Problems: Stable Matching

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1. [CLRS] Describe how to implement the Gale-Shapley algorithm so that it runs in $\mathcal{O}(n^2)$ time.
2. [CLRS] The National Resident Matching Program differs from the scenario for the stable- marriage problem set out in this section in two ways. First, a hospital may be matched with more than one student, so that hospital h takes $r_h \geqslant 1$ students. Second, the number of students might not equal the number of hospitals. Describe how to modify the Gale-Shapley algorithm to ût the requirements of the National Resident Matching Program.
3. [CLRS] Prove the following property, which is known as weak Pareto optimality: Let M be the stable matching produced by the Gale-Shapley procedure, with women proposing to men. Then, for a given instance of the stable- marriage problem there is no matching — stable or unstable — such that every woman has a partner whom she prefers to her partner in the stable matching M .
4. [CLRS] The stable-roommates problem is similar to the stable-marriage problem, except that the graph is a complete graph, not bipartite, with an even number of vertices. Each vertex represents a person, and each person ranks all the other people. The definitions of blocking pairs and stable matching extend in the natural way: a blocking pair comprises two people who both prefer each other to their current partner, and a matching is stable if there are no blocking pairs.

   Unlike the stable-marriage problem, the stable-roommates problem can have inputs for which no stable matching exists. Find such an input and explain why no stable matching exists.