

Problems: Fibonacci Heaps

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1. Analyse the running time of the Dijkstra's algorithm for single source shortest path and Prim's algorithm for computing a minimum spanning tree in a graph when the priority queue being used in a Fibonacci Heap instead of a Binary Heap. Assume that the number of vertices in the graph is n and the number of edges m .
2. Suppose in the Fibonacci Heap, we allowed an operation `ChangeKey`, which takes as input the pointer to a node in a Fibonacci Heap H and a new key value k , and changes $x.key$ to k . Note that k could be larger than, equal to or less than the current key value. Analyse the amortised running time of this operation.
3. [based on CLRS book] Suppose that we generalise the cascading-cut rule to cut a node x from its parent as soon as it loses its k^{th} child, for some integer constant k . For what value of k , we have $D(n) = O(\log n)$? What are the running times of the standard operations on Fibonacci heaps with this new definition? Give an argument with the accounting method for the amortized costs of each operation.
4. Show that the height of a tree in an n node Fibonacci heap could be $n - 1$.