Problems: Approximation Algorithms

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November 5, 2025

1. Consider the task of scheduling n jobs into m identical machines. Each job i needs to run t_i time in any machine to complete. A job once started to run in a machine cannot be stopped midway. A schedule is an assignment $\mathcal{A} : [n] \longrightarrow [m]$. The load ℓ_j of machine j in a schedule $\mathcal{A}$ is $\sum_{i \in \mathcal{A}^{-1}(j)} t_i$. The makespan of a schedule $\mathcal{A}$ is $\max_{j \in [m]} \ell_j$. Show that computing a schedule which achieves the minimum makespan is NP-complete. Design a simple 2 factor approximation algorithm for this problem. Change the algorithm suitably to improve the approximation guarantee to $\frac{3}{2}$.
2. Design a greedy $1/2$ factor deterministic approximation algorithm for the maximum cut problem on edge-weighted graphs. (Note that this is also called 2 factor approximation algorithm.)
3. Design a $7/8$ factor deterministic approximation algorithm for the weighted Max E3SAT problem. In the E3SAT problem, every clause has exactly three distinct literals.
4. Show that Every E3SAT problem instance with at most 7 clauses is always satisfiable.