Image Restoration and Reconstruction

Preview

- Goal of image restoration
  - Improve an image in some predefined sense
  - Difference with image enhancement ?
- Features
  - Image restoration v.s image enhancement
  - Objective process v.s. subjective process
  - A prior knowledge v.s heuristic process
  - A prior knowledge of the degradation phenomenon is considered
  - Modeling the degradation and apply the inverse process to recover the original image

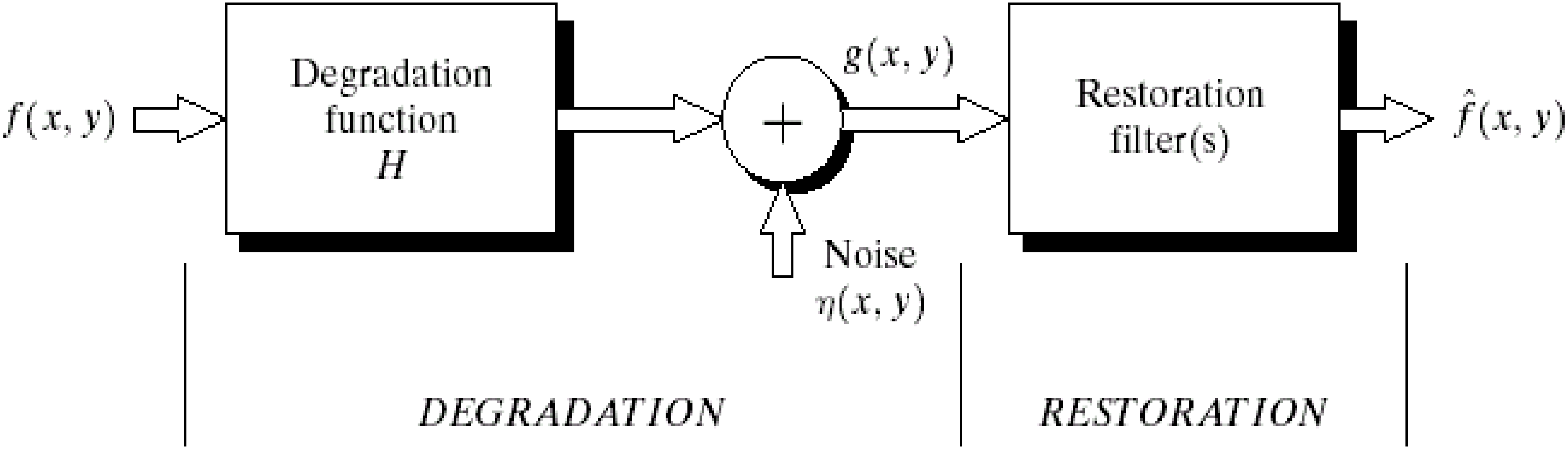
Preview (cont.)

- Target
  - Degraded digital image
  - Sensor, digitizer, display degradations are less considered
- Spatial domain approach
- Frequency domain approach

Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of noise only – spatial filtering
- Periodic noise reduction by frequency domain filtering
- Linear, position-invariant degradations
- Estimating the degradation function
- Inverse filtering

degradation/restoration process



$$\begin{cases} g(x,y)=f(x,y)*h(x,y)+\eta(x,y) \\ G(u,v)=F(u,v)H(u,v)+N(u,v) \end{cases}$$

Noise models

- Source of noise
  - Image acquisition (digitization)
  - Image transmission
- Spatial properties of noise
  - Statistical behavior of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum
  - Ex. white noise (a constant Fourier spectrum)

Noise probability density functions

- Noises are taken as random variables
- Random variables
  - Probability density function (PDF)

Gaussian noise

- Math. tractability in spatial and frequency domain
- Electronic circuit noise and sensor noise

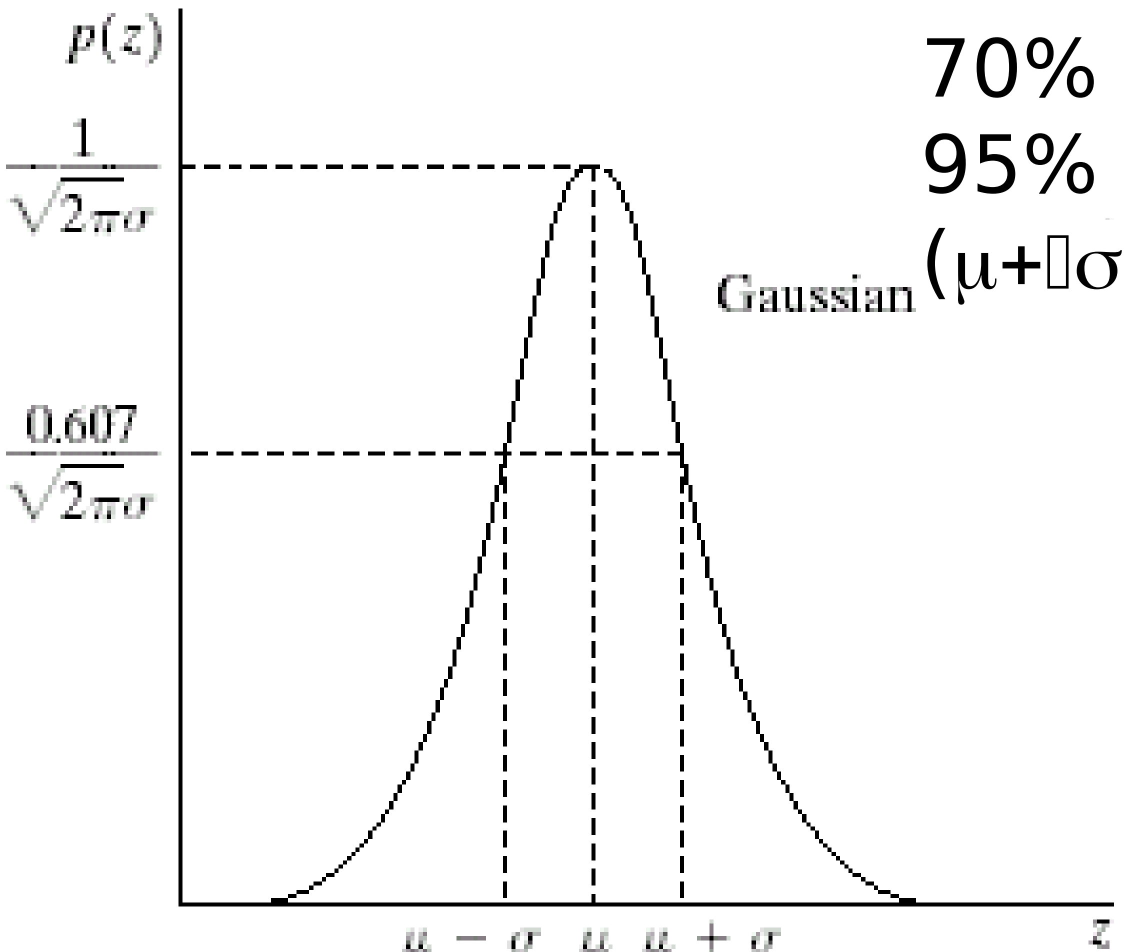
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

mean

variance

Note: $\int_{-\infty}^{\infty} p(z)\,dz = 1$

Gaussian noise (PDF)



70% in [$(\mu-\sigma)$, $(\mu+\sigma)$]
95% in [$(\mu-2\sigma)$, $(\mu+$□$\sigma)$]

Uniform noise

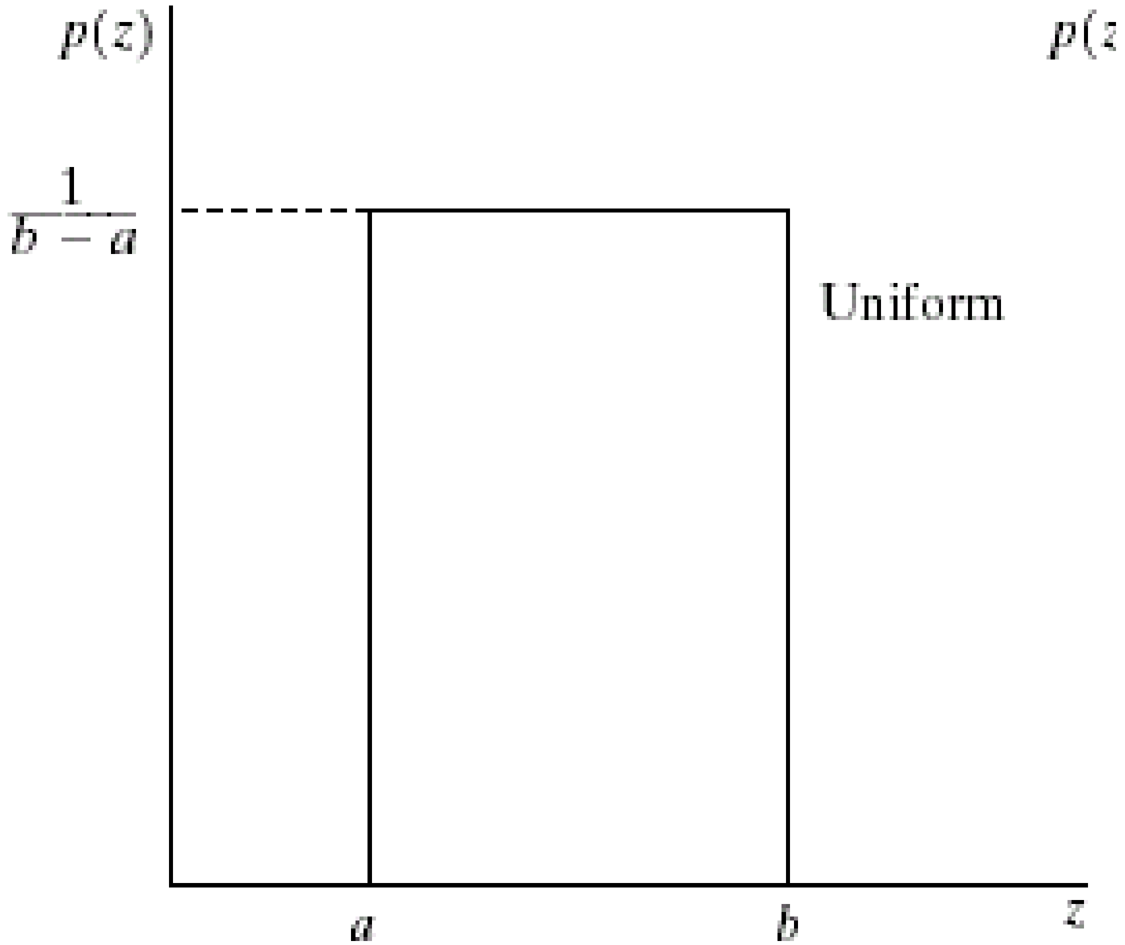
- Less practical, used for random number generator

$$p(z)=\begin{cases}\frac{1}{b-a} & \text{if } a\le z\le b\\ 0 & \text{otherwise}\end{cases}$$

