Indian Institute of Technology Kharagpur

CS31005 Algorithms II – Class Test 1

Total marks: 30 Duration: 1 hour

Answer all questions.

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1. An ordered stack is a stack where all elements in the stack are stored in increasing order. Eg: $\rightarrow -6, -4, 0, 3, 5, 9$ where $\rightarrow$ indicates the bottom of the stack.

   - ▷ The operation $\mathsf{Init}(S)$ initializes an empty ordered stack S.
   - ▷ $\mathsf{Pop}(S)$ pops out the top-most element from the stack S.
   - ▷ $\mathsf{Push}(S, x)$ pushes x onto the top of S, but then reestablishes the order of the elements by repeatedly removing the element just below x till x is the largest element in S. Eg. given $S = \rightarrow -6, -4, 0, 3, 5, 9$, $\mathsf{Push}(S, 2)$ ultimately results in $\rightarrow -6, -4, 0, 2$.
   - ▷ $\mathsf{Del}(S)$ deletes all elements on S.

   (i) What is the worst case running time of each operation? State strongest possible bound and prove your answer.

   (ii) What is the amortized running time of each operation? State strongest possible bound and prove your answer.

   [2 + (2+2+2+2) = 10 Marks]

2. Consider a dynamic array that supports only insertions. The rule is that if a table of size s is less than 3/4-th full then we simply insert a new element to the table. Otherwise, we create a new table of size $2s$, copy all elements of the old table to the new table and then insert a new element. Otherwise, What is the amortized cost of n insertion operations in such a data structure?

   [5 Marks]

3. (i) Prove or disprove: The time complexity of Ford-Fulkerson method is not polynomial in input size. Assume that all the input numbers are positive integers.

   (ii) Prove or disprove: The height of a tree in an n node Fibonacci heap is always $\mathcal{O}(\log n)$ immediately after an Extract-Min operation.

   (iii) Prove or disprove: The sum of heights of all trees in an n node Fibonacci heap is always $\mathcal{O}(\log n)$ immediately after an Extract-Min operation.

   [5+5+5=15 Marks]
