

# Indian Institute of Technology Kharagpur

CS60029 Randomized Algorithm Design, Autumn 2025

## Class Test 1

Total marks: 30

Duration: 1 hour

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**Answer all questions. Keep your answers precise.**

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1. (15 points) In the Triangle Packing problem, we are given an undirected graph  $G$  and a positive integer  $k$ , and the objective is to test whether  $G$  has  $k$ -vertex disjoint triangles. Using color coding, show that the problem admits an algorithm with running time  $2^{\Theta(k)} n^{\Theta(1)}$ .
2. (a) (5 points) Prove that the condition in Markov's inequality that the random variable under consideration must be non-negative is necessary.  
(b) (10 points) Prove the weak law of large numbers using Chebyshev inequality. The weak law of large number states that, for random variables  $X_i, i \in \mathbb{N}$  which are distributed independently and identically with mean  $\mu$  and variance  $\sigma^2$ , we have the following for any constant  $\varepsilon > 0$

$$\lim_{n \rightarrow \infty} \Pr \left[ \left| \frac{X_1 + X_2 + \cdots + X_n}{n} - \mu \right| > \varepsilon \right] = 0$$

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