Filter Design

Lecture 20: 10-Sep-12

Dr. P P Das

Filtering in Frequency Domain

$$g(x,y) = \Im^{-1}\left[H(u,v)F(u,v)\right]$$

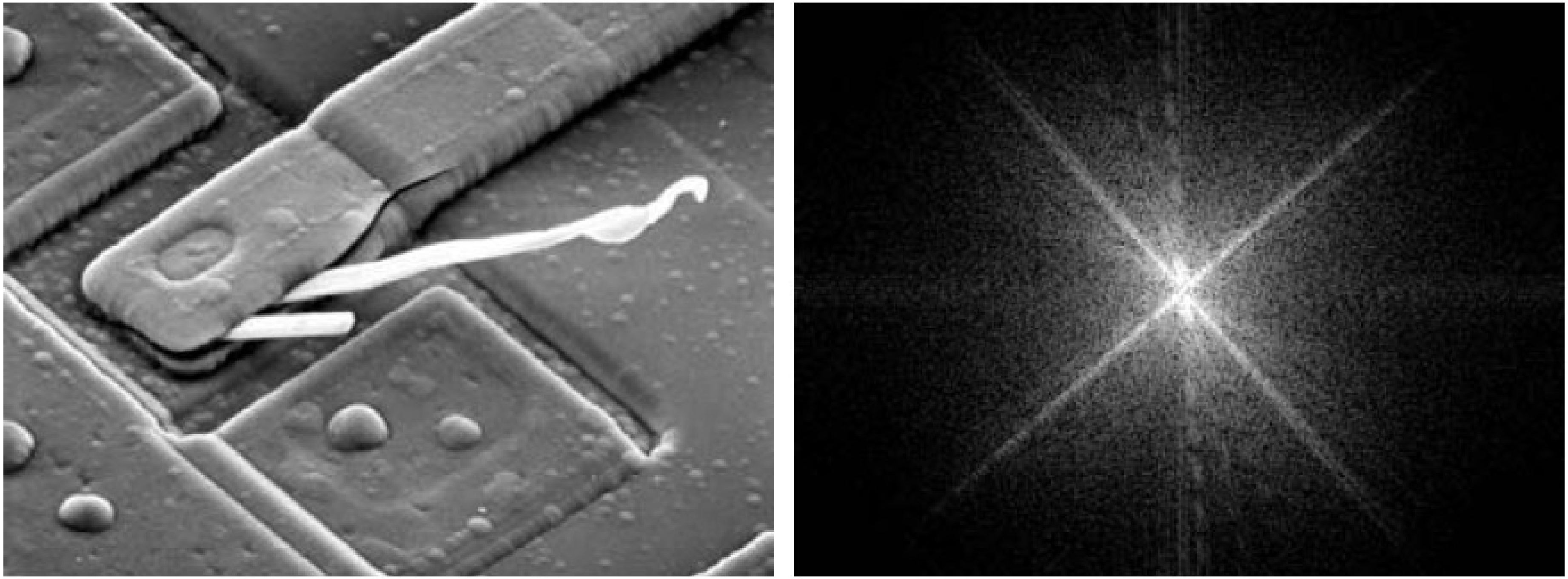
- $H(u,v)$: Filter Transfer Function
- Usually $H(u,v)$ is symmetric about center
- Needs $F(u,v)$ is centered -

  premultiply $f(x,y)$ by $(-1)^{x+y}$

Example: Frequency Domain Filter

$$H(u,v) = \begin{cases} 0, & u = P/2, v = Q/2 \\ 1, & otherwise \end{cases}$$

$$g(x,y) = \Im^{-1}\left[H(u,v)F(u,v)\right]$$



a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

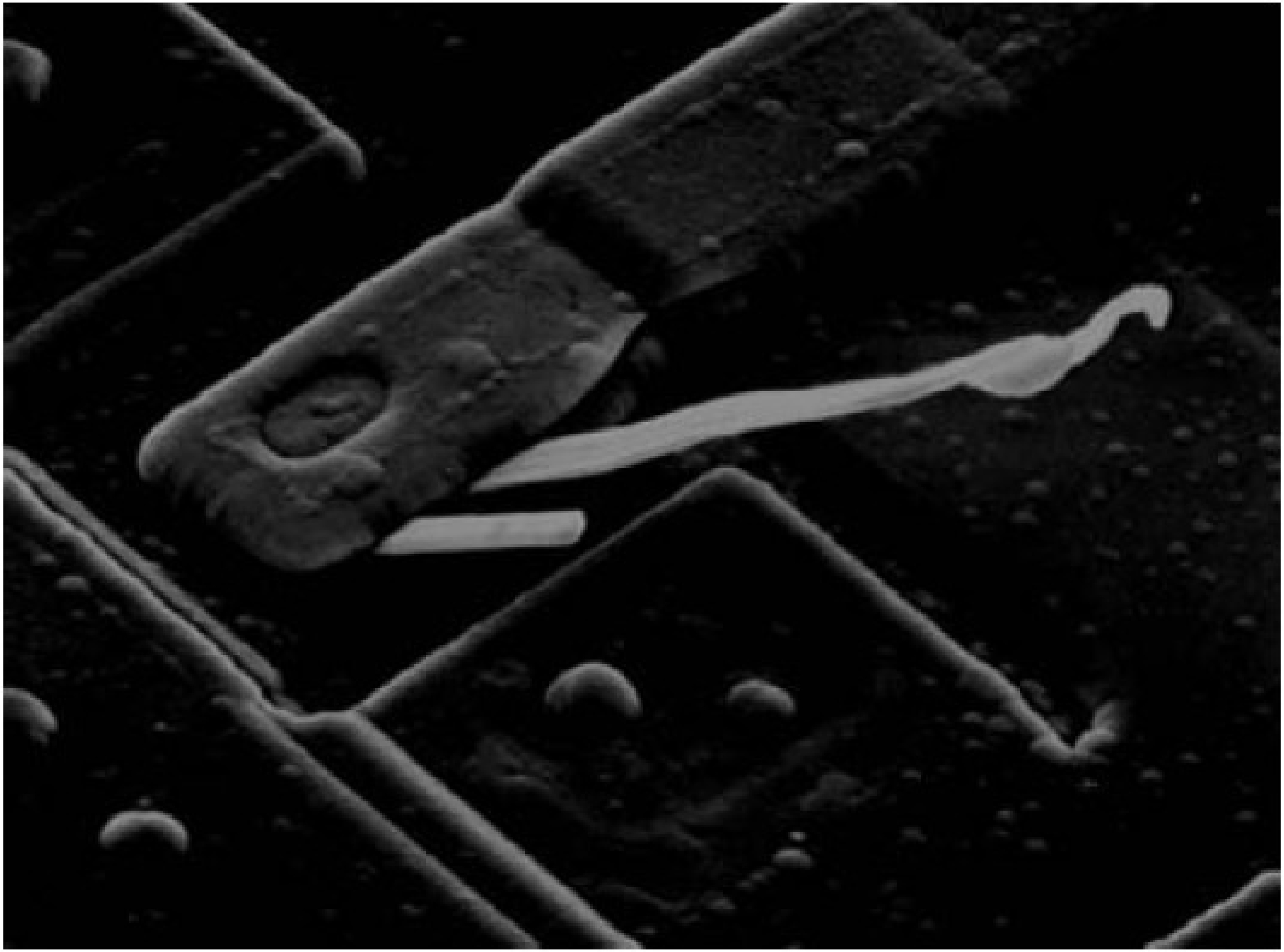
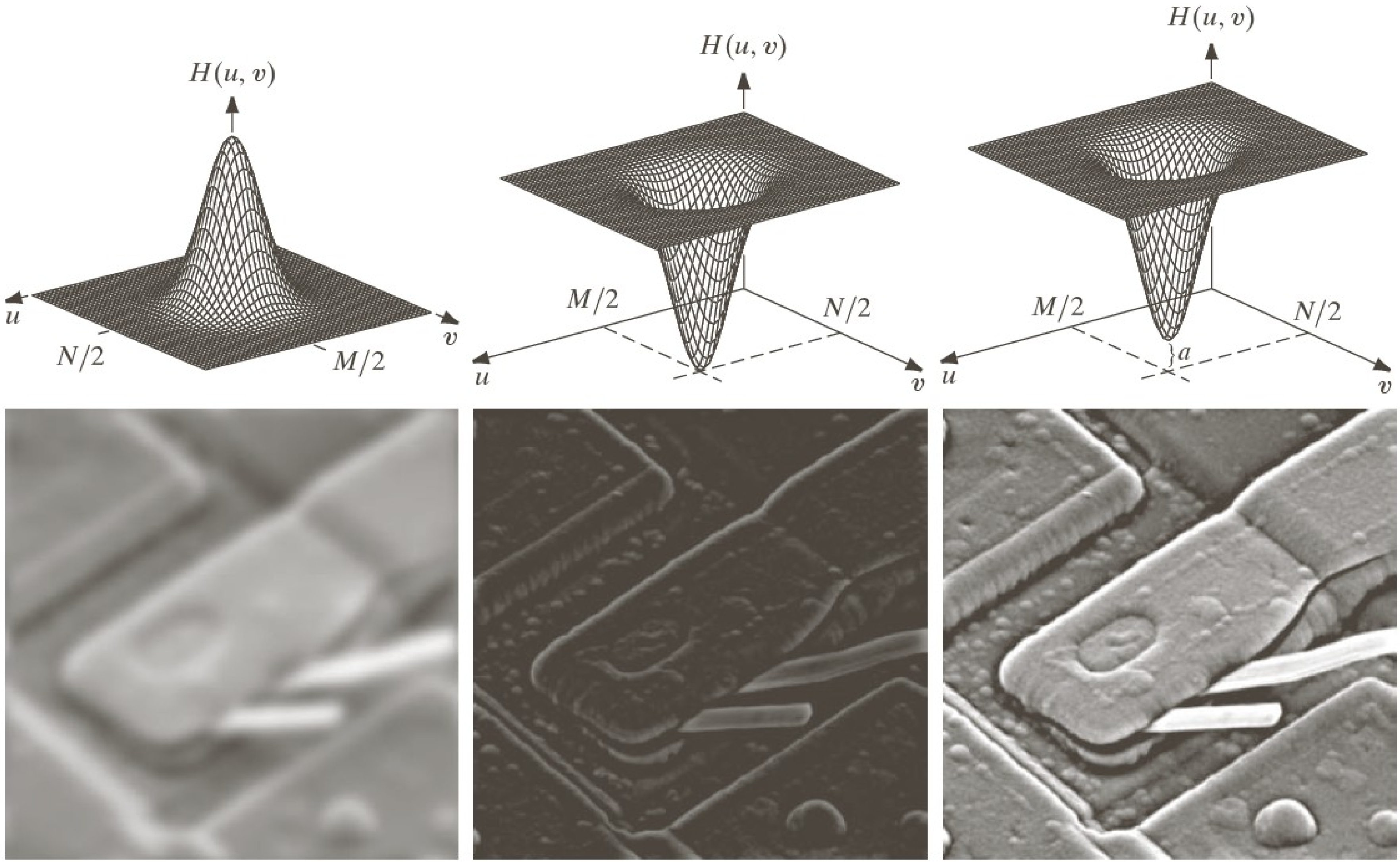
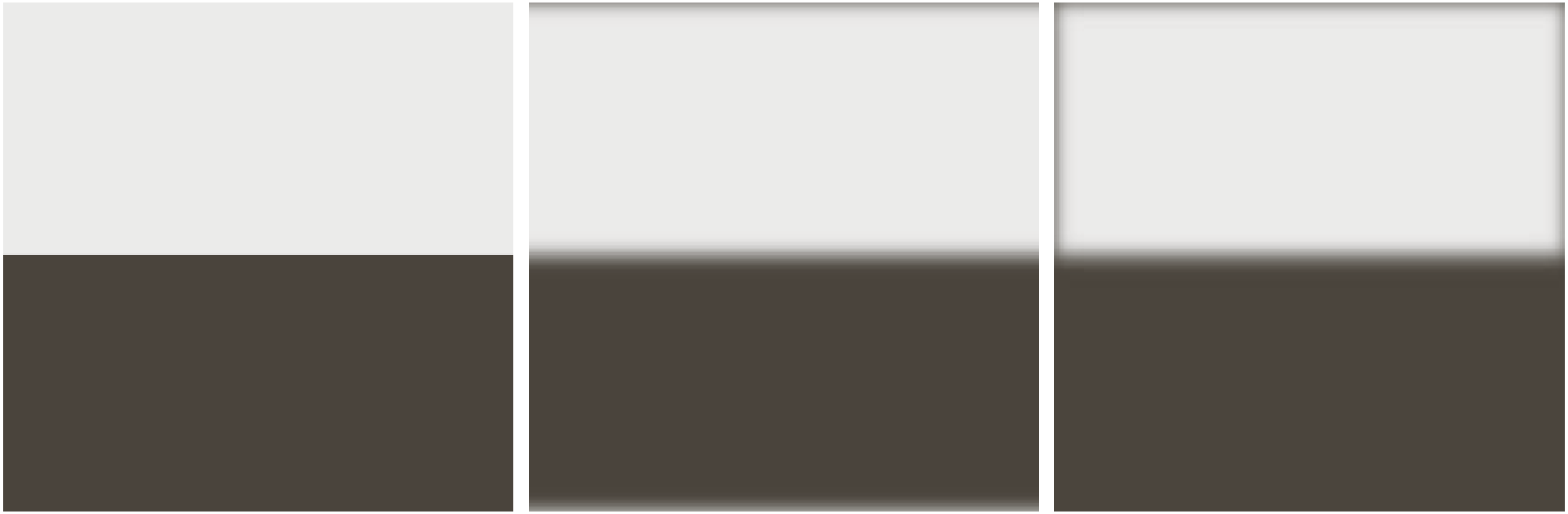


