

Indian Institute of Technology Kharagpur

CS31005 Algorithms II – Class Test 2

Total marks: 30

Duration: 1 hour

Answer all questions.

1. (10 points) Prove that the expected total number of pairwise comparisons performed by the randomized quicksort algorithm when sorting an input array of n elements is $\mathcal{O}(n \log n)$.
 2. (10 points) Give an instance of Stable Matching where the number of stable matchings is more than one. Explicitly mention at least 2 stable matchings of the instance with justification as to why they are stable.
 3. (a) (3 points) In the Knuth-Morris-Pratt algorithm, for a prefix P_i of the pattern P , $0 \leq i \leq |P|$, let $\pi(i)$ denote the length of the longest proper prefix of P_i that is also a suffix of P_i . The set $\pi^* = \{\pi(i), \pi^{(2)}(i), \dots, \pi^{(t)}(i)\}$, where t is the smallest integer such that $\pi^{(t)}(i) = 0$. What is an upper bound on the size of π^* ? Give a tight example where this upper bound is achieved.
(b) (7 points) Given two strings s_1 and s_2 , each of length n design an $\mathcal{O}(n)$ time algorithm to determine if s_1 is a cyclic rotation of s_2 . For example, “algorithm” is a cyclic rotation of “thmalgori”.
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