Practice Problems: Algorithmic Game Theory

Palash Dey Indian Institute of Technology, Kharagpur

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- 1. Prove revelation principle for BIC mechanisms.
- 2. (Taken from an exercise in [Nar14]) Let $N = \{1, 2\}, \Theta_1 = \{a_1, b_1\}, \Theta_2 = \{a_2, b_2\}, \mathcal{X} = \{x, y, z\}$ and

$$\begin{split} \mathfrak{u}_1(x,\mathfrak{a}_1) &= 100, \mathfrak{u}_1(y,\mathfrak{a}_1) = 50, \mathfrak{u}_1(z,\mathfrak{a}_1) = 0\\ \mathfrak{u}_1(x,\mathfrak{b}_1) &= 50, \mathfrak{u}_1(y,\mathfrak{b}_1) = 100, \mathfrak{u}_1(z,\mathfrak{b}_1) = 40\\ \mathfrak{u}_2(x,\mathfrak{a}_2) &= 0, \mathfrak{u}_2(y,\mathfrak{a}_2) = 50, \mathfrak{u}_2(z,\mathfrak{a}_2) = 100\\ \mathfrak{u}_2(x,\mathfrak{b}_2) &= 50, \mathfrak{u}_2(y,\mathfrak{b}_2) = 30, \mathfrak{u}_2(z,\mathfrak{b}_2) = 100 \end{split}$$

For the above environment, give an example for a social choice function for each of the following cases (EPE: Ex-Post Efficient, DSIC: Dominant Strategy Incentive Compatible, BIC: Bayesian Incentive Compatible, D: Dictatorship, ND: Non-dictatorship).

- (i) EPE, DSIC, and D
- (ii) EPE, DSIC, and ND
- (iii) Not EPE but DSIC and ND
- (iv) EPE, BIC (under suitable prior), but not DSIC
- (v) EPE but not BIC (under suitable prior)
- 3. Can a social choice function has more than one dictator if every player has a strict rational preference relation?

References

[Nar14] Y. Narahari. *Game Theory and Mechanism Design*. World Scientific Publishing Company Pte. Limited, 2014.