FIGURE 4.30 Result of filtering the image in Fig. 4.29(a) by setting to 0 the term $F(M/2, N/2)$ in the Fourier transform.



a b c
d e f

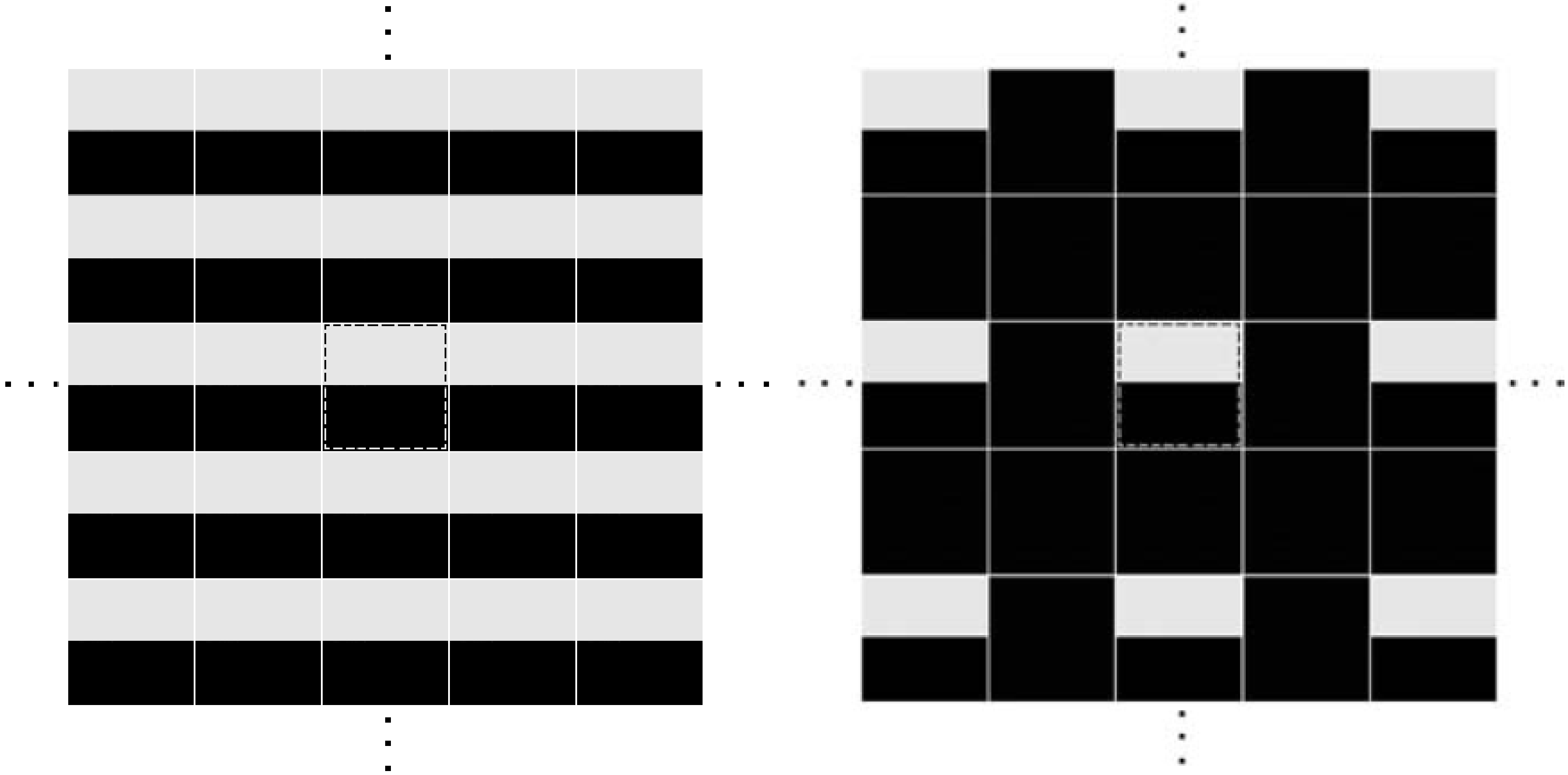
FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).



a b c

FIGURE 4.32 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with padding. Compare the light area of the vertical edges in (b) and (c).

