we can compute the following functions for every subset $S \subseteq \mathcal{U}$ together using fast subset convolution in $\mathcal{O}^\star(2^n)$ time.

$$\max_{X \subseteq S} f(X) + g(S \setminus X)$$

$$\min_{X \subseteq S} f(X) + g(S \setminus X)$$

3. Design a $\mathcal{O}^\star(2^n)$ time algorithm for computing the domantic number of an undirected graph using fast subset convolution.

4. **($\star$)**The covering product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$, denoted by $f \star_c g$ is defined for all $S \subseteq \mathcal{U}$ as

$$(f \star_c g)(S) = \sum_{X, Y \subset S : X \cup Y = S} f(X) g(Y)$$

The packing product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$, denoted by $f \star_p g$ is defined for all $S \subseteq \mathcal{U}$ as

$$(f \star_p g)(S) = \sum_{X, Y \subset S : X \cap Y = \emptyset} f(X) g(Y)$$

The intersecting covering product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$, denoted by $f \star_{ic} g$ is defined for all $S \subseteq \mathcal{U}$ as

$$(f \star_{ic} g)(S) = \sum_{X, Y \subset S : X \cup Y = S, X \cap Y \neq \emptyset} f(X) g(Y)$$

Design a $\mathcal{O}^\star(2^n)$ time algorithm for computing the covering product, packing product, and intersecting covering product of two functions $f, g : 2^{\mathcal{U}} \longrightarrow \mathbb{R}$.