INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Department of Computer Science and Engineering

CS60029 Randomized Algorithm Design | **Mid-Semester Examination, Autumn 25**
23 September 2025 | **60 Marks** | **2:00 PM to 4:00 PM**

Answer all questions.

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1. (a) (7 points) Consider the distribution of the weight of bread produced in a factory. The weight of every bread is at least 1 Kg. The mean and median of the weight distribution are respectively 1.0001 Kg and 1.00005 Kg. Show that the probability that the weight of a bread picked uniformly randomly from the set of breads produced in the factory is less than 1.0004 Kg, is at least $\frac{3}{4}$.

   (b) (8 points) Let X be the random variable denoting the maximum load of any bin in the balls and bin experiment with n balls and n bins. Show that $\mathbb{E}[X] \geqslant \frac{\ln n}{3 \ln \ln n}$.

2. (a) (3 points) Define the multi-commodity flow problem.

   (b) (12 points) Design a randomized rounding based polynomial-time $\mathcal{O}\left(\frac{\ln n}{\ln \ln n}\right)$ factor approximation algorithm for the problem. Prove its approximation guarantee.

3. Consider the following card shuffling process: insert the top card at a position chosen uniformly at random from 1 to n.

   (a) (5 points) Model the shuffling process as a Markov chain.

   (b) (10 points) Show that the unique stationary distribution is the uniform distribution and that $t_{\text{mix}}(\varepsilon) \leqslant n \ln\left(\frac{n}{\varepsilon}\right)$.

4. Let $G = (V, E)$ be any graph with $|E| > 0$. Consider the following Markov chain defined on the state space consisting of all independent sets of G. ($S \subseteq V$ is an independent set if there are no edges in the subgraph induced by S.) Let X_t denote the state at time t. Pick a vertex

$v \in V$ uniformly at random. Define

$$X_{t+1} = \begin{cases} X_t \setminus \{v\} & \text{if } v \in X_t \\ X_t \cup \{v\} & \text{if } v \notin X_t \text{ and } X_t \cup \{v\} \text{ is an independent set of } G \\ X_t & \text{otherwise} \end{cases}$$

Assume X_0 is an arbitrary independent set of G.

(a) (6 points) Argue that the above Markov chain is irreducible, aperiodic and its stationary distribution is the uniform distribution over all independent sets of G.

(b) (9 points) Let $\lambda > 0$ be some constant and I_x denote the independent set of G corresponding to state x. Use the Metropolis algorithm to modify the above chain so that the stationary distribution is given by $\pi_x = \lambda^{|I_x|}/B$ where $B = \sum_x \lambda^{|I_x|}$.