Wraparound Effect & Zero Padding



a b

FIGURE 4.33 2-D image periodicity inherent in using the DFT. (a) Periodicity without image padding. (b) Periodicity after padding with 0s (black). The dashed areas in the center correspond to the image in Fig. 4.32(a). (The thin white lines in both images are superimposed for clarity; they are not part of the data.)

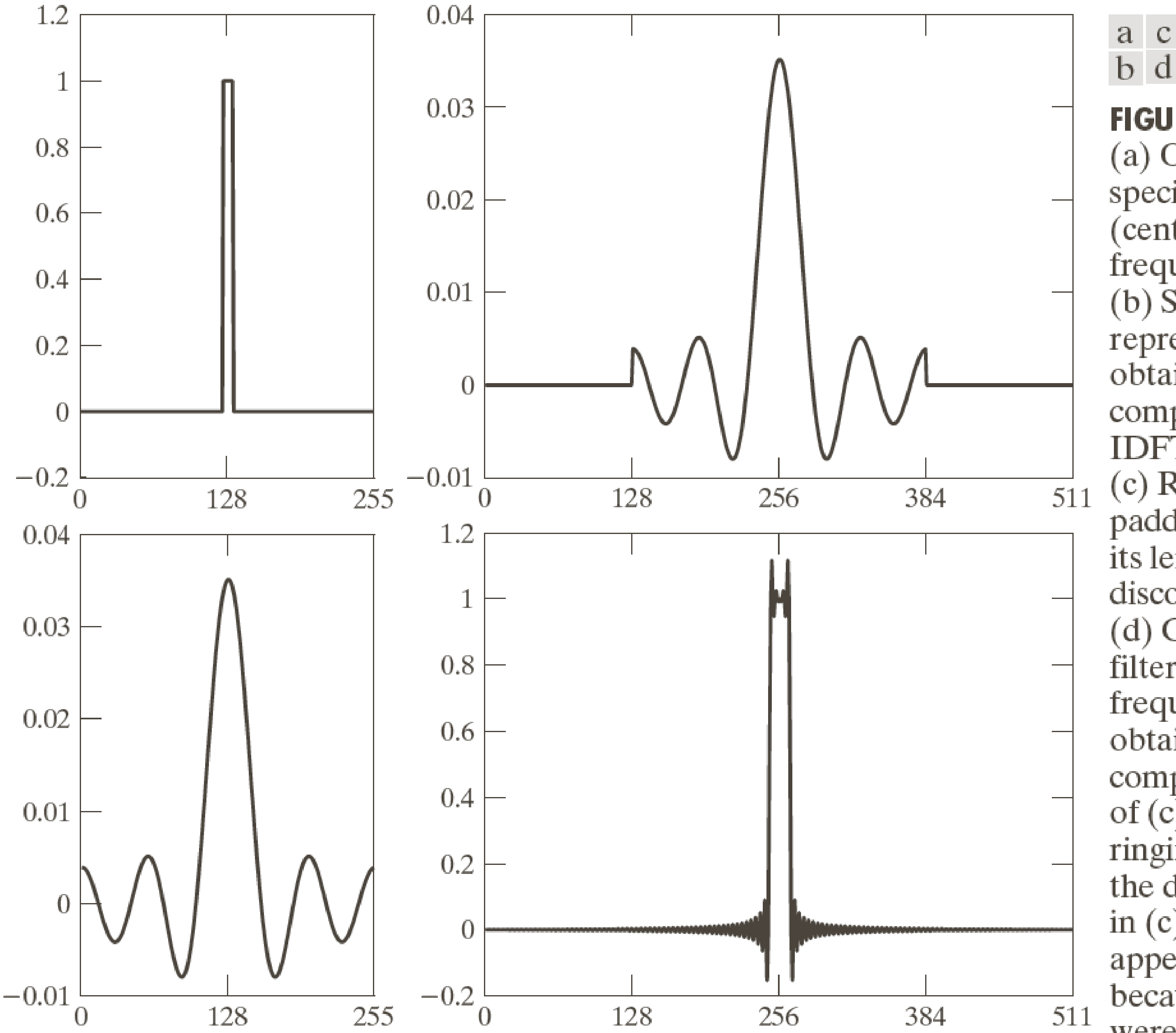


FIGURE 4.34 (a) Original filter specified in the (centered) frequency domain. (b) Spatial representation obtained by computing the IDFT of (a). (c) Result of padding (b) to twice its length (note the discontinuities). (d) Corresponding filter in the frequency domain obtained by computing the DFT of (c). Note the ringing caused by the discontinuities in (c). (The curves appear continuous because the points were joined to simplify visual analysis.)

Can we pad the filter?

Filtering Steps

- f(x,y) is MxN. Pad to PxQ. Typically, P=2M, Q=2N
- Form fp(x,y) of size PxQ by adding necessary zeros to f(x,y)
- Multiply fp(x,y) by (–1)^(x+y) to center transform
- Compute F(u,v) by DFT of fp(x,y)
- Use real symmetric filter H(u,v), of size PxQ & center at (P/2, Q/2). Form G(u,v)=H(u,v)F(u,v)
- Compute gp(x,y) by product of real part of IDFT of G(u,v) and (–1)^(x+y).
- Extract g(x,y) taking MxN at left top corner of gp(x,y)

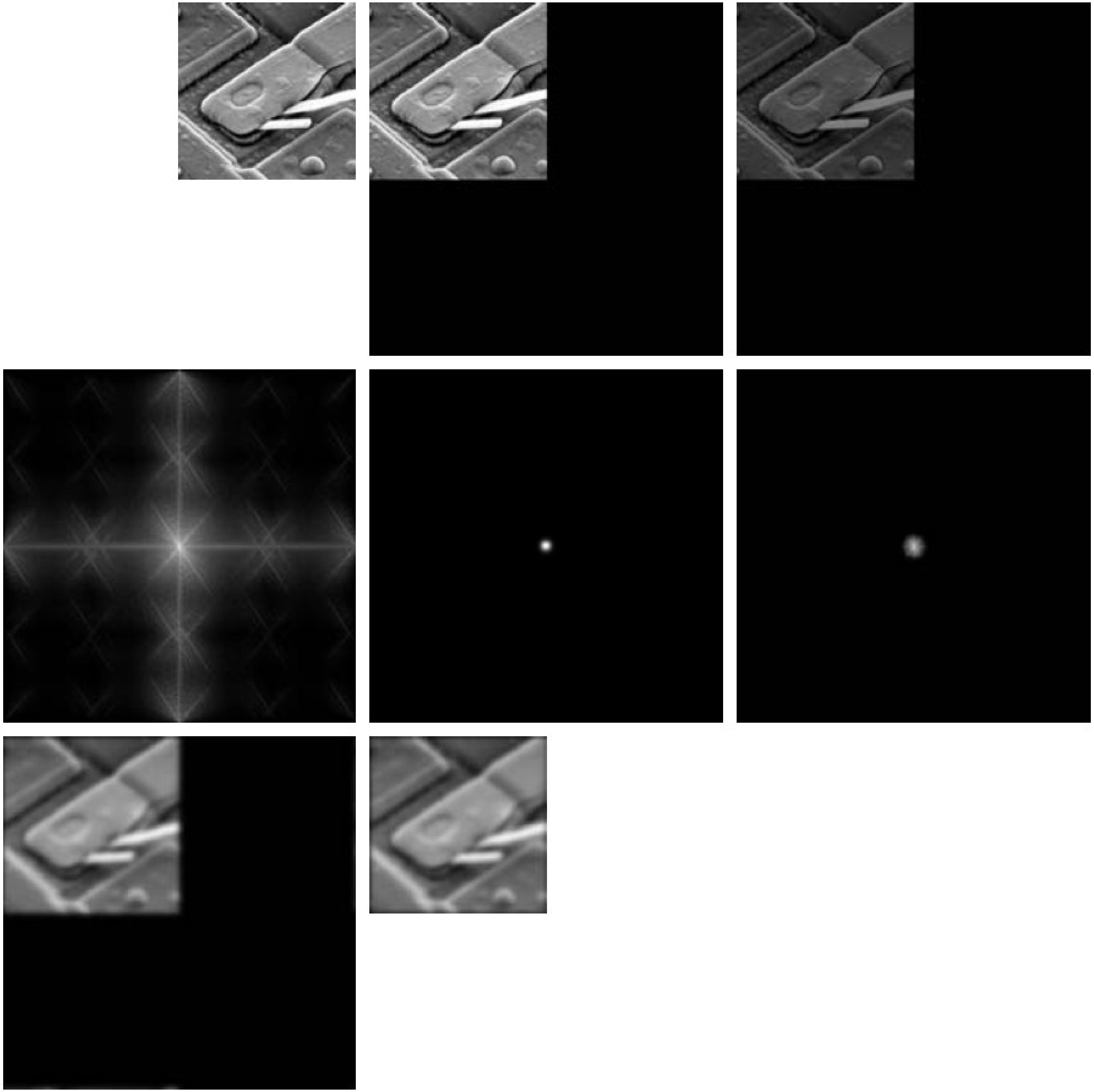


FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

Filter Design

Lecture 21-22: 11-Sep-12

Dr. P P Das

Filtering Steps

- f(x,y) is MxN. Pad to PxQ. Typically, P=2M, Q=2N
- Form fp(x,y) of size PxQ by adding necessary zeros to f(x,y)
- Multiply fp(x,y) by $(-1)^{(x+y)}$ to center transform
- Compute F(u,v) by DFT of fp(x,y)
- Use real symmetric filter H(u,v), of size PxQ & center at (P/2, Q/2). Form G(u,v)=H(u,v)F(u,v)
- Compute gp(x,y) by product of real part of IDFT of G(u,v) and $(-1)^{(x+y)}$.
- Extract g(x,y) taking MxN at left top corner of gp(x,y)

