

# Indian Institute of Technology Kharagpur

CS60029 Randomized Algorithm Design, Autumn 2025

## Class Test 2

Total marks: 30

Duration: 1 hour

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**Answer all questions. Keep your answers precise.**

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1. A hypergraph  $H$  is a pair  $(V, E)$ , where  $V$  is the set of vertices and  $E$  is the set of *hyperedges*. A hyperedge is a subset of  $V$ . In a  $r$ -uniform hypergraph, every hyperedge is of size  $r$ . For example, a 2-uniform hypergraph is a standard graph. A *dominating set* in a hypergraph  $H$  is a set of vertices  $S \subseteq V$  such that  $e \cap S \neq \emptyset$  for every edge  $e \in E$ . In other words,  $S$  hits every hyperedge in  $H$ .

Let  $H = (V, E)$  be a  $r$ -uniform hypergraph with  $n$  vertices and  $m$  edges.

- (a) (9 points) Show that there is a dominating set of size at most  $np + m(1-p)^r$  for every real number  $p \in [0, 1]$ .
- (b) (6 points) Show that there is a dominating set of size at most  $(m + n \ln r)/r$ .
2. (a) (7 points) Suppose a stream of length  $m$  over an universe of size  $n$  is distributed over  $k$  servers. Explain how using the Count-Min Sketch algorithm, you can compute the approximate frequencies of the elements with polylogarithmic communication complexity (the number of bits communicated). What is the communication complexity of your algorithm?
- (b) (8 points) Give an example of a 3-universal hash family that is not a 4-universal hash family. Prove your claim.
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