Practice Problems: Total Unimodularity

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1. Let $\mathcal{A}$ be a TU matrix. Show that duplicating a row or column preserves TU property.
2. Let $\mathcal{A}$ be a TU matrix. Show that negating a row or column preserves TU property.
3. Let $\mathcal{A}$ be a TU matrix. Show that adding a row or column with single 1 entry and all ther entries being zero, preserves TU property.
4. Show that the coefficient matrix of the following linear programing formulation of the minimum-cost flow problem is totally unimodular.

$$\begin{aligned} \text{minimize} \quad & p \cdot y \\ \text{s.t.} \quad & \forall v \in \mathcal{V}, x(\delta^+(v)) - x(\delta^-(v)) = b_v \\ & \forall e \in \mathcal{E}, x_e \leqslant c_e \\ & \qquad x \geqslant 0 \end{aligned}$$

   Do you see that we can conclude from it that there is a integral optimal flow if all the demands (b vector) are integral?

5. Give an example of a polytope $\{x : Ax \leqslant b, x \geqslant 0\}$ which is integral but the matrix A is not TU.
6. Give an example of a polytope $\{x : Ax \leqslant b, x \geqslant 0\}$ which is integral and every entry of A is either 0 or ± 1 but the matrix A is not TU.