Filtering in Spatial vis-à-vis Frequency Domains

Frequency Domain Filter : $H(u,v)$

Spatial Domain Filter : $h(x,y)$

Let $f(x,y) = \delta(x,y), F(u,v) = 1$. Hence :

Output $h(x,y) = \Im^{-1}\{H(u,v)\}$

$h(\mathrm{x,y})$: Impulse Response of $H(u,v)$

Quantities in $h(x,y)$ are finite. Hence :

$h(x,y)$: Finite Impulse Response (FIR) Filter

Example Gaussian Filter

Lowpass Filter :

$$H(u) = Ae^{-u^2/2\sigma^2}$$

$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2x^2}$$

High σ

$\Rightarrow$ Broad profile of $H(u)$

$\Rightarrow$ Narrow profile of $h(x)$

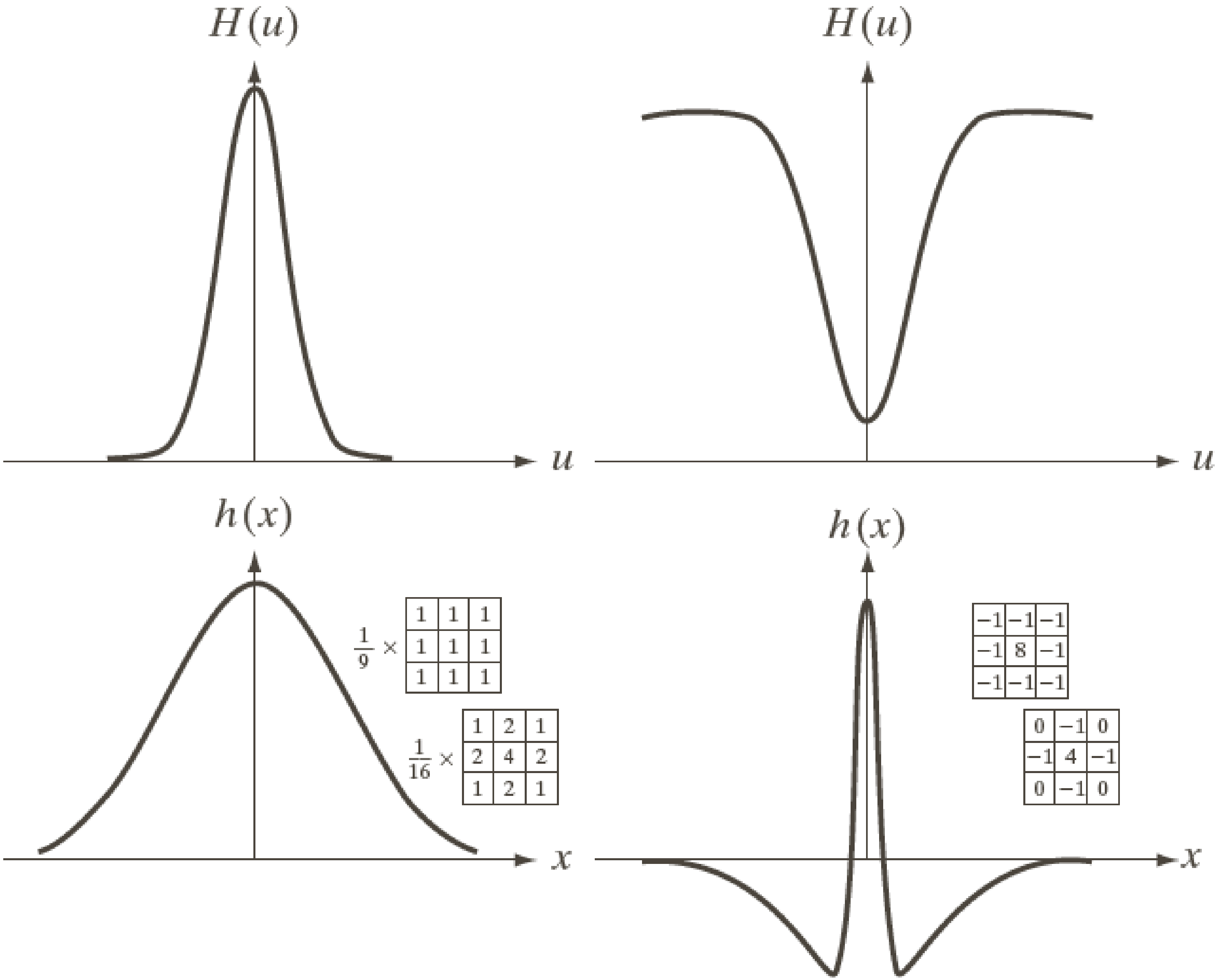
and vice - versa

Highpass Filter :

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$

$$h(x) = \sqrt{2\pi}\sigma_1 A e^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma_2 B e^{-2\pi^2\sigma_2^2x^2}$$

$A \geq B$ and $\sigma_1 > \sigma_2$



a c
b d

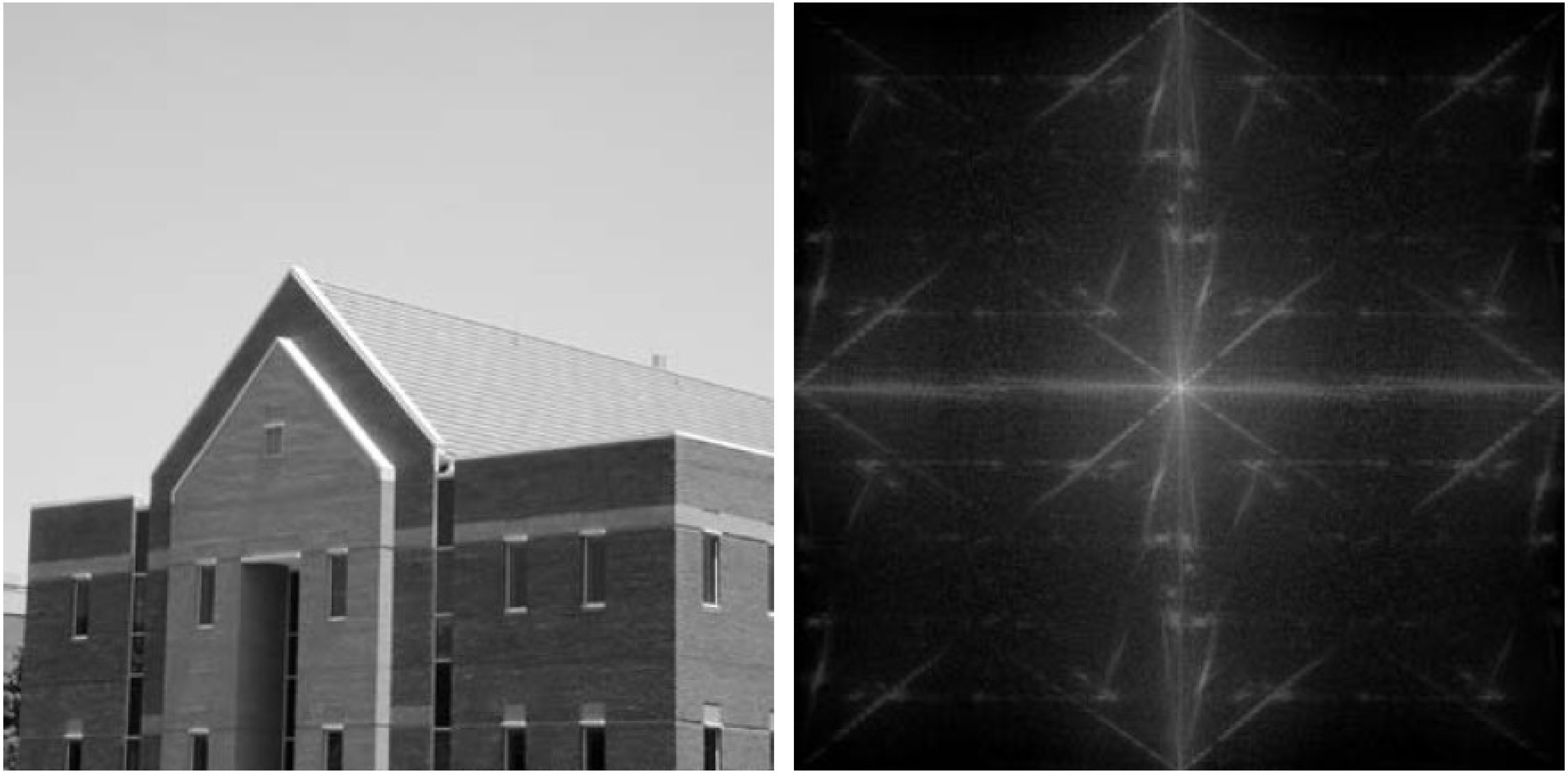
FIGURE 4.37 (a) A 1-D Gaussian lowpass filter in the frequency domain. (b) Spatial lowpass filter corresponding to (a). (c) Gaussian highpass filter in the frequency domain. (d) Spatial highpass filter corresponding to (c). The small 2-D masks shown are spatial filters we used in Chapter 3.

