Indian Institute of Technology Kharagpur
Dept. Computer Science and Engineering

Subject No.: CS31005 Subject Name: Algorithms II
Total marks: 100 Duration: 3 hours

End Semester Examination

Answer all questions.

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1. Design a deterministic polynomial-time algorithm for the Edge Cover problem.
   (a) [2 points] Define the Edge Cover problem for undirected graphs.
   (b) [4 points] Clearly describe your algorithm.
   (c) [4 points] Write the pseudocode of your algorithm.
   (d) [6 points] Prove the correctness of your algorithm.
   (e) [4 points] Analyze your algorithm's run time.
2. [20 points] Prove or disprove: If there is a polynomial-time algorithm for the decision version of the Hamiltonian Cycle problem, there is a polynomial-time algorithm to output a Hamiltonian cycle whenever it exists.
3. (a) [10 points] Show that the Vertex Cover problem many-to-one reduces to the Set Cover problem.
   (b) [10 points] Show that the Subset Sum problem many-to-one reduces to the Partition problem.
4. (a) [10 points] Prove or disprove: Halting problem is NP-hard.
   (b) [10 points] Suppose you have a randomized algorithm for outputting a minimum cut in an undirected graph whose success probability is $\Theta\left(\frac{1}{n^3}\right)$. Using this, design another randomized algorithm for outputting a minimum cut in an undirected graph whose success probability is 0.999. What will be the running time of the algorithm?

5. (a) [10 points] The problem 2-SAT takes as input a CNF-SAT instance where each clause has at most 2 literals. Either design a polynomial-time algorithm for this problem or prove that it is NP-complete.

   (b) [10 points] Given an undirected graph G and a cut $S = (U, V(G) \setminus U)$ such that $\emptyset \neq U \subsetneq V(G)$, $\mathsf{cost}(S)$ is the number of edges with one end point in U and the other end point not in U. The Max Cut problem takes as input an undirected graph G and outputs the maximum value of $\mathsf{cost}(S)$ over all cuts S of G. Design a deterministic 2-approximation algorithm for Max-Cut.