Mean: $\mu=\frac{a+b}{2}$

Variance: $\sigma^2=\frac{(b-a)^2}{12}$

Uniform PDF



Impulse (salt-and-pepper) nosie

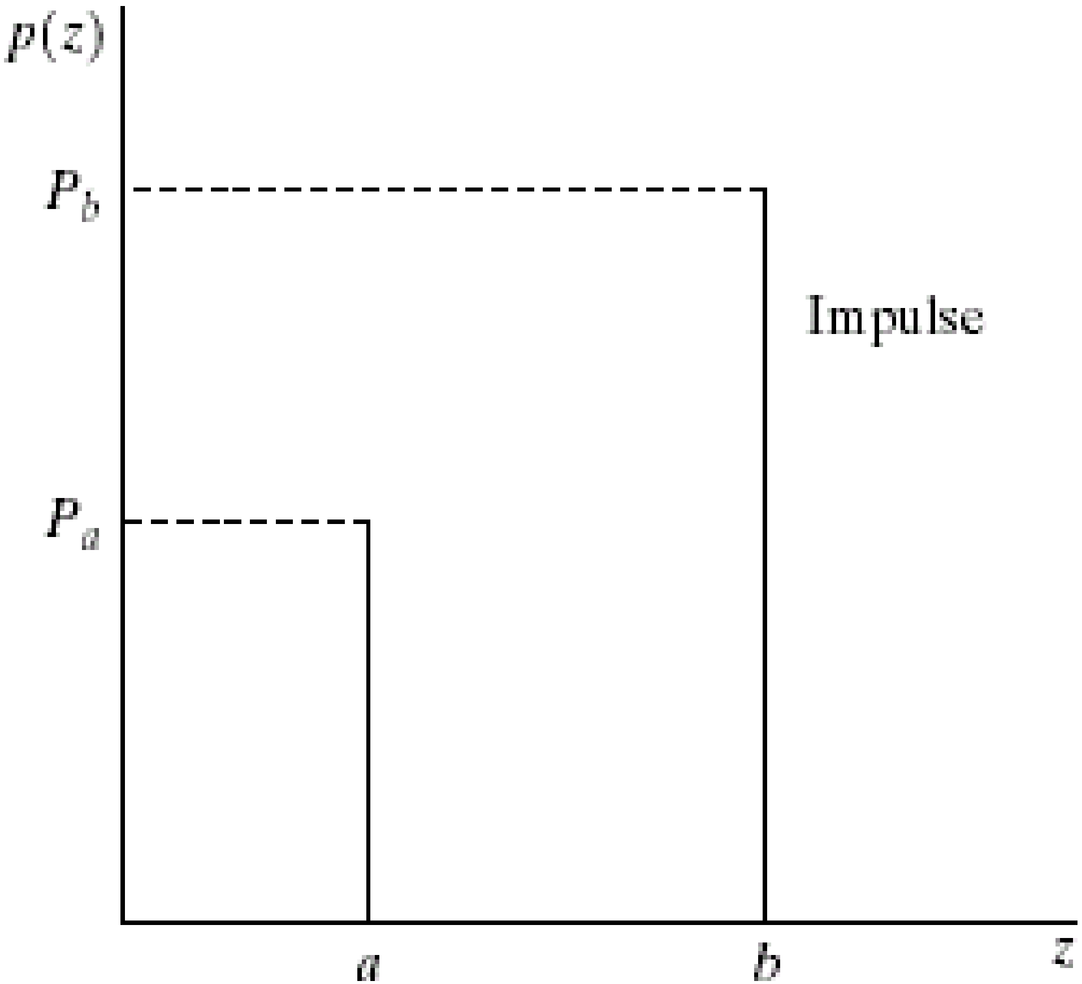
- Quick transients, such as faulty switching during imaging

$$p(z)=\left\{\begin{matrix} P_a & \text{for } z=a \\ P_b & \text{for } z=b \\ 0 & \text{otherwise} \end{matrix}\right\}$$

If either P_a or P_b is zero, it is called *unipolar.*
Otherwise, it is called bipoloar.

- In practical, impulses are usually stronger than image signals. Ex., a=0(black) and b=255(white) in 8-bit imag

Impulse (salt-and-pepper) noise PDF



PDFs of some important noise models

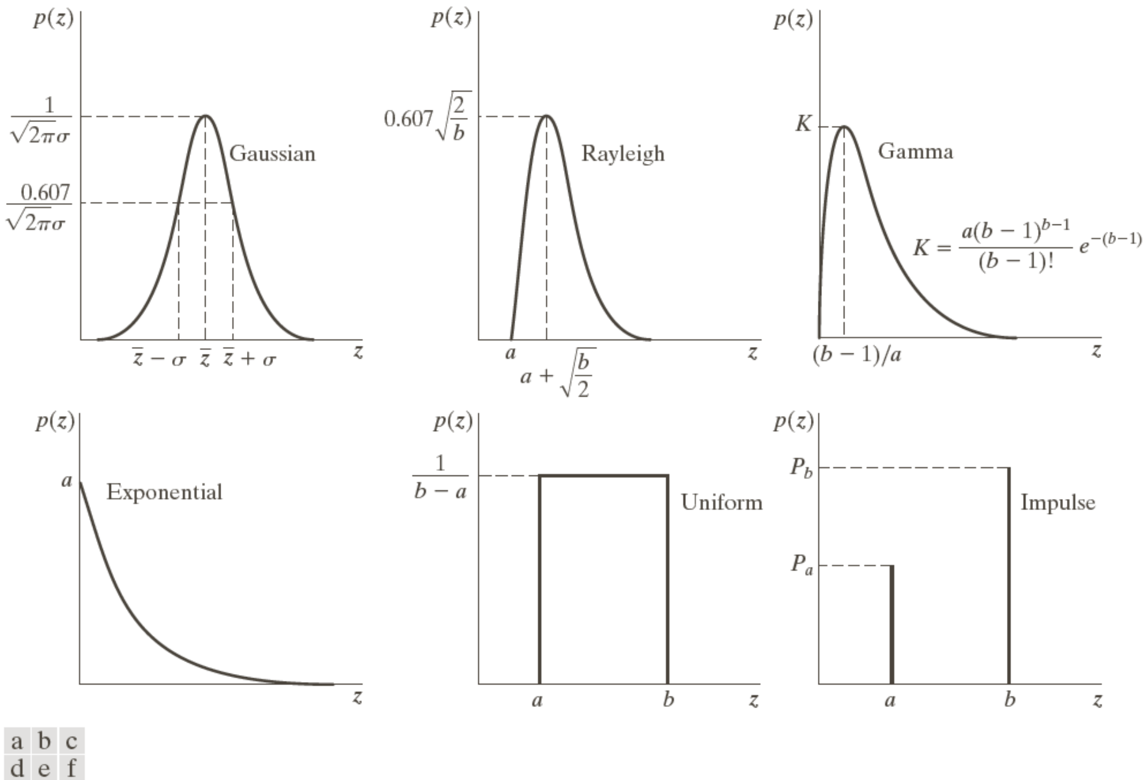
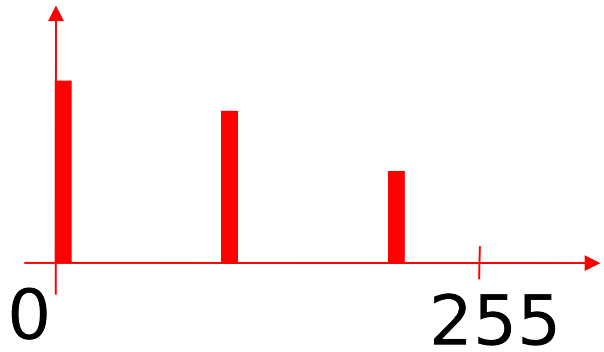


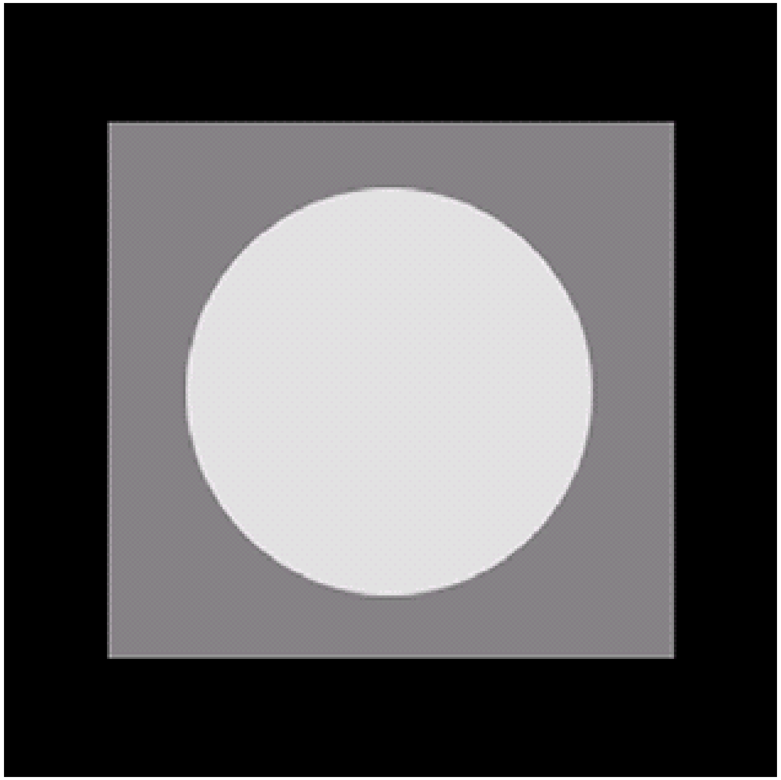
FIGURE 5.2 Some important probability density functions.

