

Hash Functions

Palash Dey
Indian Institute of Technology, Kharagpur

1. Show that a k -universal hash family is also a $(k - 1)$ -universal hash family.
2. Give an example of a k -universal hash family with its proof of k -universality.
3. Give an example of a k -universal hash family that is not a $(k + 1)$ -universal hash family for any integer $k \geq 3$. Prove your claim.
4. Suppose a stream of length m over an universe of size n is distributed over k servers. Explain how using the Count-Min Sketch algorithm, you can compute the approximate frequencies of the elements with polylogarithmic communication complexity (the number of bits communicated). What is the communication complexity of your algorithm?
5. Prove that there is no algorithm with polylogarithmic (in n and m) space complexity that outputs the frequency of any element queried at the end of an m length stream over a universe of size n .