Practice Problems: Primal-Dual Method for Approximation Algorithm

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Submit the solutions of the questions marked ($\star$) in PDF format generated using Latex by **March 21, 2025**.

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1. Show how a polynomial-time algorithm for the Transshipment problem can be used to design a polynomial-time algorithm for min-cost flow problem.
2. For a given graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ and a subset $S \subseteq \mathcal{V}$ of vertices, we define $\delta(S) = \{e \in \mathcal{E} : |e \cap S| = 1\}$. That is, $\delta(S)$ is the set of edges whose exactly one end-point belongs to S. Show that two vertices s and t are connected in a graph $\mathcal{G}$ if and only if $\delta(S)$ is non-empty for every subset $S \subset V[\mathcal{G}]$ such that $|S \cap \{s, t\}| = 1$.
3. Design a primal-dual method based approximation algorithm for the minimum weight vertex cover problem.
4. **($\star$)** Design a primal-dual based 2 factor approximation algorithm for the minimum weight $0-1$ Knapsack problem. Design a polynomial-time separation oracle for the linear programming relaxation that you have used for your 2 factor approximation algorithm.
5. Design a polynomial-time separation oracle for the linear programming relaxation of the feedback vertex set problem discussed in the class.