Assignment 2: Algorithmic Game Theory

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1. Let $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$ be a game in strategic form. Let $\sigma_i \in \Delta(S_i)$ be mixed strategies of the players and $\sigma = \prod_{i \in N} \sigma_i$. Prove that σ is a CE if and only if $(\sigma_i)_{i \in N}$ is an MSNE.

2. Let $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$ be a game in strategic form. Prove that a distribution $\sigma \in \Delta(\prod_{i \in N} S_i)$ is a CE if and only if the following holds for every $i \in N$ and every $\delta_i : S_i \longrightarrow S_i$.

$$\mathbb{E}_{s \sim \sigma}[u_i(s)] \geqslant \mathbb{E}_{s \sim \sigma}[u_i(\delta_i(s_i), s_{-i})]$$

3. Give an example of a game which has a PSNE but the best response dynamics can run forever.

4. Let α be a correlated equilibrium of a matrix game. Prove that $u_1(\alpha)$ (the utility of the row player) is equal to the value of the game in mixed strategies.

5. Compute all correlated equilibrium of the following coordination game.
   - ▷ The set of players $(N) : \{1, 2\}$
   - ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$
   - ▷ Payoff matrix:

		Player 2	
		A	B
Player 1	A	$(2, 2)$	$(0, 6)$
	B	$(6, 0)$	$(1, 1)$

6. Compute all correlated equilibrium of the following coordination game.
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7. Prove that as the degree p of the cost function in the bottom link of Pigou's network goes to ∞, the price of anarchy of Pigou's network tends to ∞ as $\frac{p}{\ln p}$.

8. Prove that in a selfish load balancing game with 3 tasks and 2 identical machines, the PoA with respect to PSNE is 1.