### INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR Advanced Graph Theory: Second Class Test 2018-19

Date of Examination: 23 October 2019 Duration: 50 minutes Full Marks: 20 Subject No: CS60025 Subject: Advanced Graph Theory Department/Center/School: Computer Science and Engineering Special instruction (if any): You do not need to prove anything that is already proved in the class. Any result which is not proved in the class and you want to use it, you have to prove it.

# Answer all question.

1. Prove or disprove: every graph can be embedded in a 3 dimensional Euclidean space (that is  $\mathbb{R}^3$ ) such that no two edges cross each other.

## [10 Marks]

2. (i) Show that, in every planar graph, there always exists a vertex of degree at most 5.

#### [5 Marks]

(ii) Using (i), show that every planar graph is 6 colorable. That is, for every planar graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , there is a mapping  $\chi : \mathcal{V} \longrightarrow \mathcal{C}$  such that  $|\mathcal{C}| = 6$  and, for every edge  $\{u, v\} \in \mathcal{E}$ , we have  $\chi(u) \neq \chi(v)$ .

## [5 Marks]