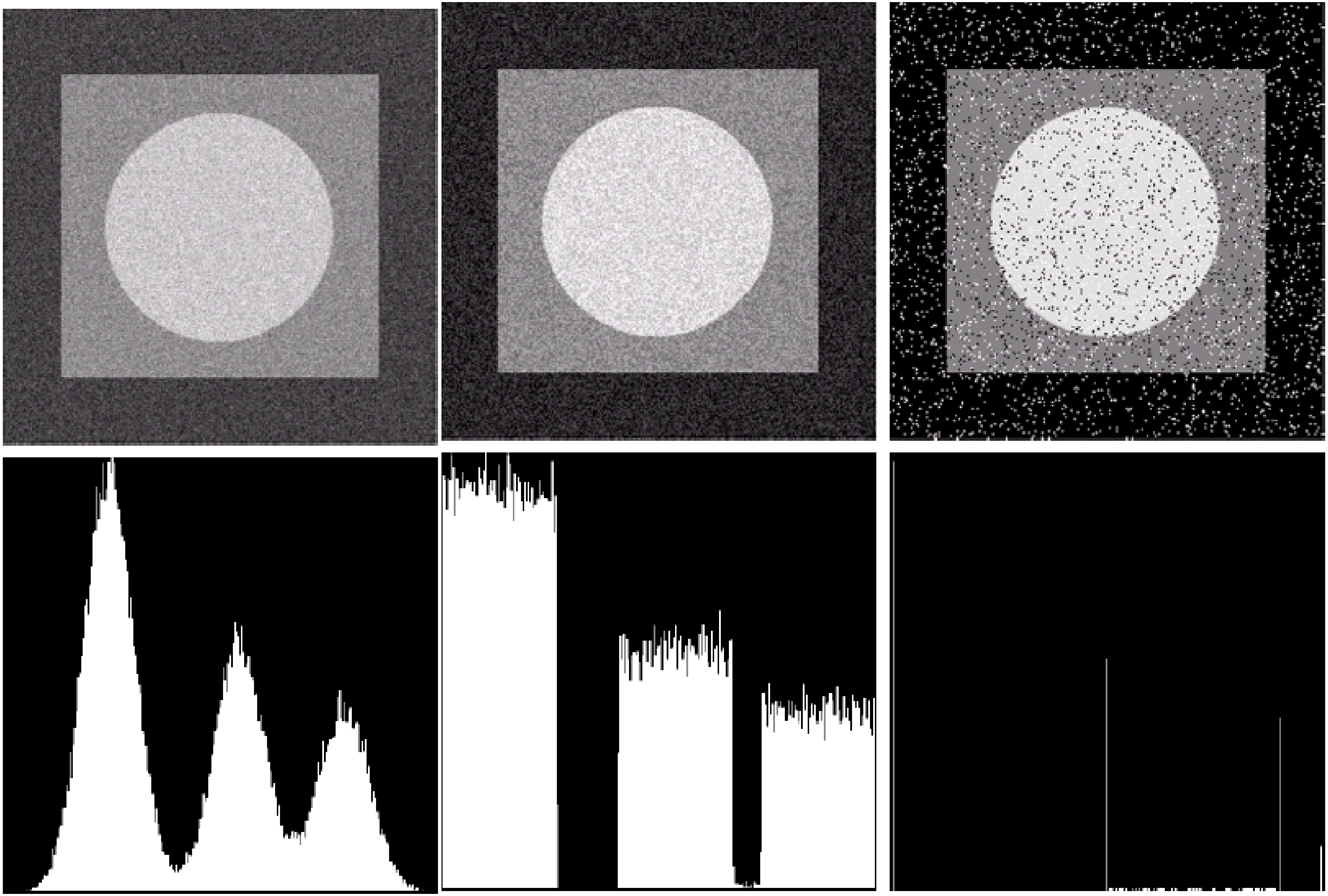
Test for noise behavior

- Test pattern

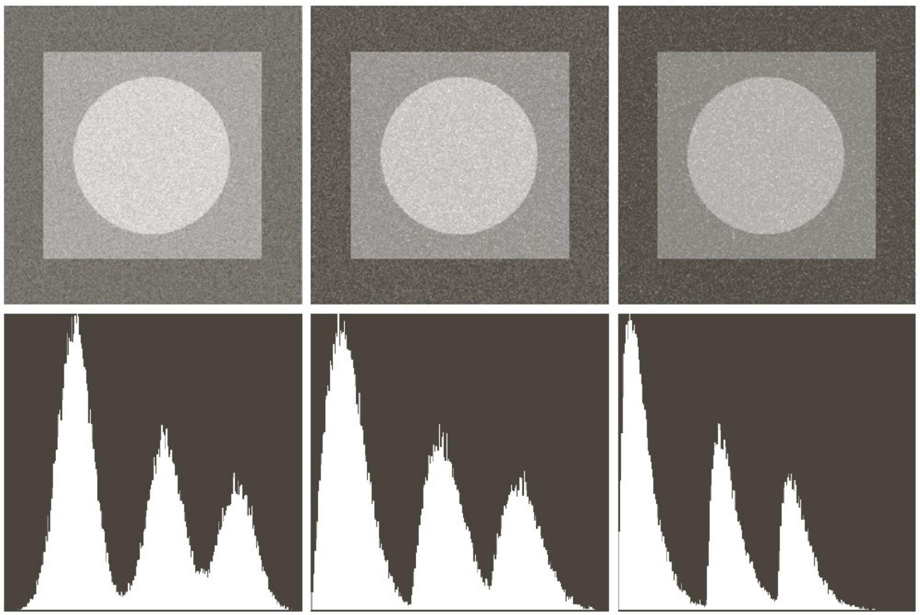
Its histogram:







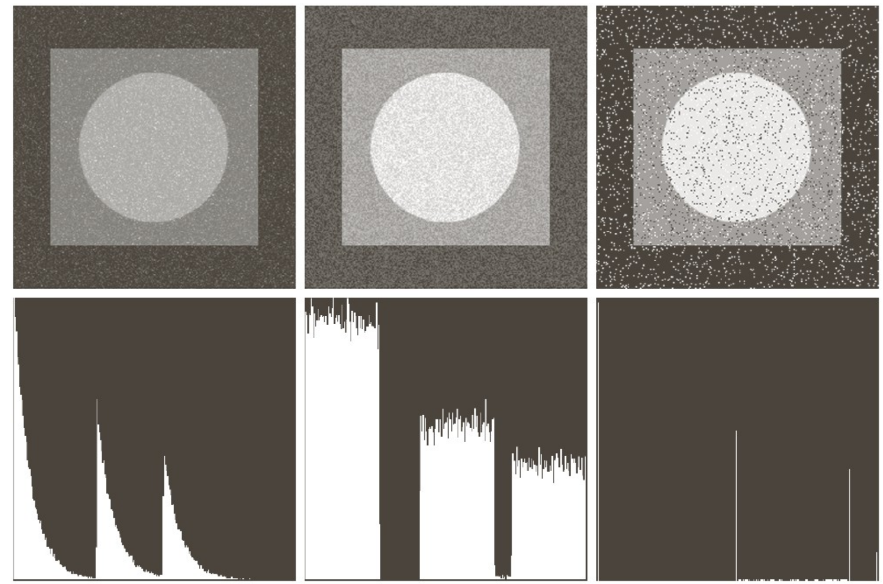
Gaussian Uniform Salt & Pepper



Gaussian Rayleigh Gamma

a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.



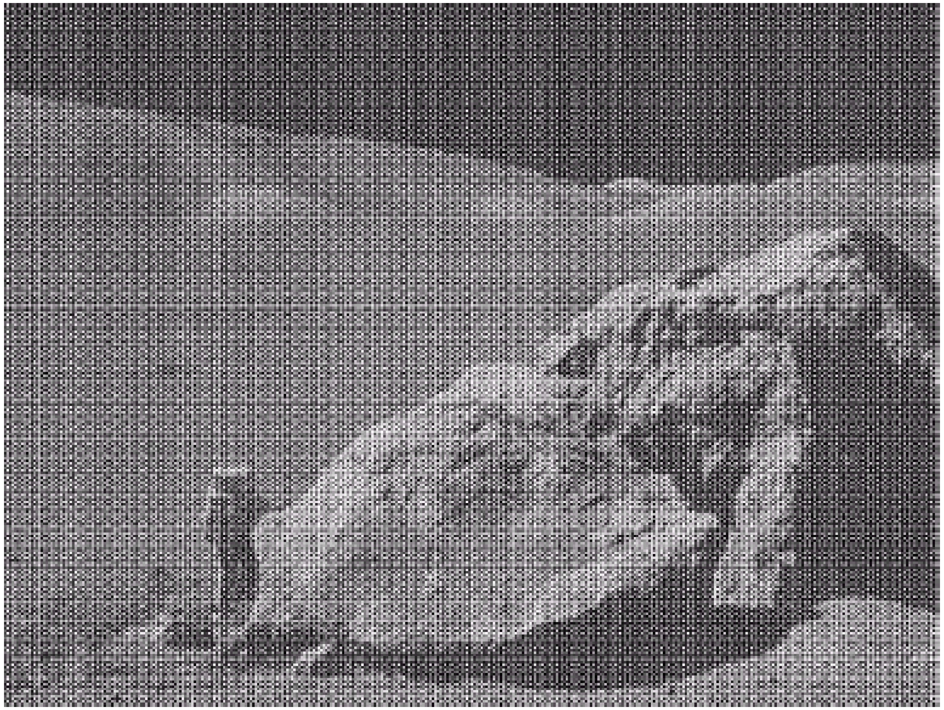
Exponential Uniform Salt & Pepper

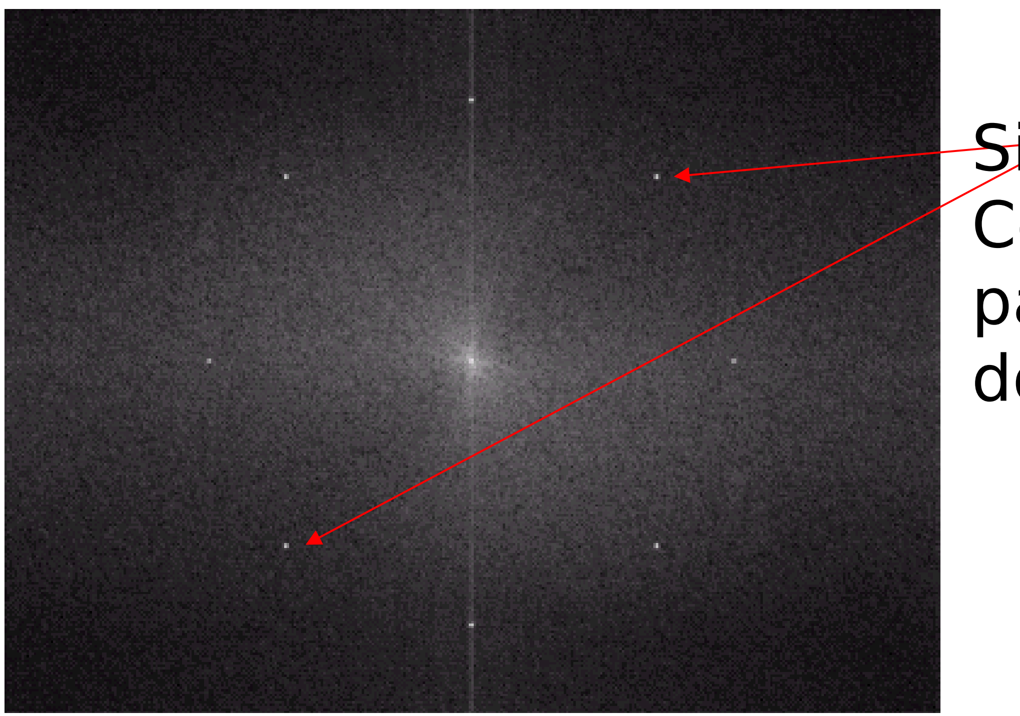
g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Periodic noise

- Arise from electrical or electromechanical interference during image acquisition
- Spatial dependence
- Observed in the frequency domain



