INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Stamp / Signature of the Invigilator

EXAMINATION (Mid Semester)	**SEMESTER (Autumn)**

Roll Number										**Section**		**Name**	

Subject Number	C	S	3	1	0	0	5	**Subject Name**	*Algorithms II*

Department / Center of the Student		**Additional sheets**	

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession, even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer script and do not tear off any page. **Use last page(s) of the answer script for rough work.** Report to the invigilator if the answer script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the examination hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and the consumption of any kind of beverages are strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer script to the invigilator. **In any case, you are not allowed to take away the answer script with you.** After the completion of the examination, do not leave the seat until the invigilators collect all the answer scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as **'unfair means'**. Do not adopt unfair means and do not indulge in unseemly behavior.

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be filled in by the examiner

Question Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Marks Obtained											

Marks obtained (in words)	**Signature of the Examiner**	**Signature of the Scrutineer**

1. Consider the problem of insertions and deletions of positive integers into a table that is solved by the following algorithm $\mathscr{A}$: Let the current table size be i.

   - Elements are inserted into the current table till the table is full. When we try to insert an element, but it is full and the table already has i elements, a new table of size $2i$ is created and all elements including the new element are inserted into the new table while the old table is forgotten.
   - Elements may be deleted from the current table up to the point where the table contains more than $i/4$ elements. When an element has to be deleted, but the table has only $i/4$ elements, then a new table of size $i/4$ is created and all elements minus the element to the currently deleted are copied onto the new table while the old table is forgotten.

   (a) (5 points) Analyze the amortized running time of an insertion operation in the algorithm $\mathscr{A}$.

   (b) (5 points) Analyze the amortized running time of a deletion operation in the algorithm $\mathscr{A}$.

2. Consider the problem Area-Union, that takes as input a set of axis-parallel rectangles and outputs the area of the union of these rectangles. A rectangle is axis-parallel if each side is parallel to the x-axis or y-axis. **Also assume that your input instance is such that any rectangle can intersect with at most one other rectangle.**
   (a) (5 points) Design a sweep line algorithm for Area-Union. [Hint: What should be the events of the sweep line? What area is covered between events?]
   (b) (5 points) What is the running time of the algorithm? Explain your answer.

3. Design an algorithm to check if a flow network has a unique minimum capacity $s-t$ cut and if not, then output two different minimum capacity $s-t$ cuts.
   (a) (3 points) Clearly explain the algorithm in English language
   (b) (2 points) Write a pseudo-code of the algorithm.
   (c) (3 points) Prove its correctness.
   (d) (2 points) Analyze its running time.

4. (a) (2 points) What is layered graph in the context of max-flow problem?
   (b) (2 points) Design an $O(mn)$ time algorithm to compute a blocking flow in a layered graph.
   (c) (2 points) Design an algorithm for computing a maximum $s-t$ flow in a flow network.
   (d) (2 points) Prove correctness of your algorithm.
   (e) (2 points) Analyze its running time.

5. (a) (3 points) Clearly explain Edmond's blossom algorithm for computing a maximum-cardinality matching in a graph.

   (b) (3 points) Write pseudo-code of Edmond's blossom algorithm.

   (c) (4 points) Show the execution of Edmond's blossom algorithm in a connected, non-bipartite graph with 10 vertices, and the degree of every vertex is at least 3.