Tutorial 4: CS21003 Algorithms I

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1. Given two strings, $X = [x_1, x_2, \ldots, x_n]$ (having length n) and $Y = [y_1, y_2, \ldots, y_m]$ (having length m), the shortest common supersequence (SCS) is a minimum length string Z such that both X and Y are subsequences of Z. For example, if X = [abcbdab] (length 7) and Y = [bdcaba] (length 6), a SCS is Z = [abdcabdab] (length 9). Your task is to find out the length of the SCS from two input strings of length n and m. Answer the following five parts:
   (a) Provide a recursive definition to compute the length of the SCS as given in the problem statement.
   (b) Develop a recursive algorithm translating the above definition, without declaring additional space. Also, derive the time-complexity of your algorithm in asymptotic Big-O notation.
   (c) Improve this top-down recursive algorithm with the help of Memoization (using additional space).
   (d) Now, propose an iterative (bottom-up) algorithm for the same problem. Also, provide the time and space complexity of your algorithm in asymptotic Big-O notation (give tight bounds).
   (e) Clearly show the working steps of your proposed iterative bottom-up algorithm (above) in the given example strings, X = [abcbdab] and Y = [bdcaba].
2. Solve the Coins Problem by Dynamic Programming and memoization. Show the working on the example of $S = \{8, 6, 5, 2, 1\}$ and $V = 11$. Analyse the complexity of the Algorithm.
3. Solve the k-peg n-disk Tower of Hanoi Problem by Dynamic Programming to determine the optimal number of moves and present the top-down and bottom-up algorithms with memoization. Analyse the complexity of the algorithm. How will you find the actual minimum sequence of moves? Show how you will get the solution for $k = 5$ and $n = 7$.