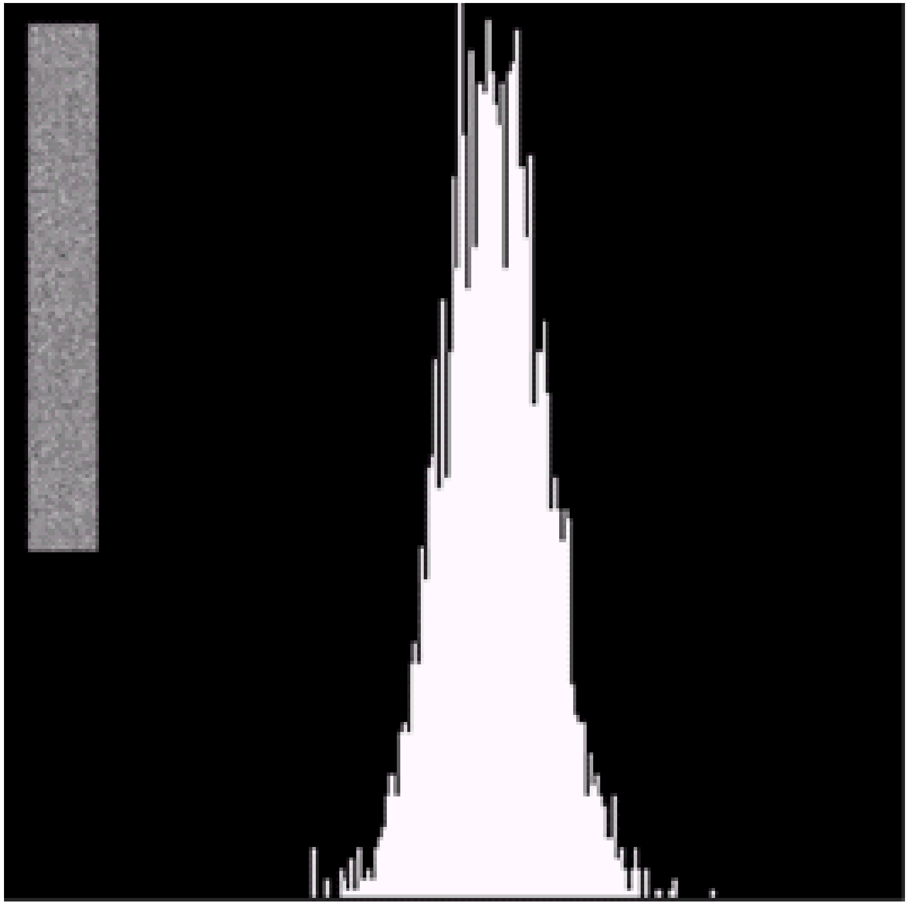


Sinusoidal noise:
Complex conjugate
pair in frequency
domain

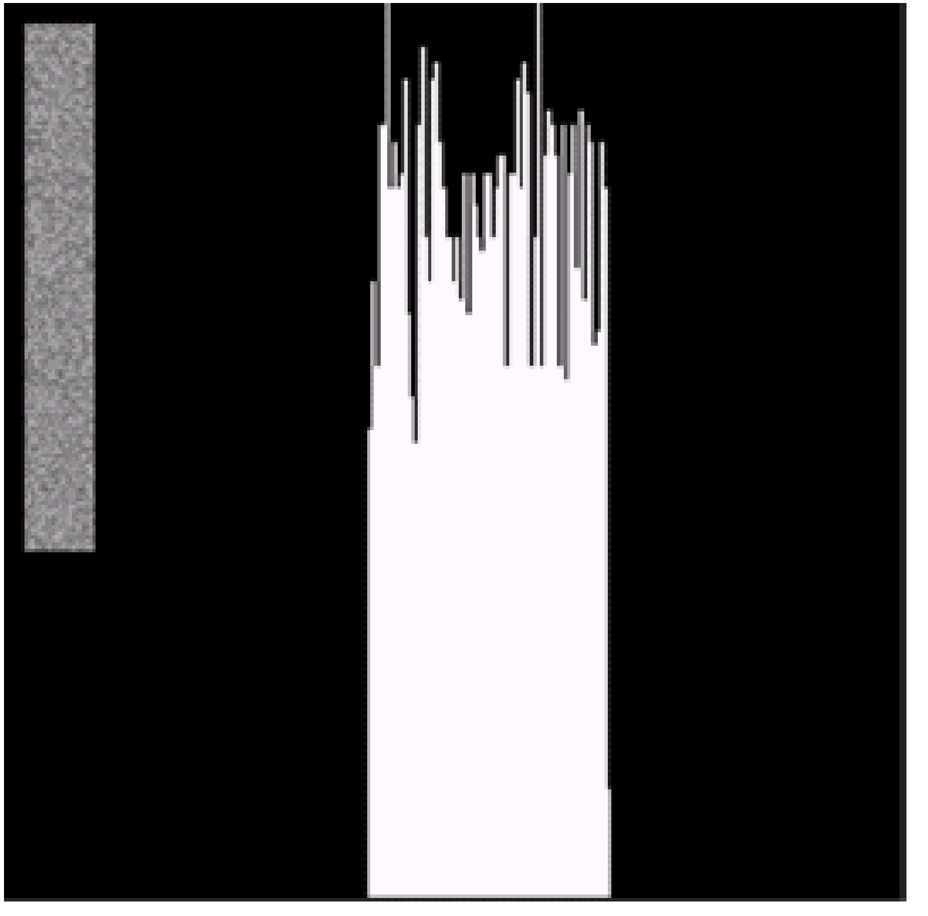
Estimation of noise parameters

- Periodic noise
  - Observe the frequency spectrum
- Random noise with unknown PDFs
  - Case 1: imaging system is available
    - Capture images of "flat" environment
  - Case 2: noisy images available
    - Take a strip from constant area
    - Draw the histogram and observe it
    - Measure the mean and variance

Observe the histogram



Gaussian



uniform

Measure the mean and variance

- Histogram is an estimate of PDF

$$\begin{cases} \mu = \sum_{z_i \in S} z_i \, p(z_i) \\ \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 \, p(z_i) \end{cases}$$

$\Leftrightarrow$ $\begin{cases} \text{Gaussian: } \mu, \sigma \\ \text{Uniform: } a, b \end{cases}$

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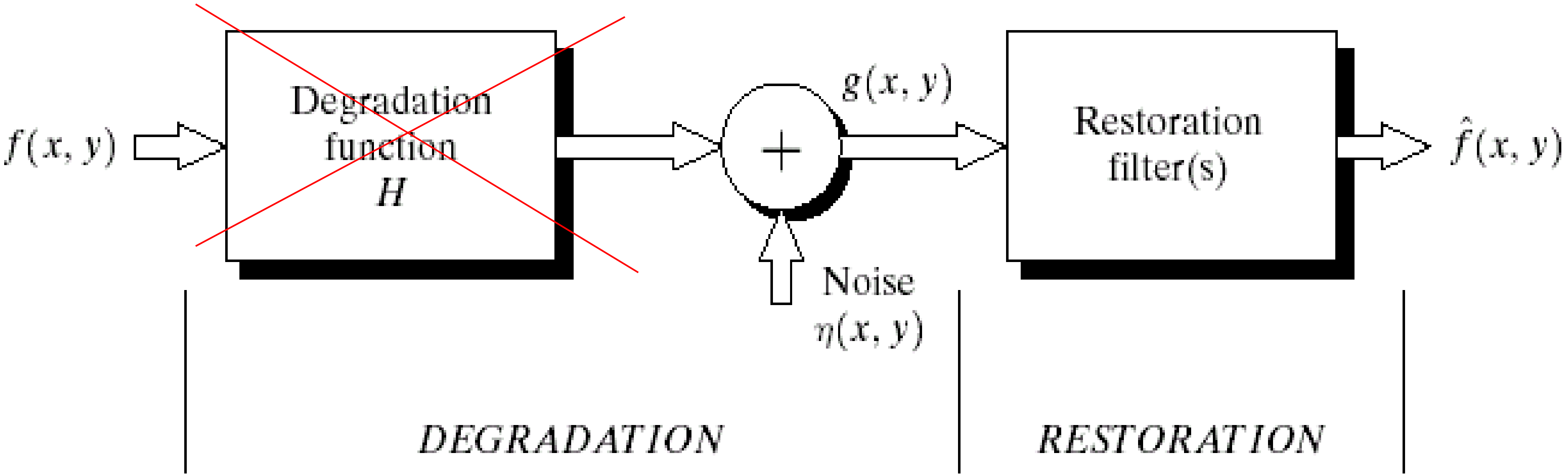
Additive noise only



$$\begin{cases} g(x,y)=f(x,y)+\eta(x,y) \\ G(u,v)=F(u,v)+N(u,v) \end{cases}$$

Spatial filters for de-noising additive noise

- Skills similar to image enhancement
- Mean filters
- Order-statistics filters
- Adaptive filters

Mean filters

- Arithmetic mean

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$$

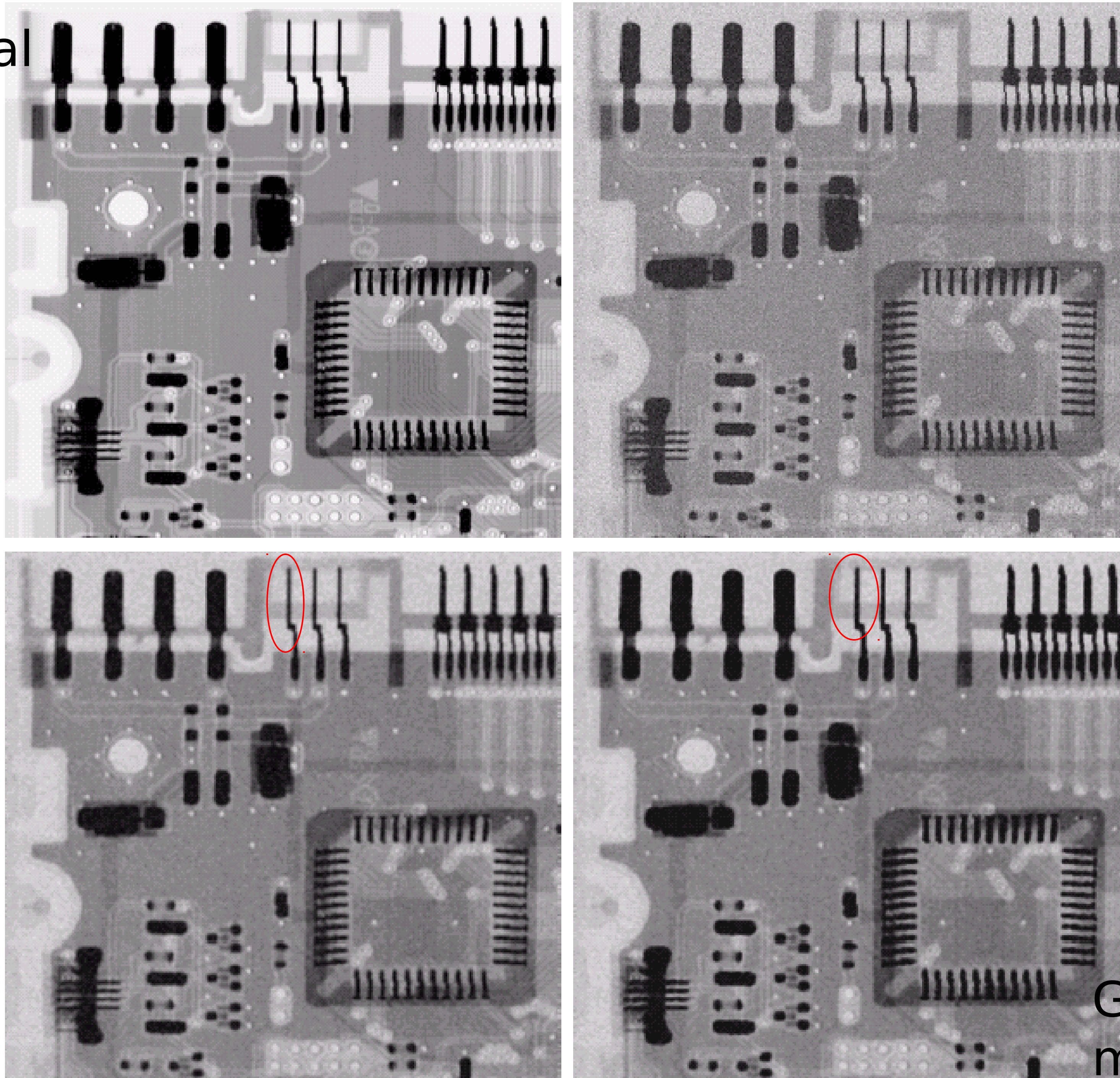
Window centered at (x,y)

- Geometric mean

$$g(s,t)1/mn\{\hat{f}(x,y)=¿$$

original

Noisy
Gaussian
$\mu=0$
$\sigma=2$



Arith. mean

Geometric mean

Mean filters (cont.)

