Network Centrality
Part 1

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Node centrality

- Relative importance of a node in a network

- Importance varies according to application
  - How influential a person is within a social network
  - How important a webpage is in the Web
  - Which persons to vaccinate when a disease is spreading

- There is an analogous concept of edge centrality, but we will focus on node centrality

Node centrality measures

- Many proposed centrality measures
  - Network structure based
  - Activity based (e.g., number of times a user is mentioned on Twitter or Facebook)
  - Temporal (e.g., Test-of-Time awards to research papers)
  - Hybrid
  - … and more

- We will focus on the first two types of measures

Degree centrality

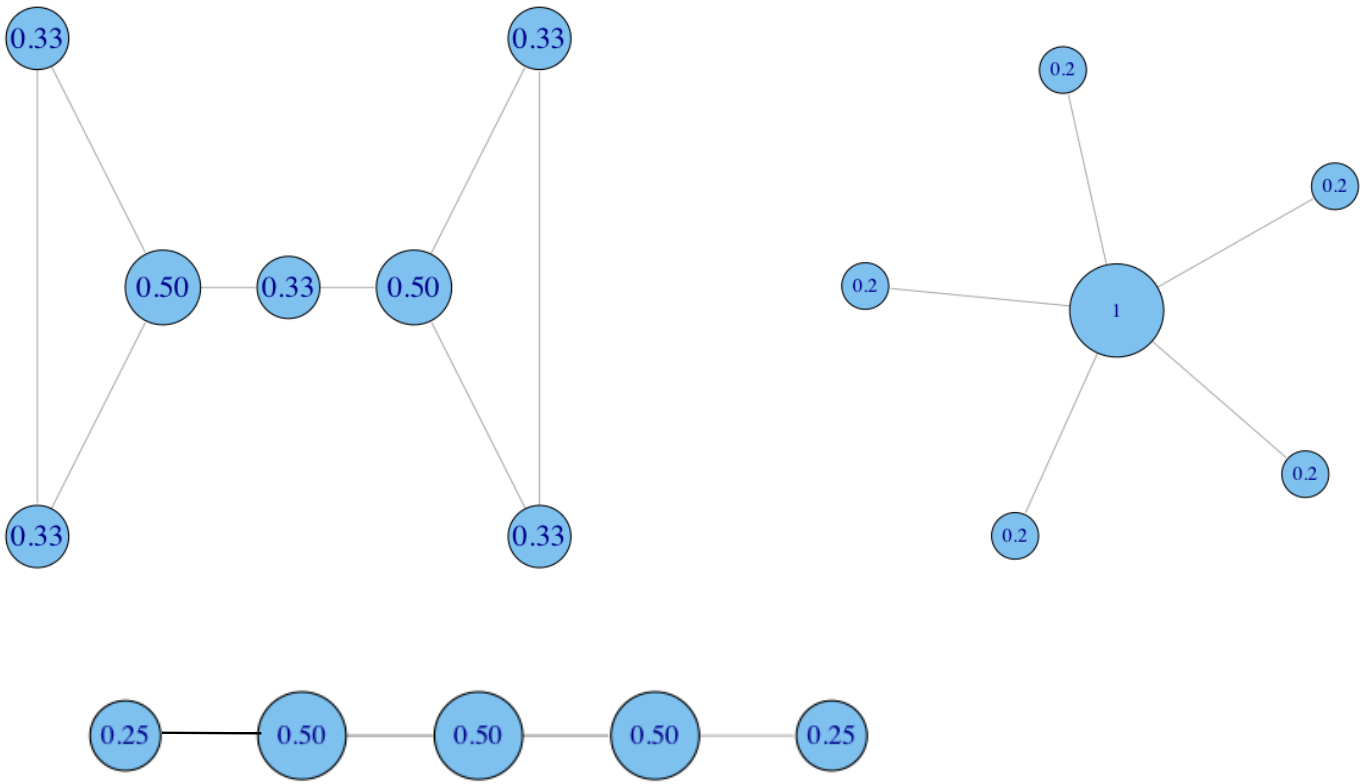
- Simply, centrality measured by degree of a node
  - A node of higher degree is more important