$$H(u) = Ae^{-u^2/2\sigma^2}$$

$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2 x^2}$$

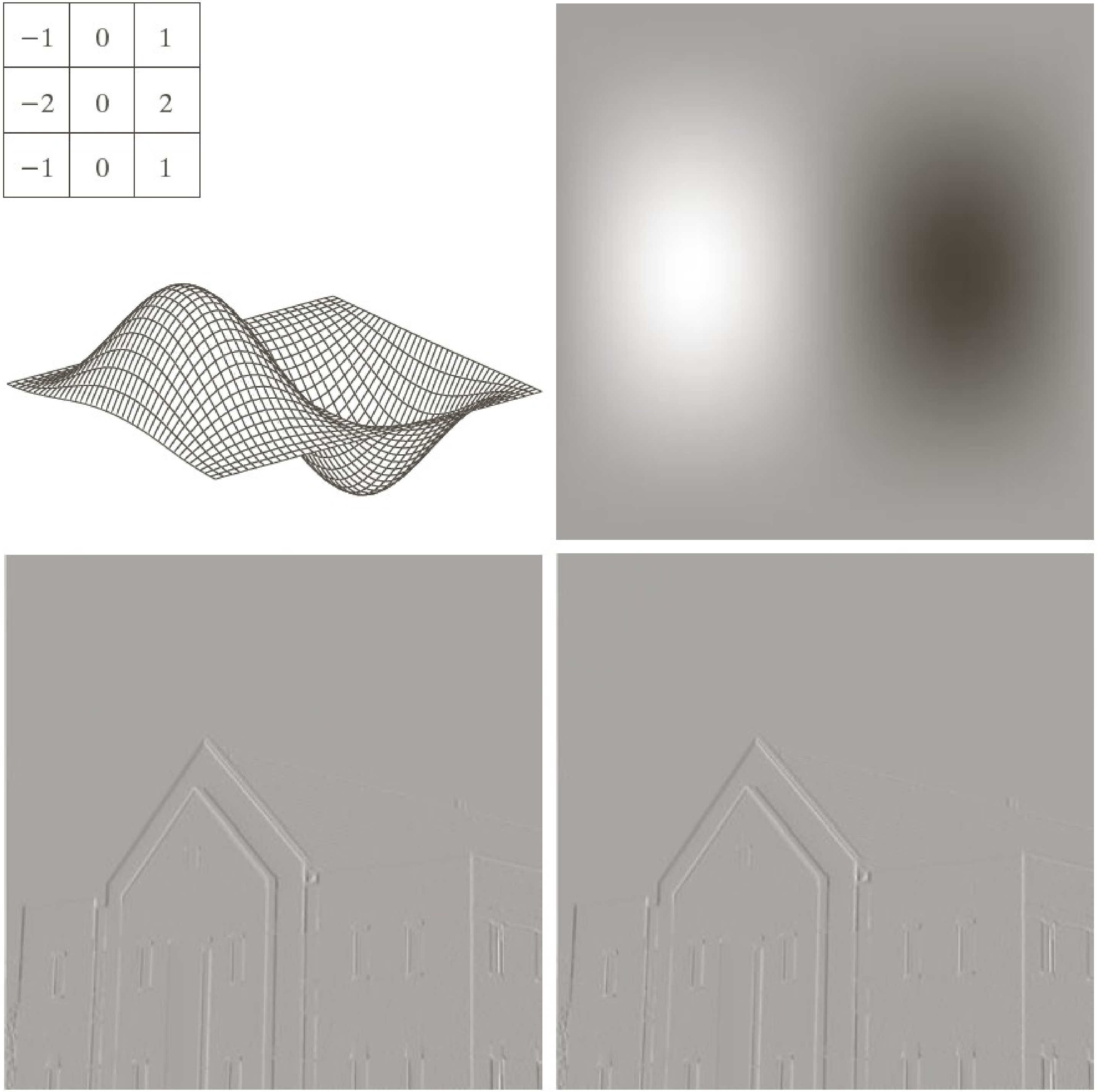
$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$

$$h(x) = \sqrt{2\pi}\sigma_1 A e^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\sigma_2 B e^{-2\pi^2\sigma_2^2 x^2}$$



a b

FIGURE 4.38
(a) Image of a building, and (b) its spectrum.



−1	0	1
−2	0	2
−1	0	1

a b
c d

FIGURE 4.39 (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

Image Smoothing

Frequency Domain Filters

Image Smoothing – Low-pass Filter

- Low-pass Filtering
  - Ideal: Very sharp
  - Butterworth
  - Gaussian: Very Smooth
- Butterworth Filter is parameterized by Filter Order
  - High Order → Ideal
  - Low Order → Gaussian

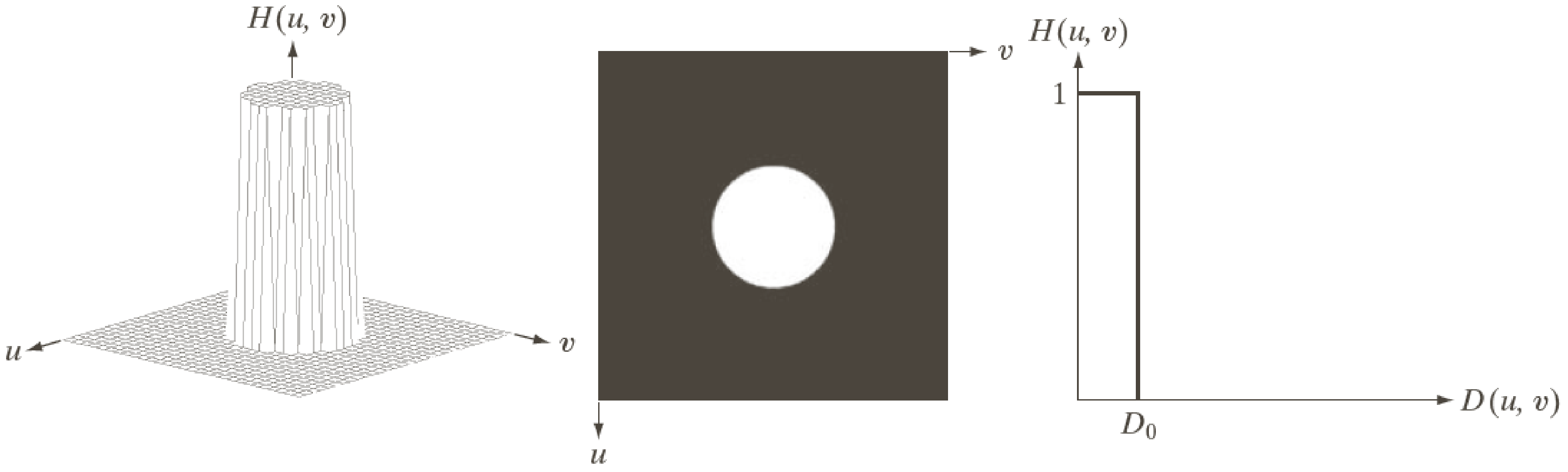
Ideal Low-Pass Filter (ILPF)

$$H(u,v) = \begin{cases} 1, & D(u,v) \le D_0 \\ 0, & D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