- Harmonic mean filter

$$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$$

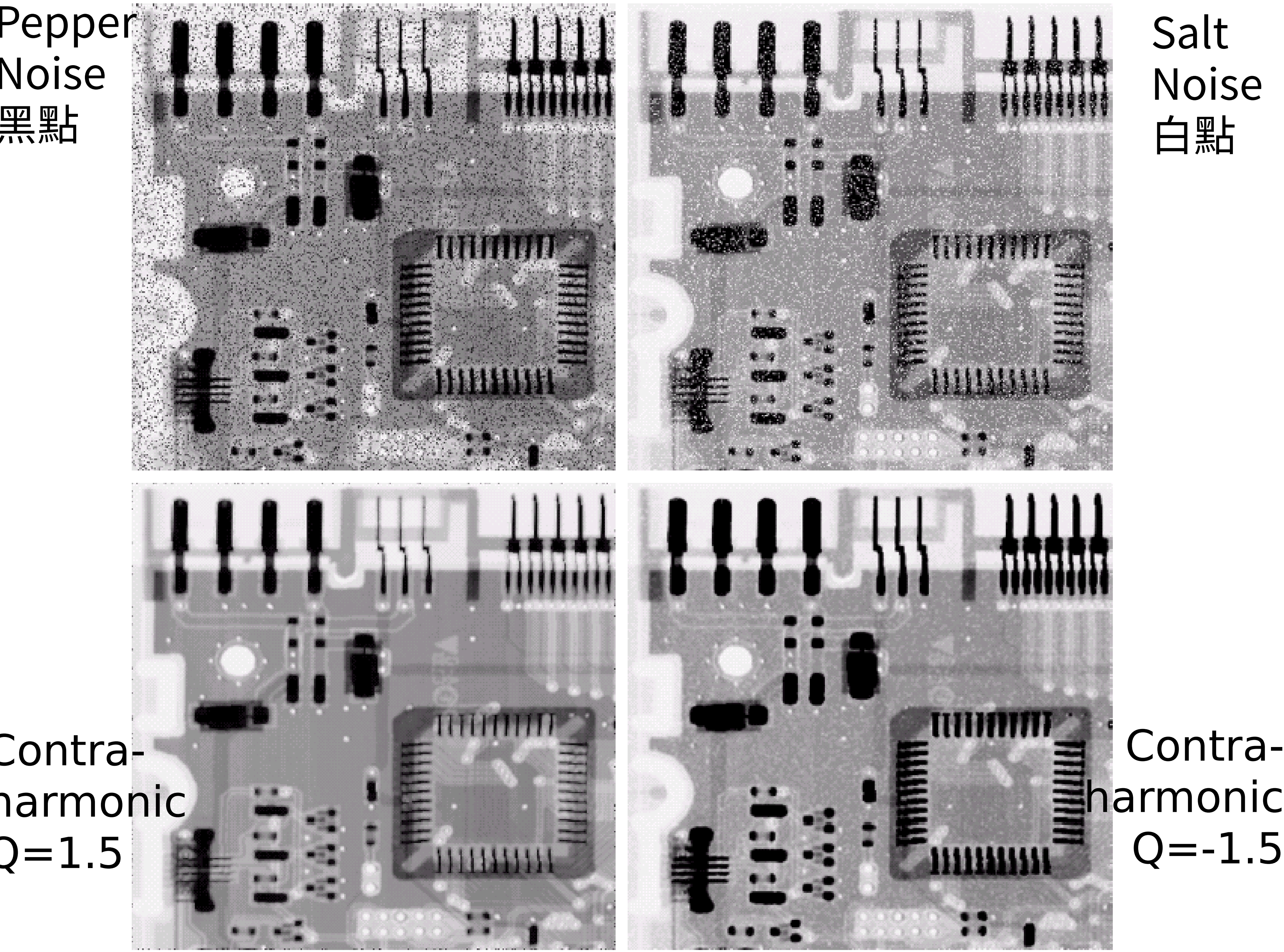
- Contra-harmonic mean filter

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

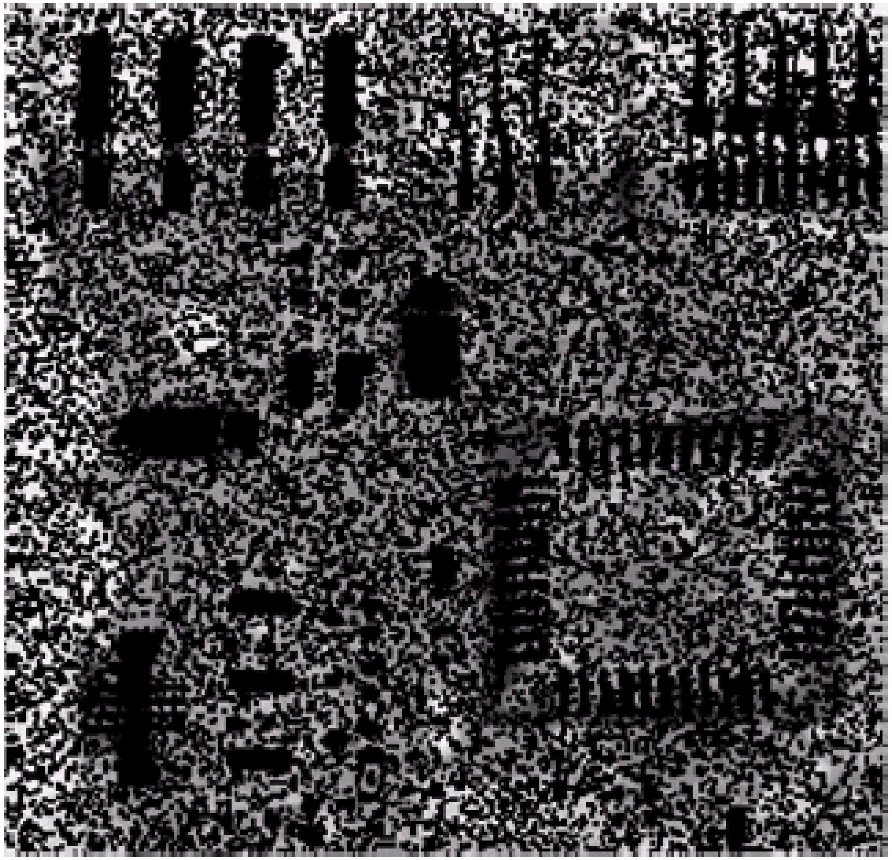
Q=-1, harmonic

Q=0, airth. mean

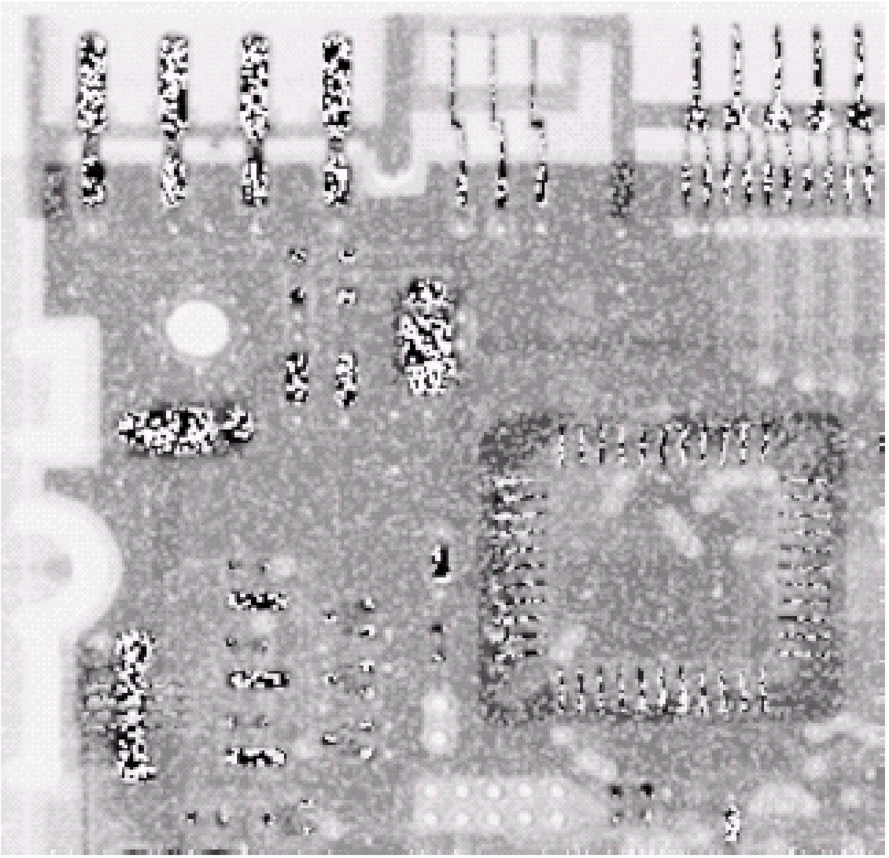
Q=+, ?


Pepper Noise 黑點
Salt Noise 白點
Contra-harmonic Q=1.5
Contra-harmonic Q=-1.5

Wrong sign in contra-harmonic filtering



Q=-1.5



Q=1.5

Order-statistics filters

- Based on the ordering(ranking) of pixels
  - Suitable for unipolar or bipolar noise (salt and pepper noise)
- Median filters
- Max/min filters
- Midpoint filters
- Alpha-trimmed mean filters