- Undirected graphs
  - Number of friends of a user in Facebook
  - Important stations in railway networks

- Directed graphs: usually indegree of node
  - Number of pages linking to a given page in the Web
  - Number of followers of a user in Twitter

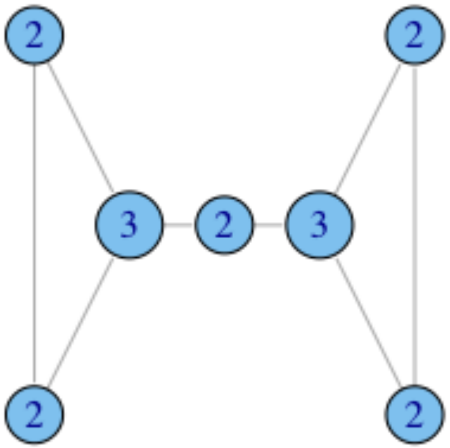
Normalized degree centrality

divide degree by the max. possible, i.e. (N-1)



When degree isn't everything

In what ways does degree fail to capture centrality in the following graphs?



- Ability to broker between groups
- Likelihood that information originating anywhere in the network reaches you…

Closeness centrality

- Intuition
  - Farness of node *s* : sum of its shortest distances to all other nodes
  - Closeness of node *s* : inverse of farness

Closeness centrality

Closeness is based on the length of the average shortest path between a vertex and all vertices in the graph

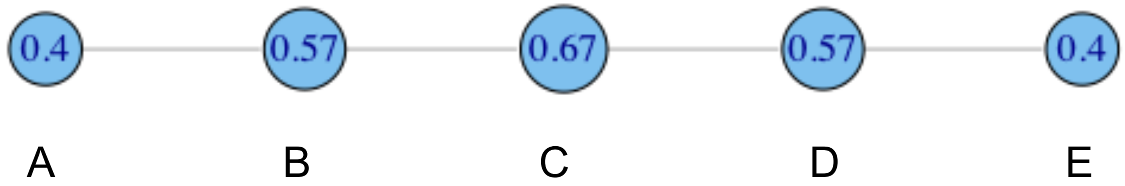
Closeness Centrality:

$$C_c(i) = \frac{1}{\sum_{\{j = 1\}}^{N} d(i,j)}$$

Normalized Closeness Centrality

$$C'_c(i) = \frac{(N - 1)}{\sum_{\{j = 1\}}^{N} d(i,j)} = \left[\frac{\sum_{\{j=1\}}^{N} d\,(i,j)}{N - 1}\right]^{-1}$$

Closeness centrality: toy example



$$C_c'(A) = \left[\frac{\sum_{j=1}^{N} d(A,j)}{N-1} \right]^{-1} = \left[\frac{1+2+3+4}{4} \right]^{-1} = \left[\frac{10}{4} \right]^{-1} = 0.4$$

Closeness centrality

- Higher the closeness centrality of *s*, the lower is its total distance to all other nodes

- Applications
  - Where to set up a hospital in a town?
  - How fast can information spread from *s* to all other nodes?

Betweenness centrality

- Intuition
  - How many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- Betweenness of node s:
  - For each pair of vertices (u, v), find the shortest paths between them (u or v is not s itself)
  - Compute the fraction of these shortest paths which pass through node s
  - Sum this fraction for all pairs of nodes (u, v)

Betweenness centrality: definition

$$C_B(i) = \sum_{j<k} g_{jk}(i)/g_{jk}$$

Where g_{jk} = the number of geodesics connecting *jk*, and
g_{jk} (i) = the number of these geodesics that actor *i* is on.