D_0 : Cut-off Frequency

Ideal Low-Pass Filter (ILPF)



a b c

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

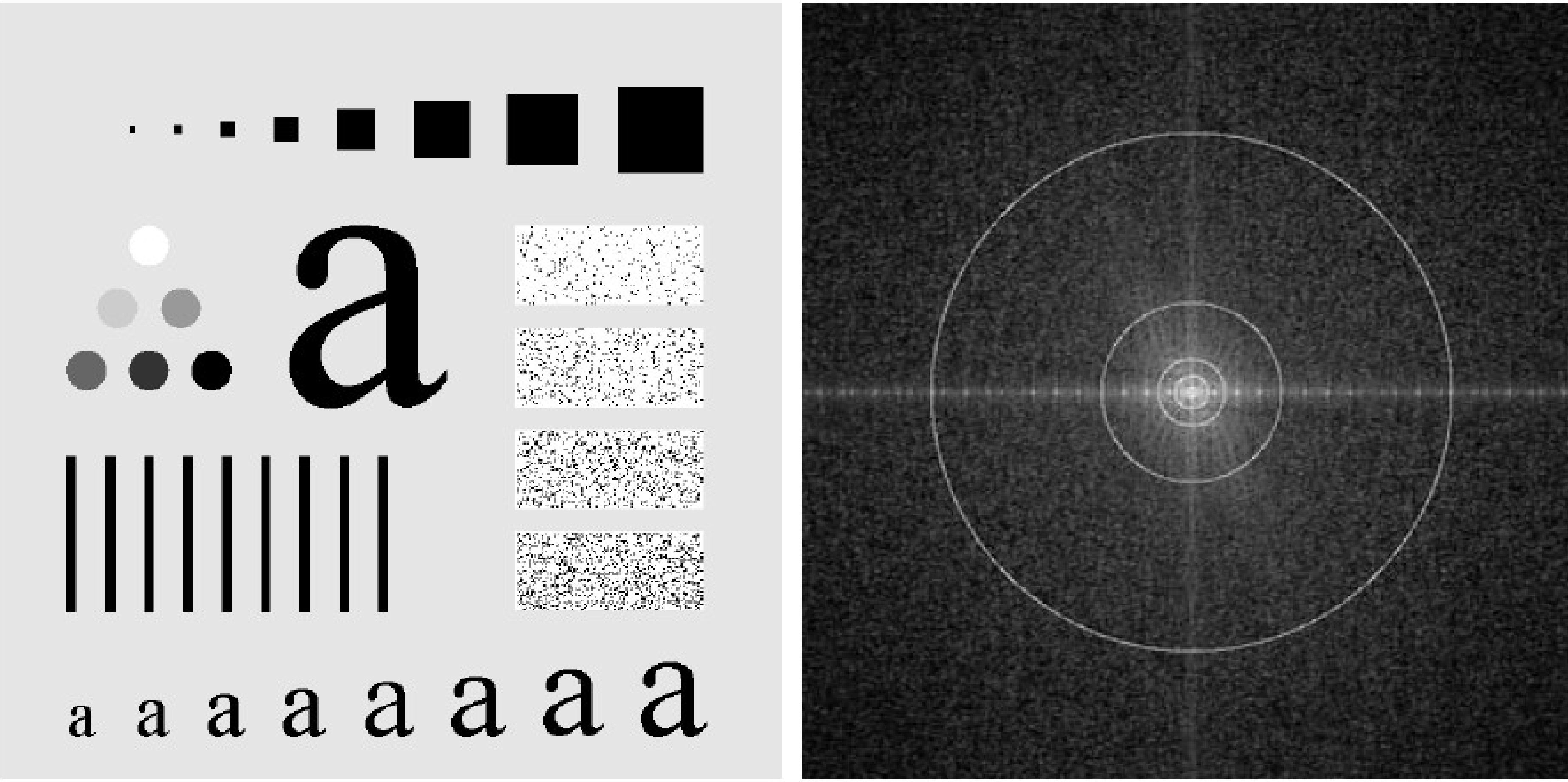
Ideal Low-Pass Filter (ILPF)

$$\text{Total Image Power}: P_T = \sum_{u=0}^{P-1} \sum_{v=0}^{Q-1} P(u,v)$$

$$\text{where } P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

$$\text{Percent of power cut-off by filter}:$$

$$\alpha = 100 \left[\sum_{\{u,v:\, D(u,v) \le D_0\}} P(u,v) / P_T \right]$$



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

Ideal Low-Pass Filter (ILPF)

ILPF

Radius	Power (α)
10	87.0
30	93.1
60	95.7
160	97.8
460	99.2

Blurring & Ringing decreases with increase of radius / power

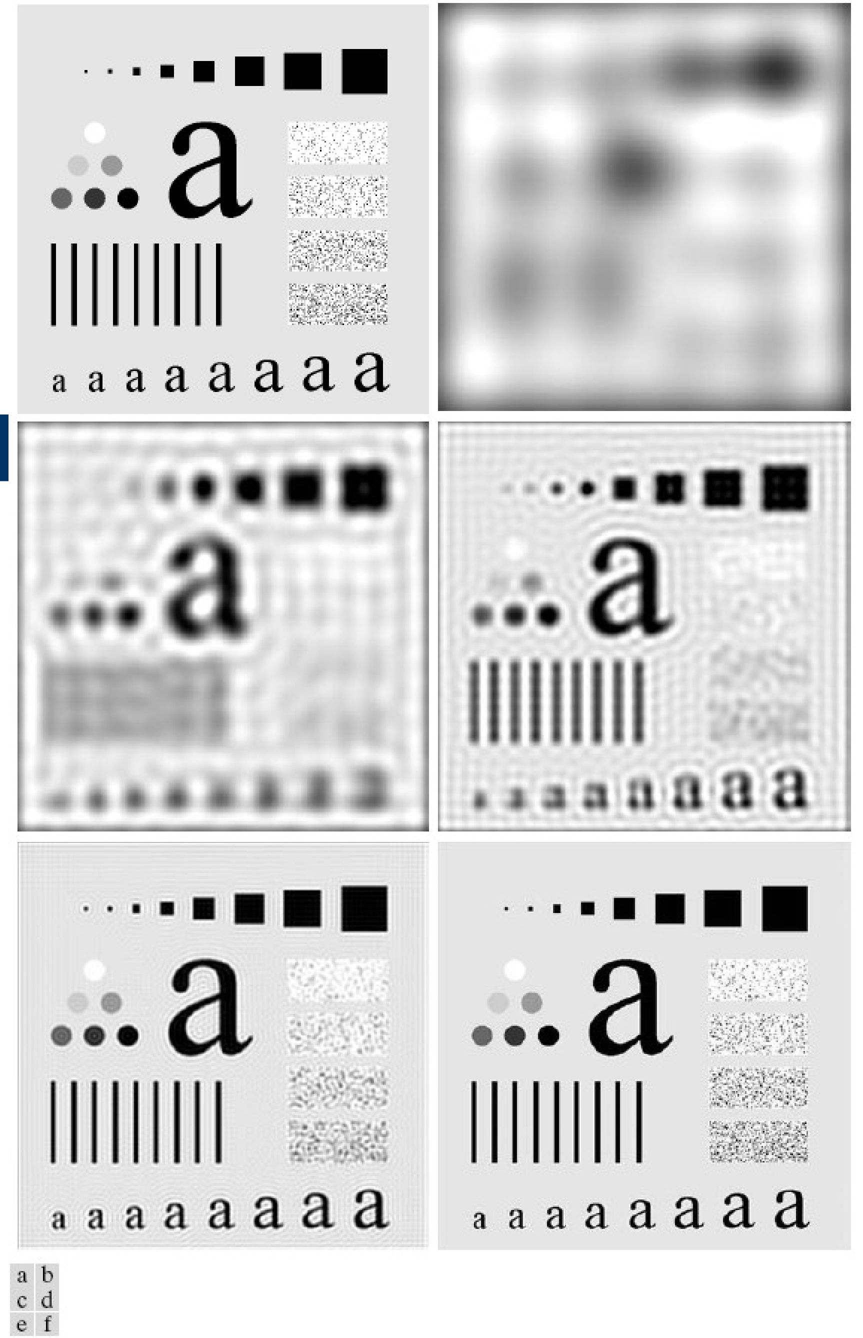
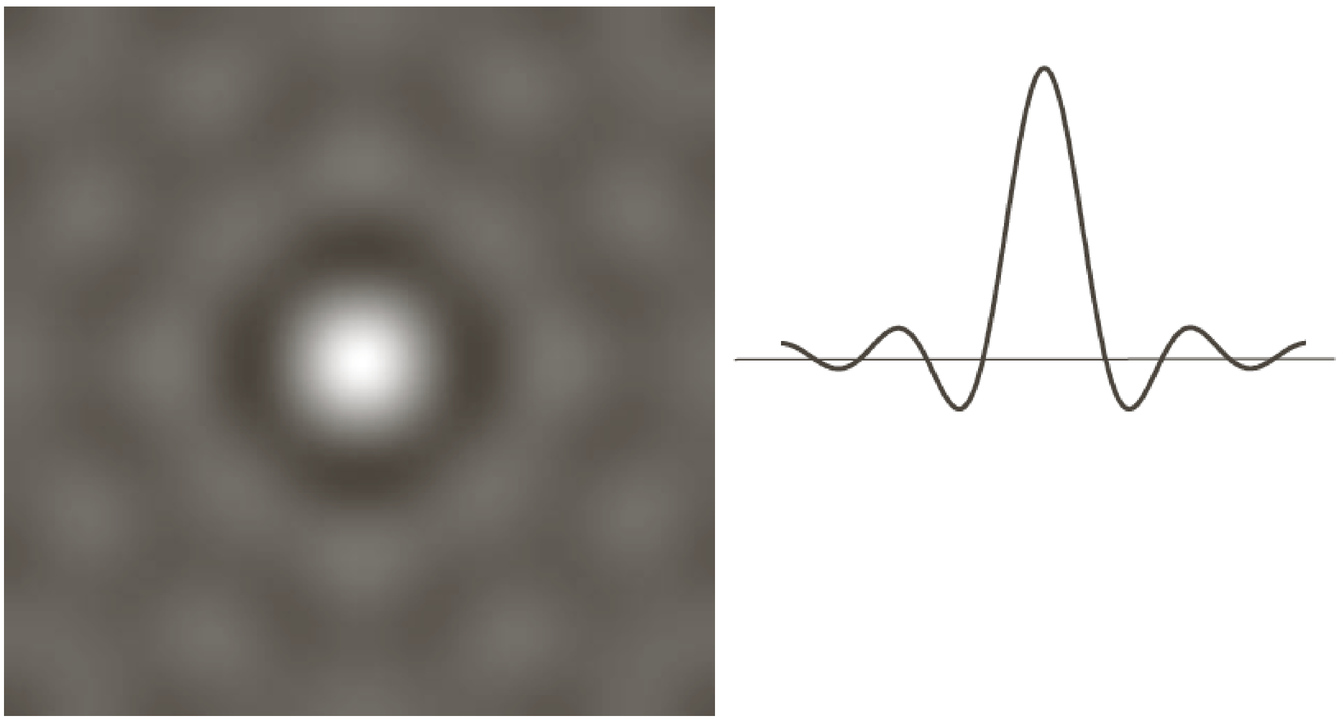