Order-statistics filters

- Median filter

$$\left\{ g\left(s,t \right) \right\}$$

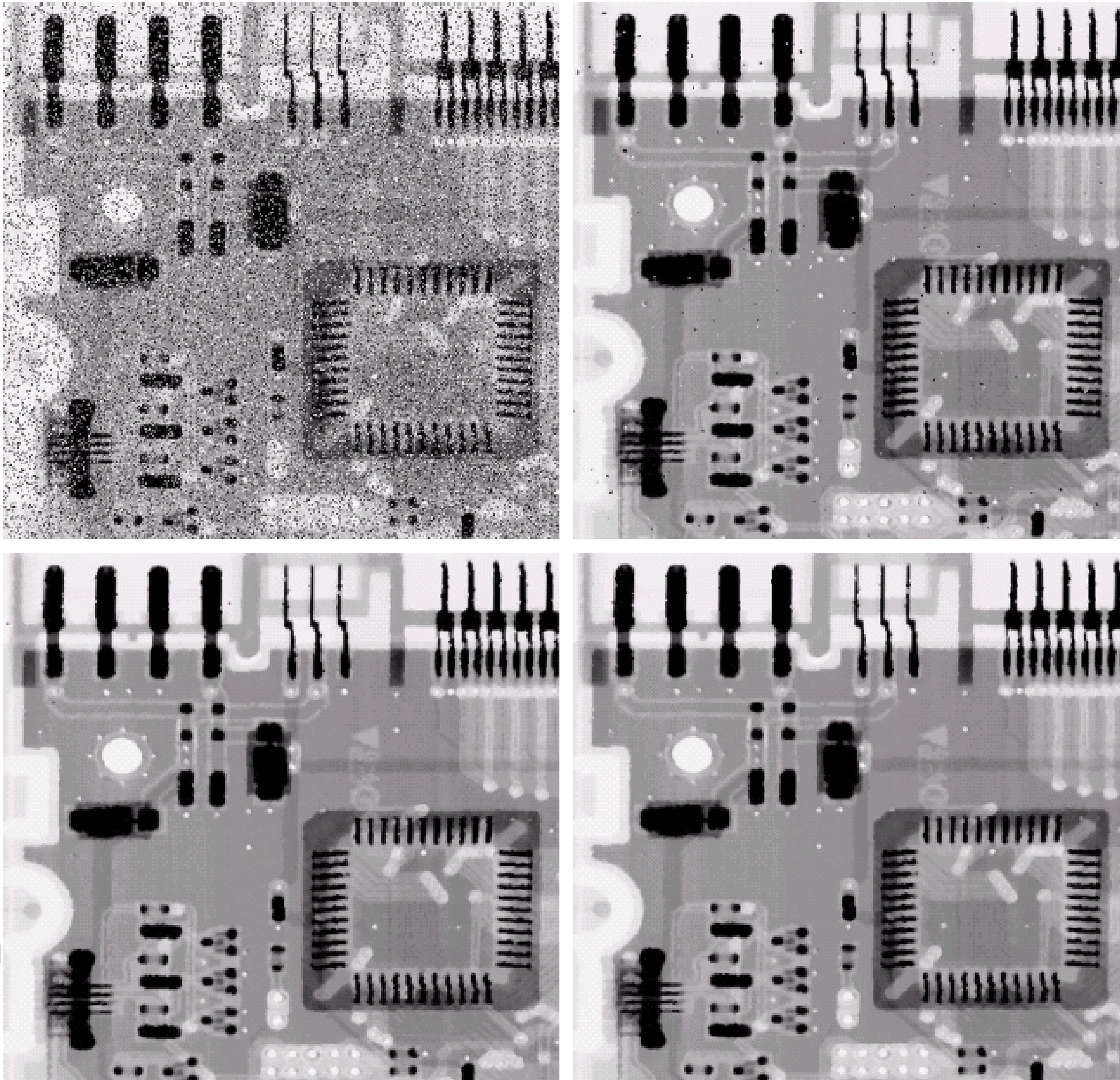
- Max/min filters

$$\left\{ g\left(s,t \right) \right\}$$
$$\left\{ g\left(s,t \right) \right\}$$

bipolar Noise

$P_a = 0.1$

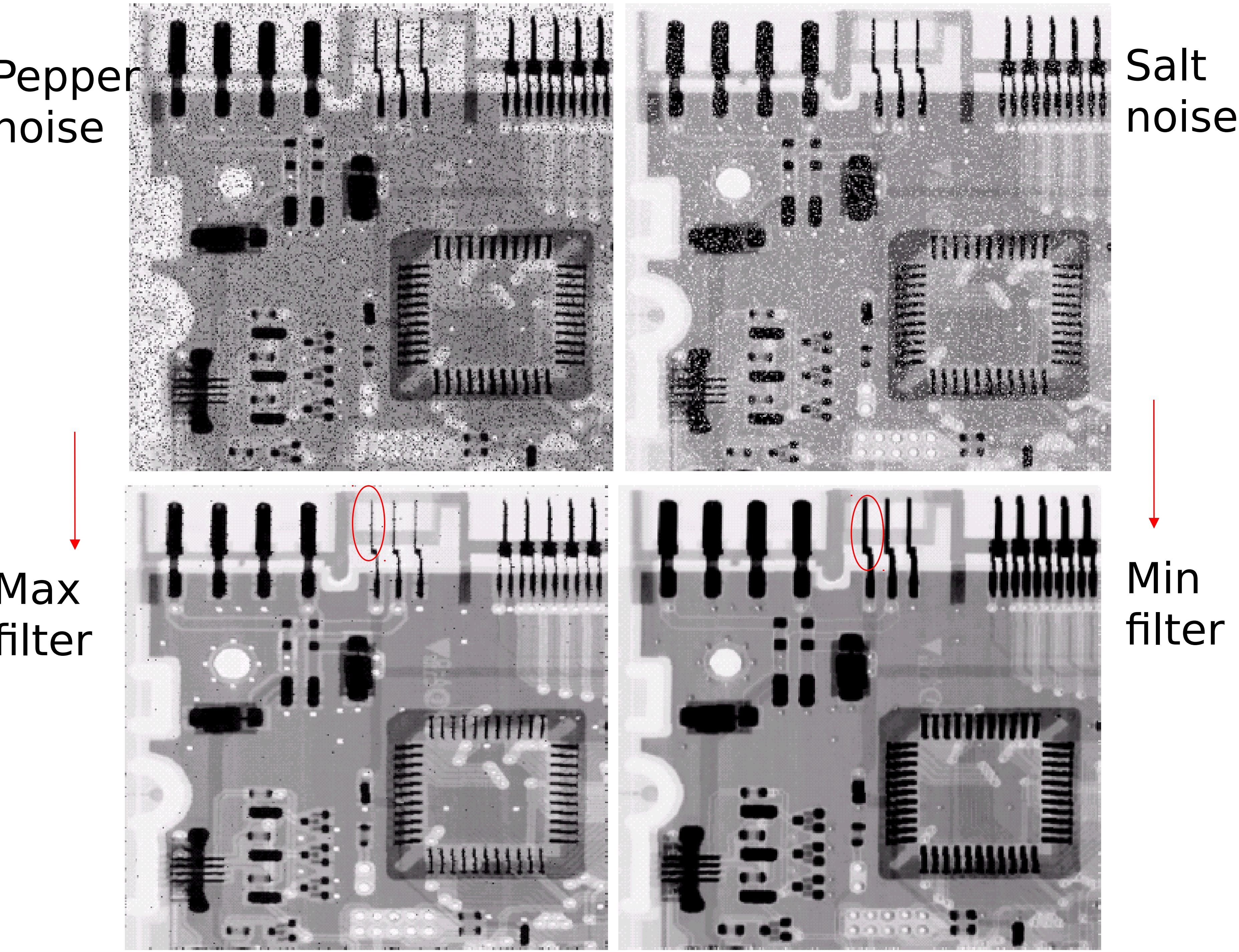
$P_b = 0.1$



3x3 Median Filter Pass 1

3x3 Median Filter Pass 2

3x3 Median Filter Pass 3


Pepper noise
Salt noise
Max filter
Min filter

Order-statistics filters (cont.)

- Midpoint filter

$$\{\, g(s,t) \,\}$$

- Alpha-trimmed mean filter
  - Delete the d/2 lowest and d/2 highest gray-level pixels

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Middle (mn-d) pixels

Uniform noise

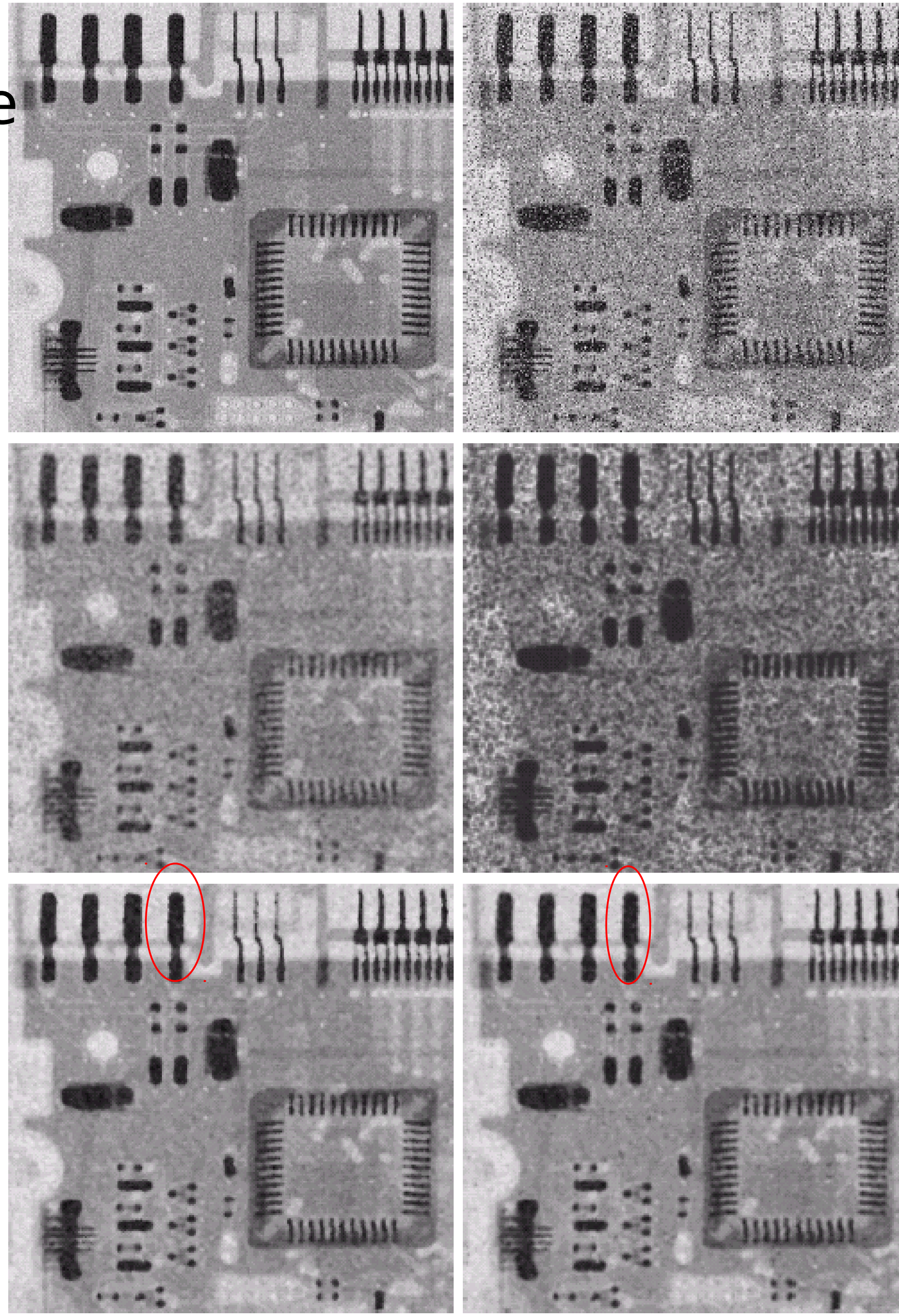
$\mu=0$

$\sigma^2=800$

Left +
Bipolar Noise

$P_a = 0.1$

$P_b = 0.1$



5x5
Arith. Mean
filter

5x5
Geometric
mean

5x5
Median
filter

5x5
Alpha-trim.
Filter
d=5

Adaptive filters

- Adapted to the behavior based on the statistical characteristics of the image inside the filter region S_{xy}
- Improved performance v.s increased complexity
- Example: Adaptive local noise reduction filter

Adaptive local noise reduction filter

- Simplest statistical measurement
  - Mean and variance
- Known parameters on local region S_{xy}
  - g(x,y): noisy image pixel value
  - σ^2_{η}: noise variance (assume known a prior)
  - m_L : local mean
  - σ^2_L : local variance

Adaptive local noise reduction filter (cont.)

