INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Randomized Algorithm Design: Mid Semester Examination 2018-19

Date of Examination: 25 February 2019
Session (FN/AN): Mid Semester Examination
Duration: 2 hours
Full Marks: 60
Subject No: CS60029
Subject: Randomized Algorithm Design
Department/Center/School: COMPUTER SCIENCE AND ENGINEERING
Specific charts, graph paper, log book etc., required: NO
Special instruction (if any): NA

Answer question 5 and any 3 from rest (total 4).

1. State and prove Schwartz-Zippel lemma. Reduce the problem of finding a perfect matching in a bipartite graph to the polynomial identity testing problem.

[10+5 Marks]

2. Show that the expected number of balls one needs to through randomly into m bins to have every bin at least one ball is $\mathcal{O}(m \log m)$.

[15 Marks]

3. Show that the mixing time of a random walk on an n dimensional hypercube is at most $n \ln n + n \ln(1/\varepsilon)$.

[15 Marks]

4. Suppose we have a program that takes as input a number x on the real interval $[0, 1]$ and outputs $f(x)$ for some bounded function f taking on values in the range $[1, b]$. We want to estimate

$$\int_{x=0}^{1} f(x)dx$$

Assume we can generate random numbers $X_i, i \in [m]$ independently and uniformly from $[0, 1]$. Then show that $\sum_{i=1}^{m} \frac{f(X_i)}{m}$ gives an (ε, δ)-approximation for the integral for a suitable value of m.

[15 Marks]

5. A cat and a mouse each independently take a random walk on a connected, undirected, non-bipartite graph $\mathcal{G}$. They start at the same time on different nodes, and each makes one transition at each time step. The cat eats the mouse if they are ever at the same node at some time step. Let n and m denote. respectively. the number of vertices and edges of $\mathcal{G}$. Show an upper bound of $\mathcal{O}(m^2 n)$ on the expected time before the cat eats the mouse. (Hint: Consider a Markov chain whose states are the ordered pairs (a, b), where a is the position of the cat and b is the position of the mouse.)

[15 Marks]