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

Blurring & Ringing in ILPF



a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.

ILPF → Box Filter → Sinc in Spatial Domain

Butterworth Low-Pass Filter (BLPF)

$$H(u,v) = \frac{1}{1 + \left[D(u,v) / D_0 \right]^{2n}}$$

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

D_0 : Cut-off Frequency

n : Order of BLPF

Butterworth Low-Pass Filter (BLPF)

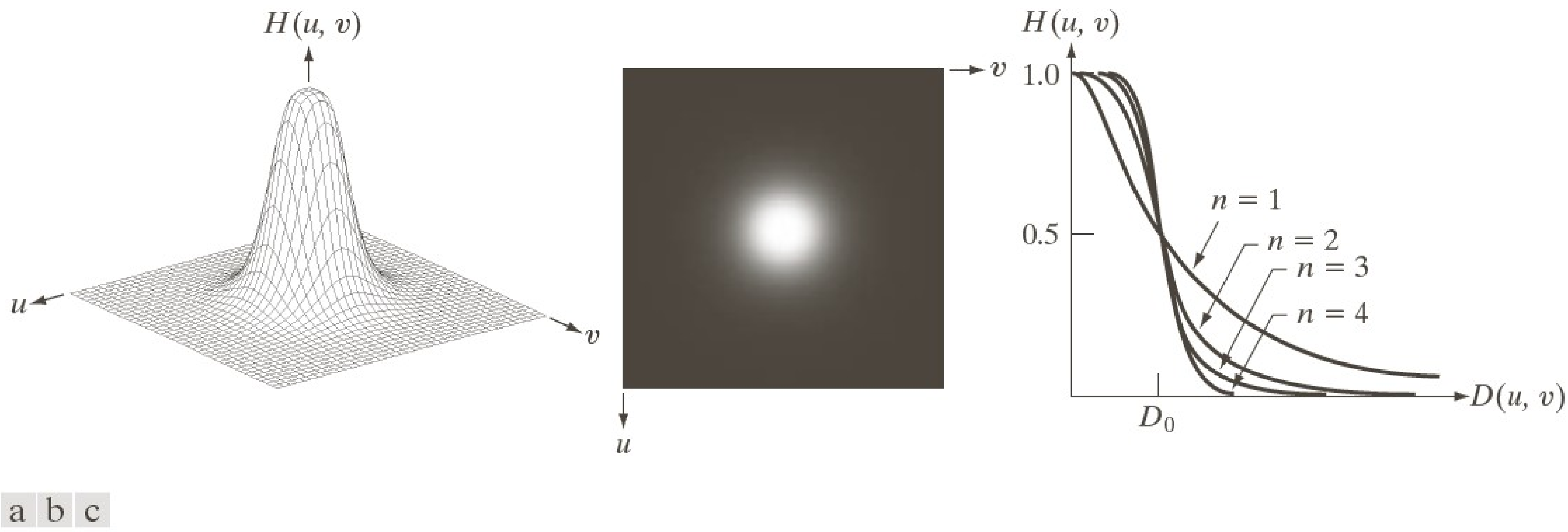
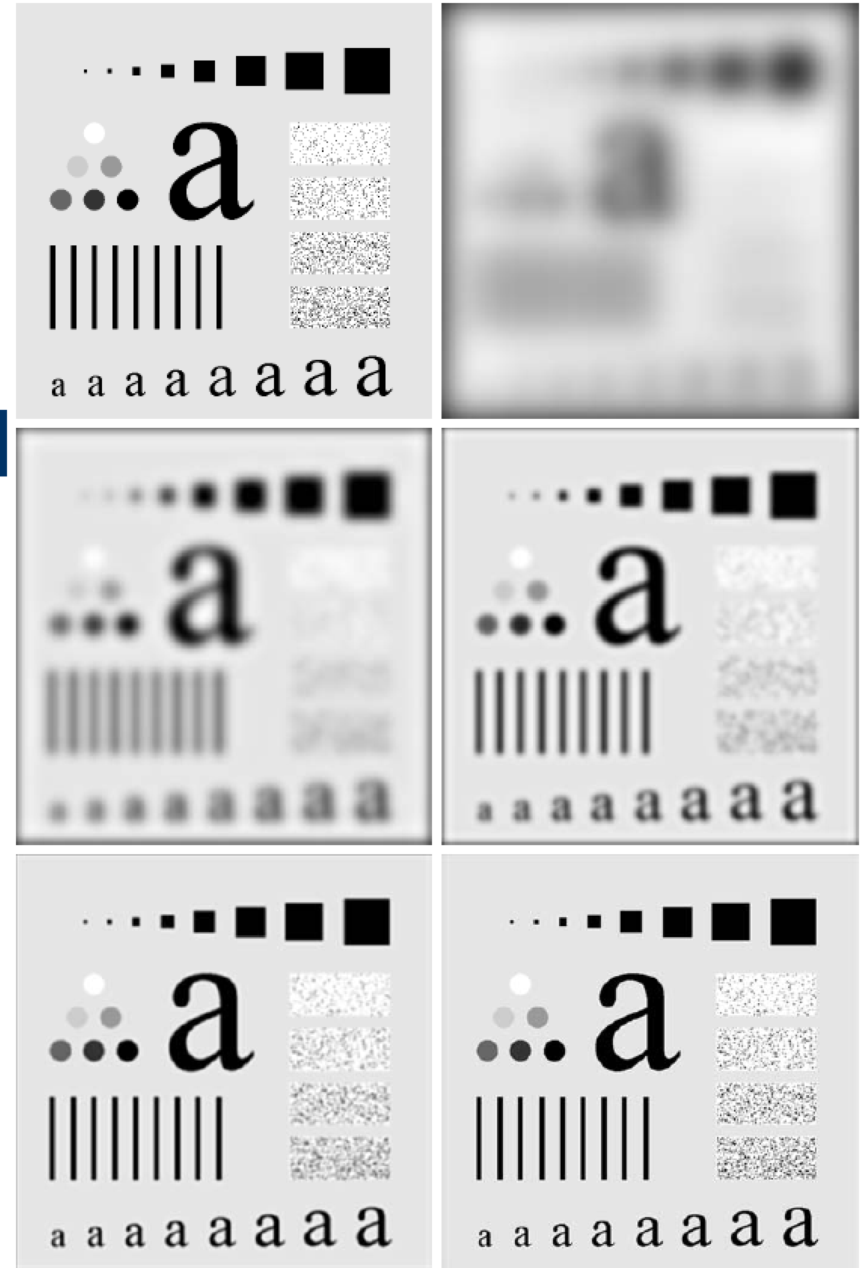


FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Unlike ILPF, no sharp cut-off