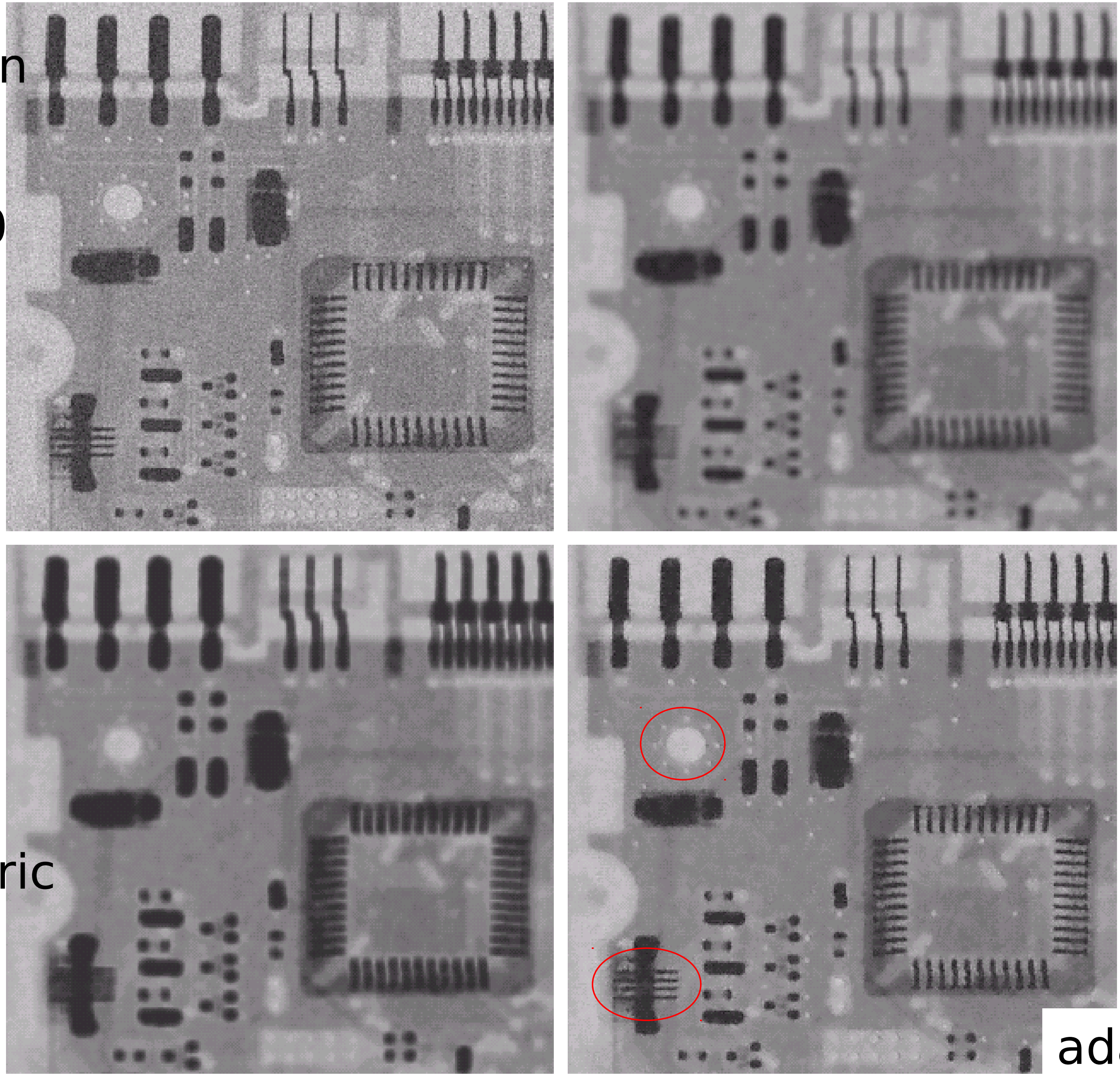
- Analysis: we want to do
  - If σ^2_η is zero, return g(x,y)
  - If $\sigma^2_L > \sigma^2_\eta$, return value close to g(x,y)
  - If $\sigma^2_L = \sigma^2_\eta$, return the arithmetic mean m_L
- Formula

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

Gaussian noise

$\mu=0$

$\sigma^2=1000$

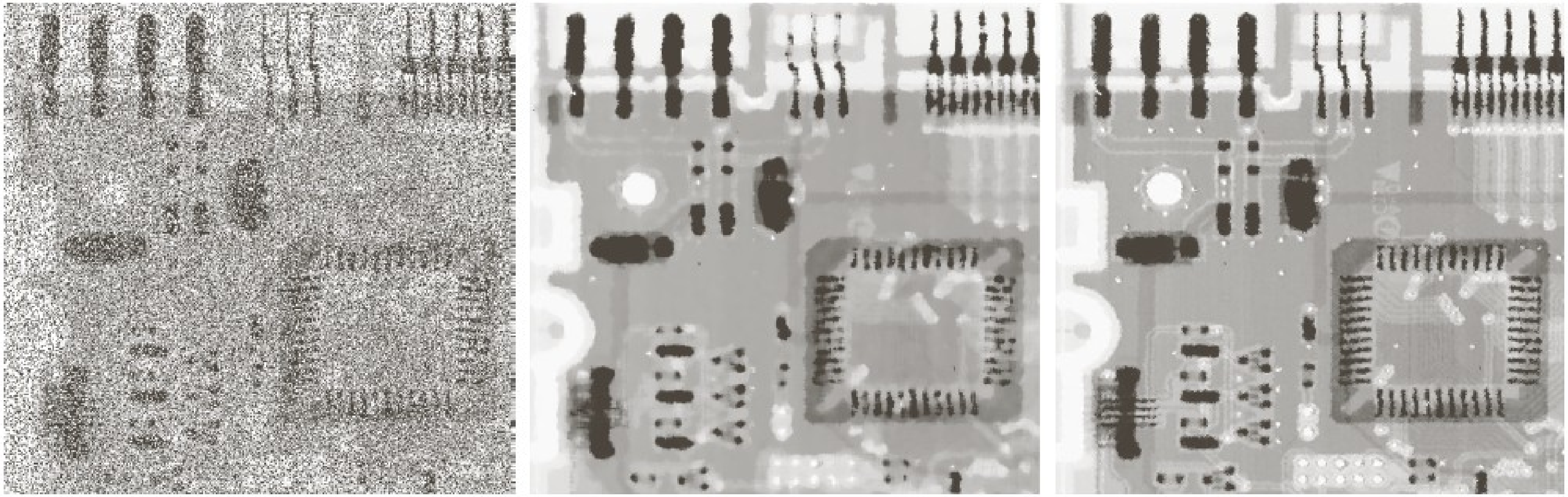


Arith. mean 7x7

Geometric mean 7x7

adaptive

Adaptive Median Filter



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

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Periodic noise reduction

- Pure sine wave
  - Appear as a pair of impulse (conjugate) in the frequency domain

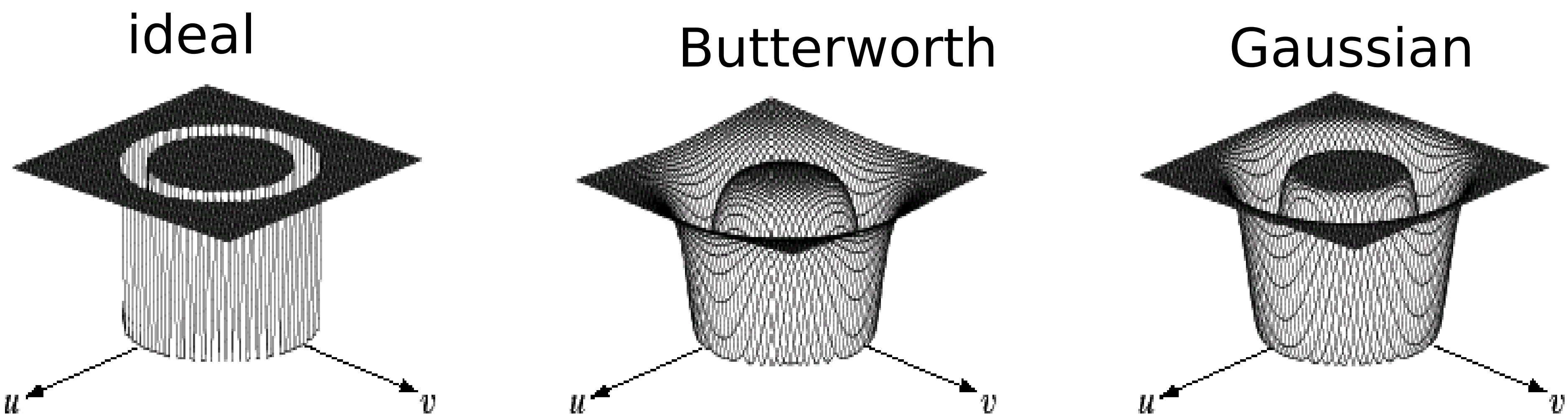
$$\begin{cases} f(x,y) = A\sin(u_0 x + v_0 y) \\ F(u,v) = -j\frac{A}{2}\left[\delta\left(u - \frac{u_0}{2\pi}, v - \frac{v_0}{2\pi}\right) - \delta\left(u + \frac{u_0}{2\pi}, v + \frac{v_0}{2\pi}\right)\right] \end{cases}$$

Periodic noise reduction (cont.)

- Bandreject filters
- Bandpass filters
- Notch filters
- Optimum notch filtering

