Practice Problems: Inclusion-Exclusion for Designing Exponential Time Exact Algorithms

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Submit the solutions of the questions marked ($\star$) in PDF format generated using Latex by **April 4, 2025**.

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1. **($\star$)** In the Steiner Tree problem, we are given an undirected graph $\mathcal{G}$, a set $\mathcal{K} \subseteq \mathcal{V}[\mathcal{G}]$ of terminals. The goal is to find a tree $\mathcal{T}$ with the minimum number of edges that includes every vertex in $\mathcal{K}$. Design an inclusion-exclusion based algorithm for the Steiner Tree problem that runs in time $\mathcal{O}^\star\left(2^{|\mathcal{K}|}\right)$ and polynomial space.

2. The domatic number of a graph $\mathcal{G}$ is the minimum integer k such that $\mathcal{V}[\mathcal{G}]$ can be partitioned into k sets $\mathcal{V}_1, \ldots, \mathcal{V}_k$ such that each $\mathcal{V}_i$ is a dominating set of $\mathcal{G}$. Design an inclusion-exclusion based algorithm for computing the domatic number of any graph in $\mathcal{O}^\star(2^n)$ time and polynomial space, where n is the number of vertices of $\mathcal{G}$.