Indian Institute of Technology Kharagpur

CS29003: Algorithms Laboratory, Spring 2022

Assignment 5: Dynamic Programming

2PM – 5PM 15th February, 2022

General Instructions (to be followed strictly)

Submit a single C/C++ source file.
Do not use global variables unless you are explicitly instructed so.
Do not use Standard Template Library (STL) of C++.
Use proper indentation in your code and include comments.
Name your file as <roll_no>_a5.<extn>

Write your name, roll number, and assignment number at the beginning of your program.

Chef Rijo Dassarp has 5 different kinds of sauces s_1, s_2, s_3, s_4, s_5. Interestingly, mixing any two of the sauces yields one of the 5 sauces i.e., a sauce in the set $S = \{s_1, s_2, s_3, s_4, s_5\}$. Two sauces $s_i, s_j \in S$ are mixed (in that order) as follows: s_i is heated in a saucepan; once it starts to boil s_j is stirred into it, resulting in some sauce $s_k \in S$. The mixture of s_i with s_j is written as $s_i s_j$. Mixing the same two sauces in the opposite order may result in a sauce different from s_k. Mixing a sauce s_i with itself would still yield s_i. Rijo knows the results of mixing all 25 possible pairs of sauces.

A particular sequence of sauces can be mixed in different ways. For example, $s_2 s_5 s_1 s_3$ can be mixed in 5 different ways: $((s_2 s_5) s_1) s_3$, $(s_2 (s_5 s_1)) s_3$, $s_2((s_5 s_1) s_3)$, $s_2(s_5(s_1 s_3))$, $(s_2 s_5)(s_1 s_3)$. Here, $(s_2 s_5) s_1$ denotes the mixing of s_2 with s_5 and the resulting sauce being mixed with s_1. So the number of ways to mix a sequence (represented as a string over S) of sauces is the number of ways it can be paranthesised.

Rijo tests one of his apprentices, Inala Ono, by giving her a sequence/string of sauces $\sigma \in S^*$ to mix. He asks her to find out the number of different sauces that can be obtained by mixing σ and the number of mixings that result in each possible sauce. Your task is to help Inala by solving the problem for her.

- Write a function *possible* that takes as input $\ell \in \{1, 2, 3, 4, 5\}$, $\sigma \in S^*$, $M \in \{1, 2, 3, 4, 5\}^{5 \times 5}$. The 5×5 matrix M contains the results of mixing all possible pairs of sauces from S. The (i, j)-th entry of M is k if mixing $s_i s_j$ results in s_k. The function outputs 1 if s_ℓ can be obtained by mixing σ and 0 otherwise.
- Write a function *countways* that takes as input $\sigma \in S^*$, $M \in \{1, 2, 3, 4, 5\}^{5 \times 5}$ that for each $s_i \in S$, computes and prints the number of ways in which s_i can be obtained by mixing σ.

If $|\sigma| = n$, then both the functions must run in $O(n^3)$ time and use $O(n^2)$ space.

In the *main()* function, first read the 5×5 matrix M and then read the string σ as a sequence of indices from $\{1, 2, 3, 4, 5\}$. Call $possible(i, \sigma, M)$ for each $i \in \{1, 2, 3, 4, 5\}$ and print whether or not it is possible to obtain s_i by mixing σ. Then call $countways(\sigma, M)$ to print the number of ways each s_i can be obtained by mixing σ.

Sample Output

```
Enter M:

1 3 4 3 4
5 2 5 3 1
2 5 3 1 2
3 1 5 4 1
4 2 4 2 5

Enter string: 31442

1: Possible
2: Possible
3: Possible
4: Not Possible
5: Possible

Number of ways in which mixing 31442 results in
1: 1
2: 5
3: 3
4: 0
5: 5
```