Problems: Push Relabel Preflow Algorithm

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1. Design/use suitable data structure so that the push relabel algorithm taken $\mathcal{O}(n^3)$ time. Implement the push relabel algorithm using your data structure and compare its performance against Edmond-Karp's algorithm and Dinic's algorithm.

2. [CLRS] Suppose that all edge capacities in the flow network G are in the set $\{1, 2, \ldots, k\}$. Analyse the running time of the push relabel algorithm in terms of the number n of vertices, the number m of edges in G as well as k.

3. Given a flow network $\mathcal{G}$ and a flow f show that the underlying undirected graph of $\mathcal{G}$ is connected if and only if the underlying undirected graph of $\mathcal{G}_f$ is connected.

4. Consider a run of push-relabel algorithm. For a residual graph G_f, let $\delta_f(u, v)$ be the number of edges in the shortest path from u to v.

   ▷ Show that for any vertex u, $\delta_f(u, t) \geqslant h(u)$ where h denotes the current height function.
   ▷ Is it possible that $h(s) \geqslant h(u) > \delta_f(u, s)$? Construct examples.