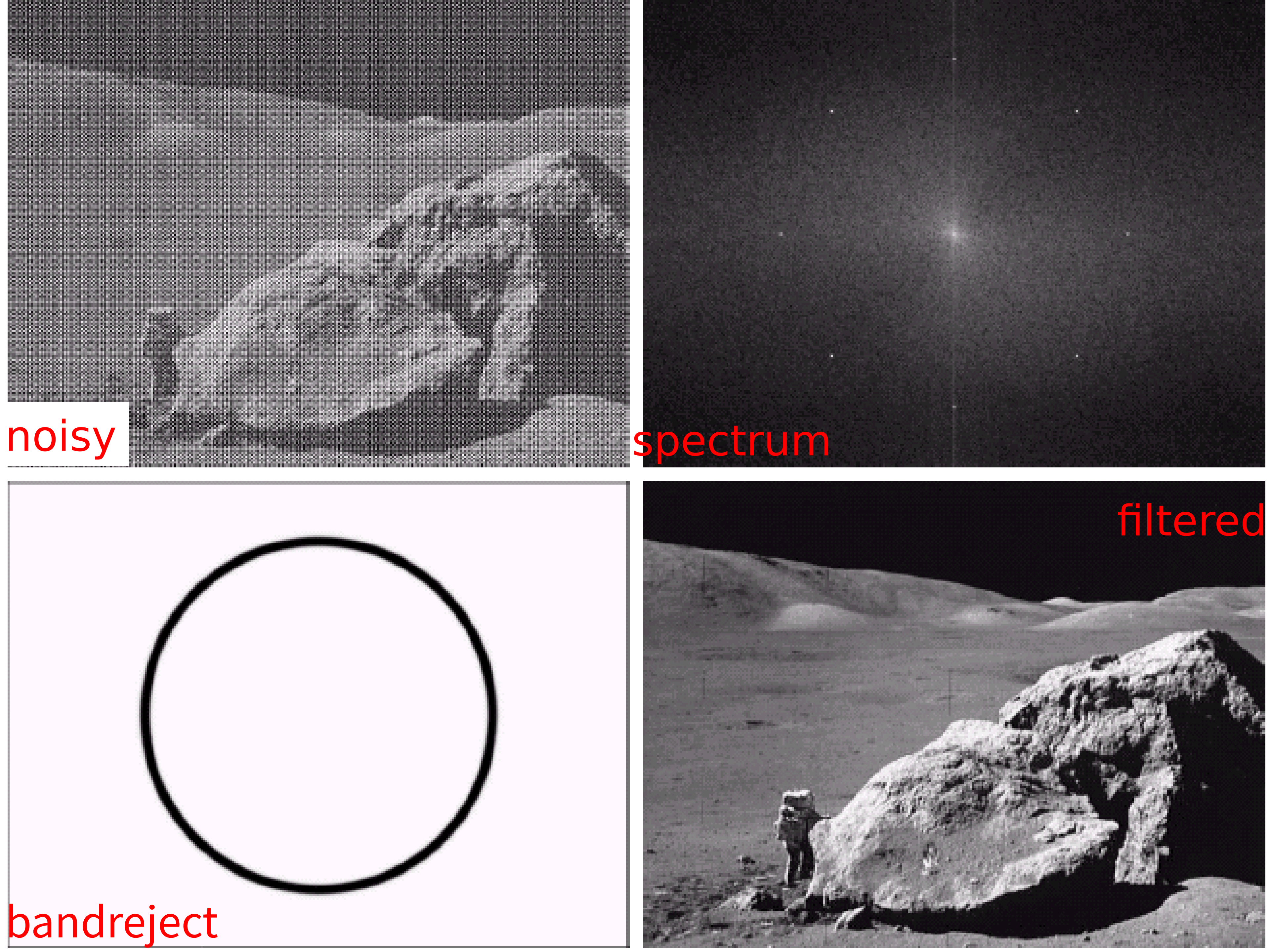
Bandreject filters

* Reject an isotropic frequency



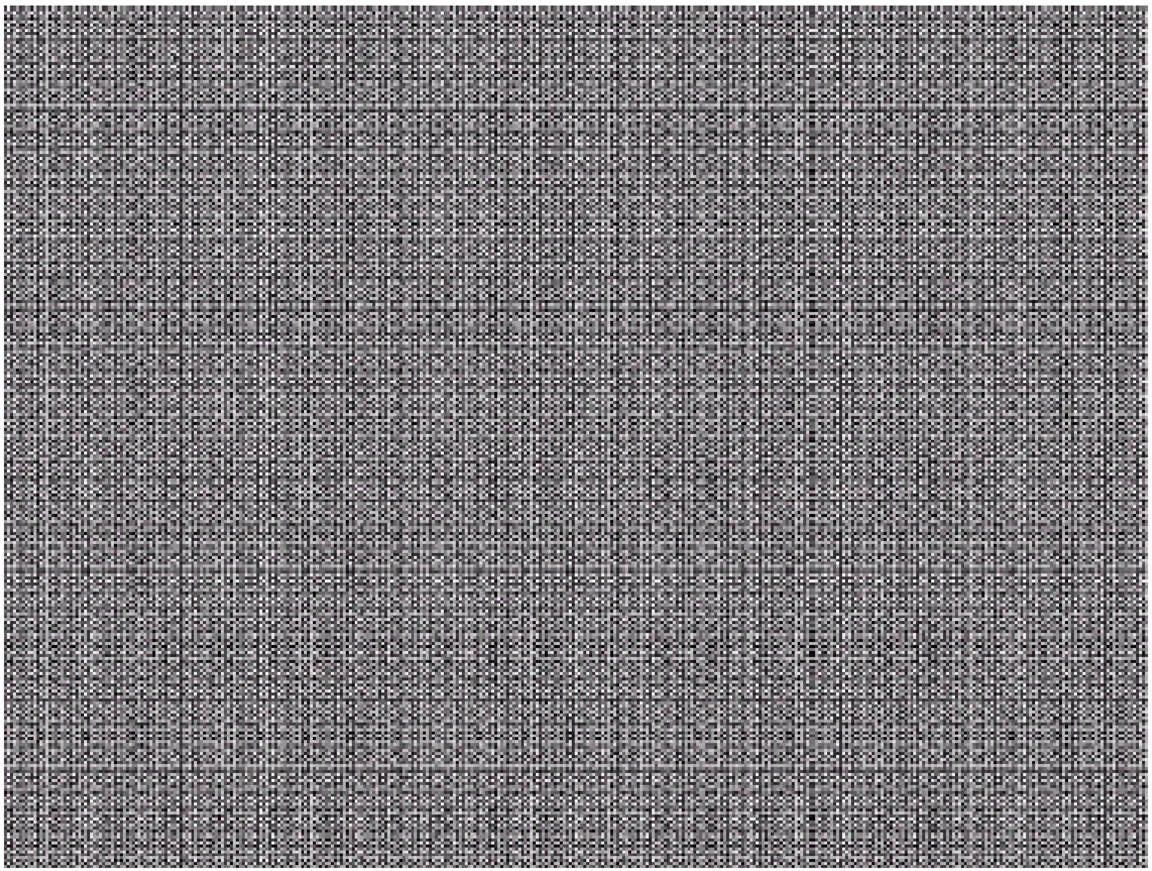
a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.


noisy
spectrum
bandreject
filtered

Bandpass filters

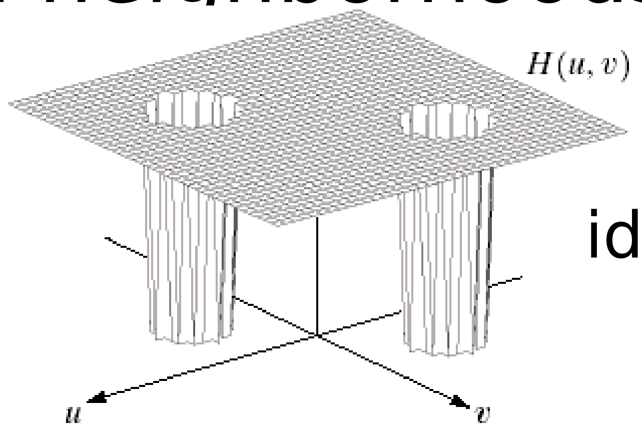
- $H_{bp}(u,v) = 1 - H_{br}(u,v)$



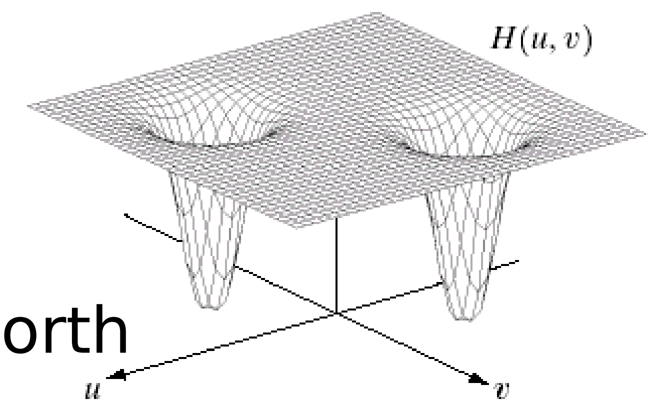
$$\Im^{-1}\left\{G(u,v)H_{bp}(u,v)\right\}$$

Notch filters

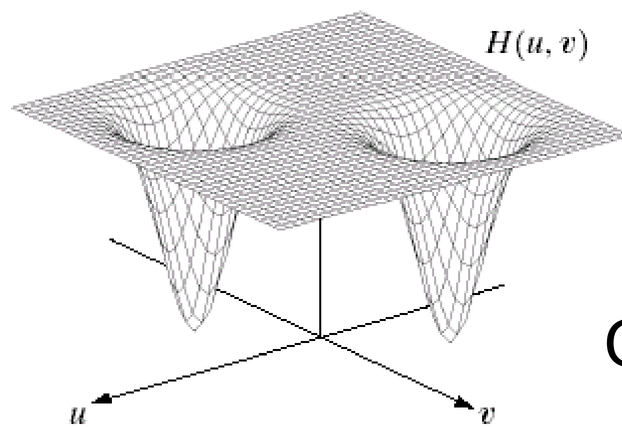
- Reject(or pass) frequencies in predefined neighborhoods about a cer



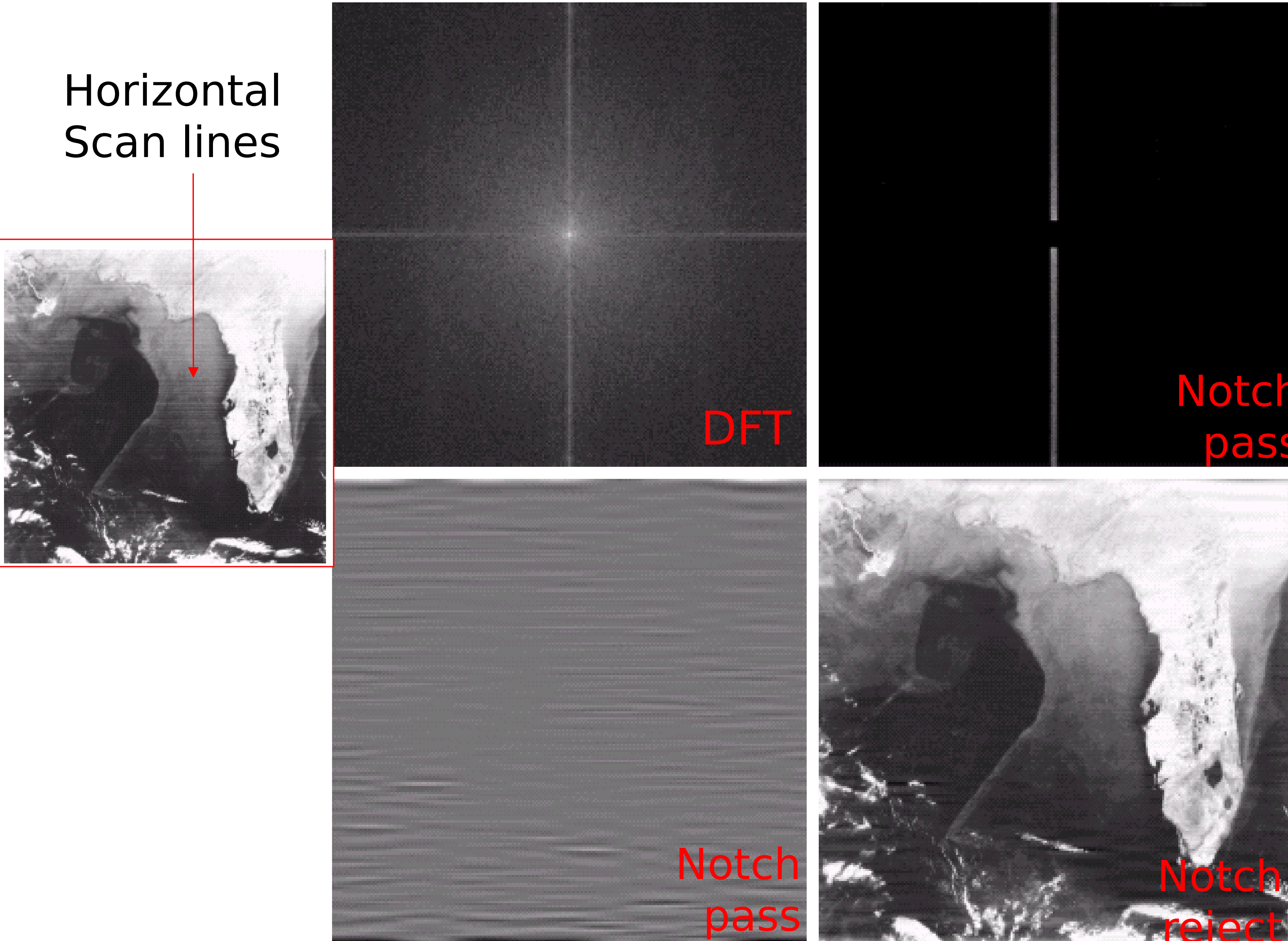
ideal



Butterworth



Gaussian


Horizontal
Scan lines
DFT
Notch
pass

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