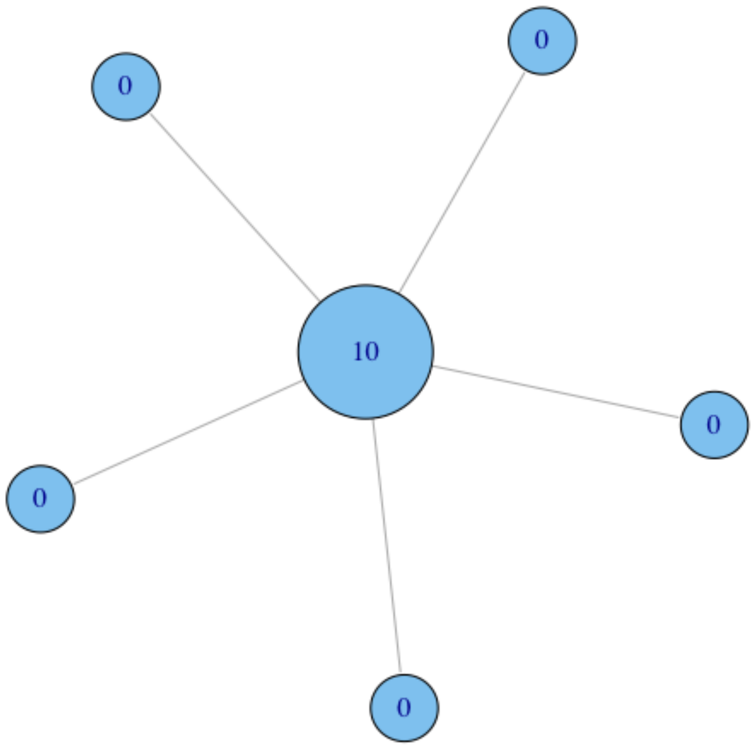
Can be normalized by:

$$C'_B(i) = C_B(i)/[(n-1)(n-2)/2]$$

number of pairs of vertices excluding the vertex itself

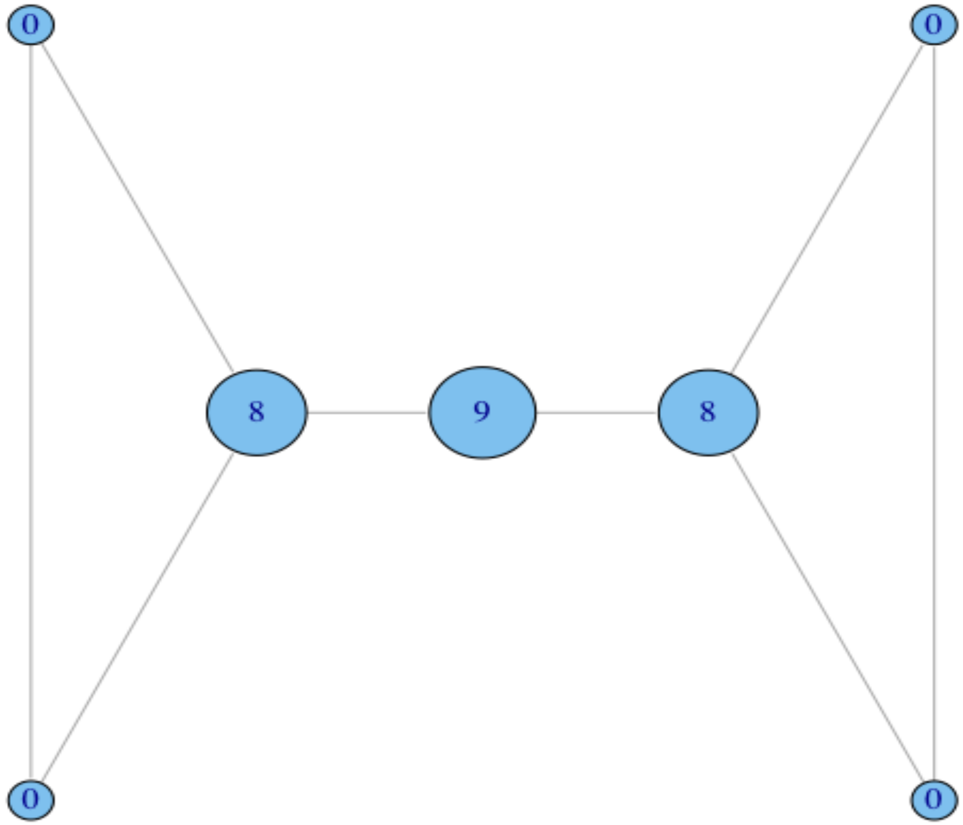
Betweenness on toy networks

- non-normalized version:

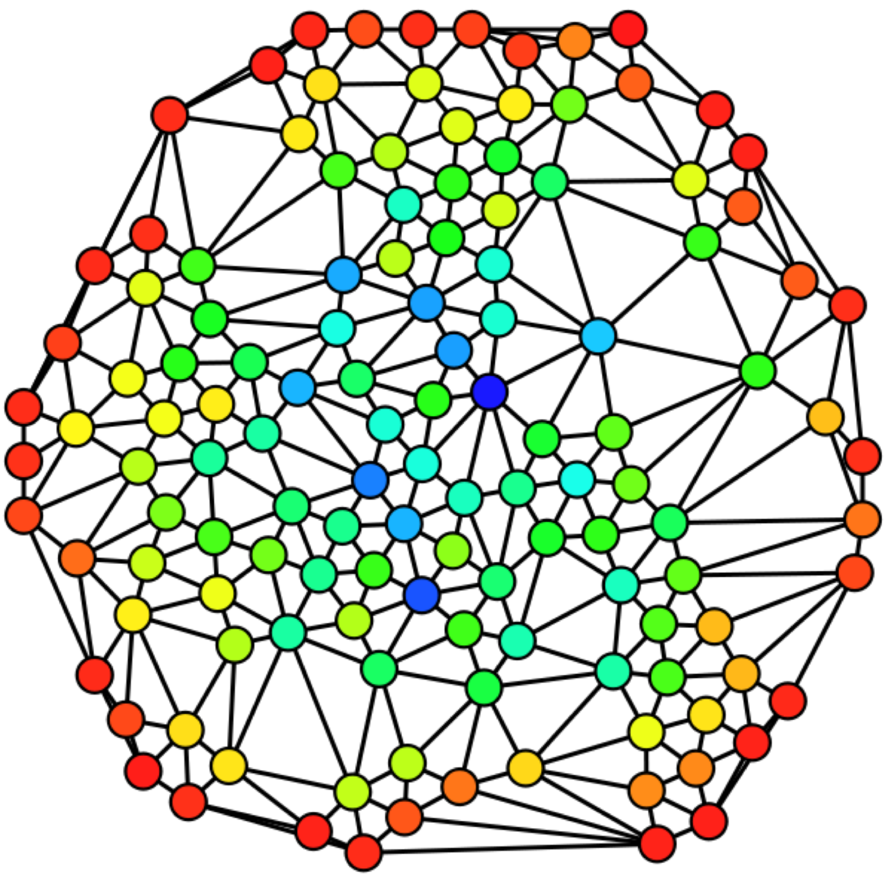


Betweenness on toy networks

- non-normalized version:



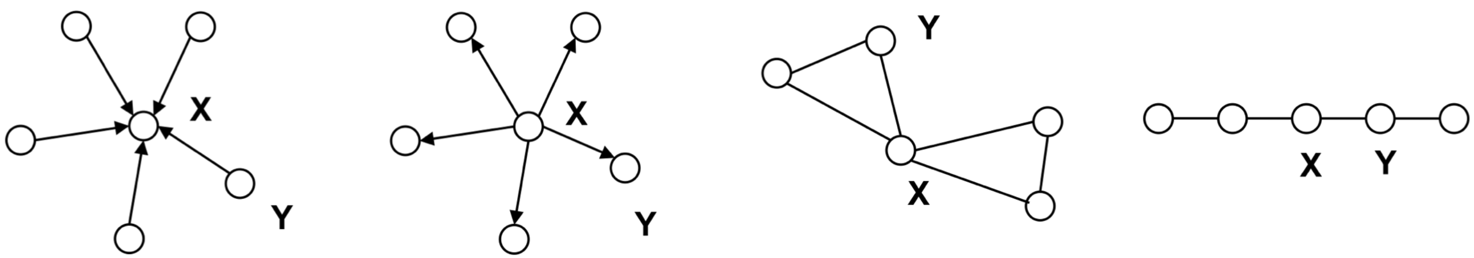
Example of betweenness centrality



Betweenness centrality coded by color

Red: 0 betweenness
Blue: maximum betweenness

Centrality measures - visual comparison



indegree outdegree betweenness closeness

In each of the following networks, X has higher centrality than Y according to a particular measure