# Practice Problems: LP Rounding for Approximation Algorithm

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1. In the maximum weighted cut problem, we are given an edge-weighted undirected graph and we need to compute a paritition $(U, W)$ of the vertices of the graph which maximizes the sum of the weights of the edges between $U$ and $W$. Design a $\frac{1}{2}$ factor approximation algorithm for this problem.

2. In the maximum weighted directed cut problem, we are given an edge-weighted directed graph and we need to compute a paritition $(U, W)$ of the vertices of the graph which maximizes the sum of the weights of the edges from $U$ to $W$. Design a $\frac{1}{4}$ factor approximation algorithm for this problem.

3. Show that the following integer linear program models the maximum weighted directed cut problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{e \in \mathcal{E}} w_e z_e \\ \text{s.t.} \quad & \forall (u,v) \in \mathcal{E}, z_{uv} \leqslant x_u \\ & \forall (u,v) \in \mathcal{E}, z_{uv} \leqslant 1 - x_v \\ & \forall e \in \mathcal{E}, z_e \in \{0,1\} \\ & \forall v \in \mathcal{V}, x_v \in \{0,1\} \end{aligned}$$

   Consider the randomized rounding algorithm which takes an optimal solution $(x^*, z^*)$ of the linear programming relaxation of the ILP above and puts every vertex $v$ in $U$ with probability $\frac{1}{4} + \frac{1}{2}x_v^*$. Show that this randomized algorithm achieves an approximation factor of $\frac{1}{2}$.

4. Consider the non-linear randomized rounding algorithm for MAXSAT discussed in the class. Let $(y^*, z^*)$ be an optimal solution of the LP relaxation of the standard ILP formulation of MAXSAT. Show that setting $x_i$ to true with probability $f(y_i^*) = \frac{1}{2}y_i^* + \frac{1}{4}$

$$f(y_i^*) = \begin{cases} \frac{3}{4}y_i^* + \frac{1}{4} & \text{if } 0 \leqslant y_i^* \leqslant \frac{1}{3} \\ \frac{1}{2} & \text{if } \frac{1}{3} \leqslant y_i^* \leqslant \frac{2}{3} \\ \frac{3}{4}y_i^* & \text{if } \frac{2}{3} \leqslant y_i^* \leqslant 1 \end{cases}$$

   gives a $\frac{3}{4}$ factor approximation algorithm for MAXSAT.

5. Show that the total expected facility cost of the randomized rounding algorithm discussed in the class is at most LP-OPT.

6. Design a randomized rounding based algorithm for set cover which returns a $\mathcal{O}(\log n)$ factor approximate solution with probability at least $\frac{2}{3}$.

7. You are given a vertex weighted undirected graph and a valid $k$ coloring of it. Design a polynomial-time $2\left(1 - \frac{1}{k}\right)$ factor approximation algorithm for minimum weighted vertex cover of this graph.

8. Derandomize all the randomized approximation algorithms discussed in the class.