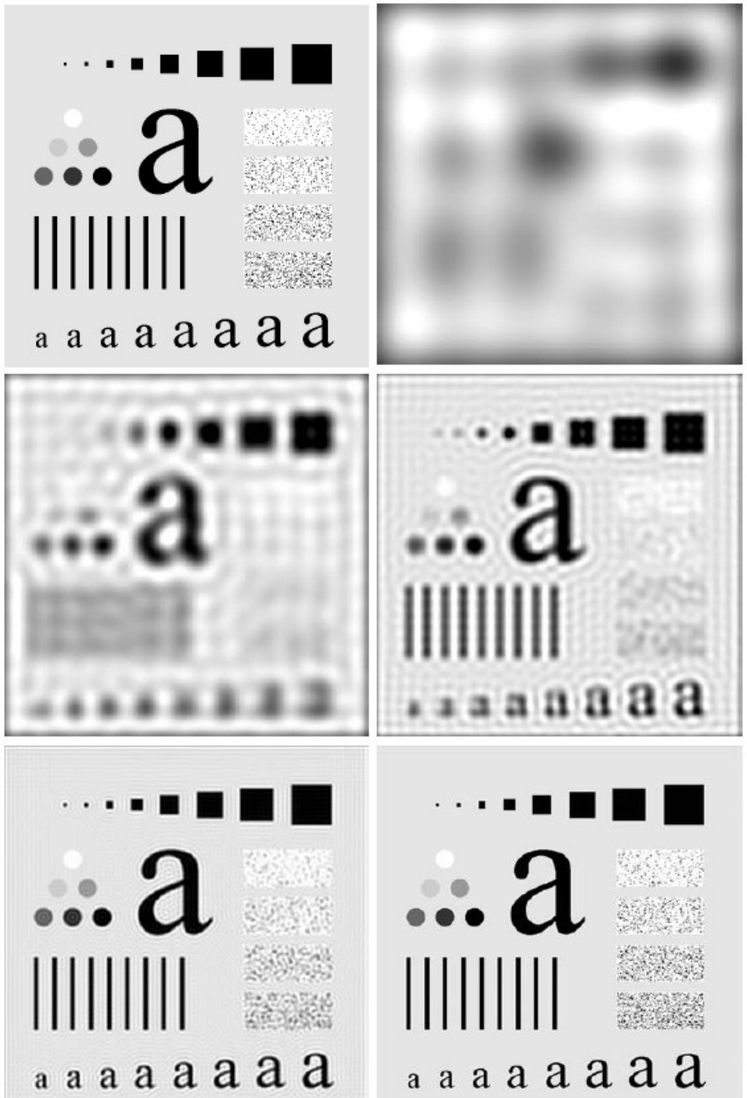
BLPF

Radius
10
30
60
160
460



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

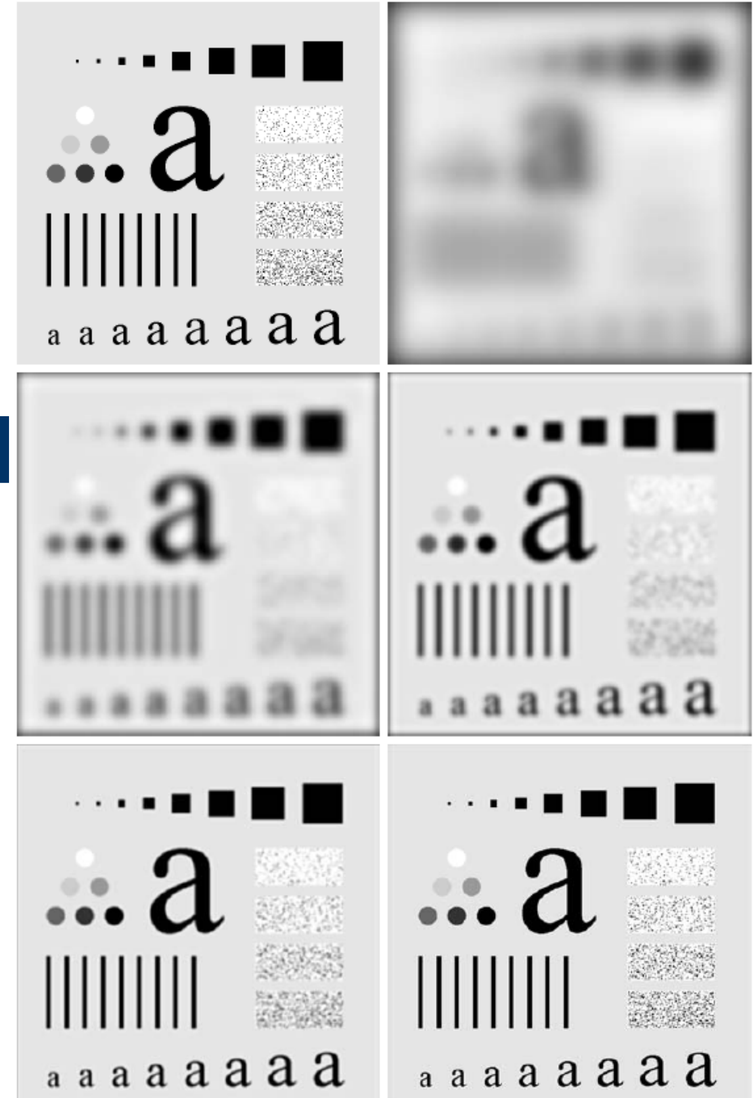


a b
c d
e f

ILPF

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

- Sharp transition in response causing heavy blurring & ringing

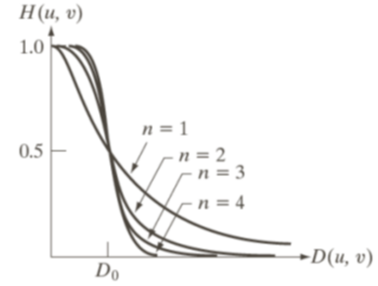


a b
c d
e f

BLPF

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

- Smooth transition in blurring
- No ringing



BLPF: Ringing increases with order

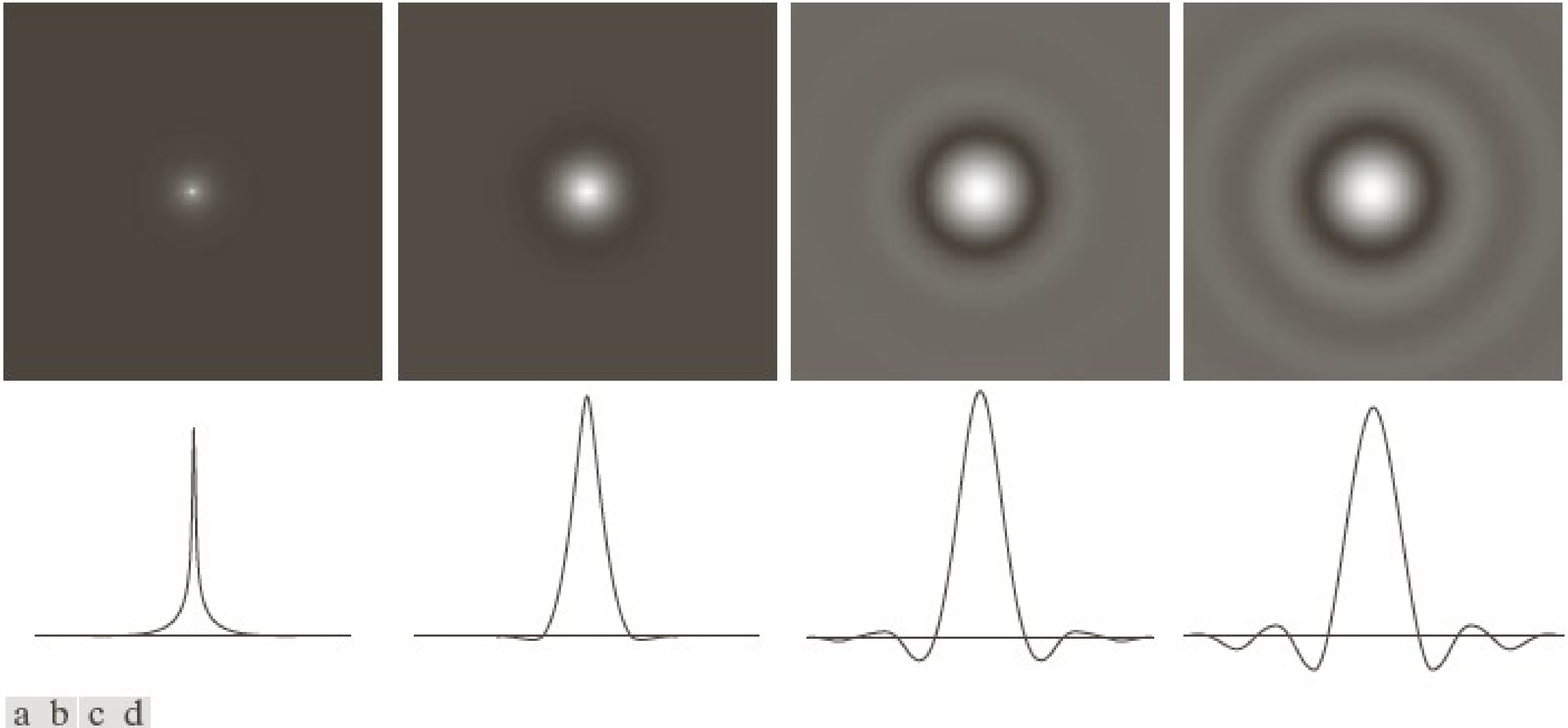


FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Gaussian Low Pass Filter (GLPF)

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

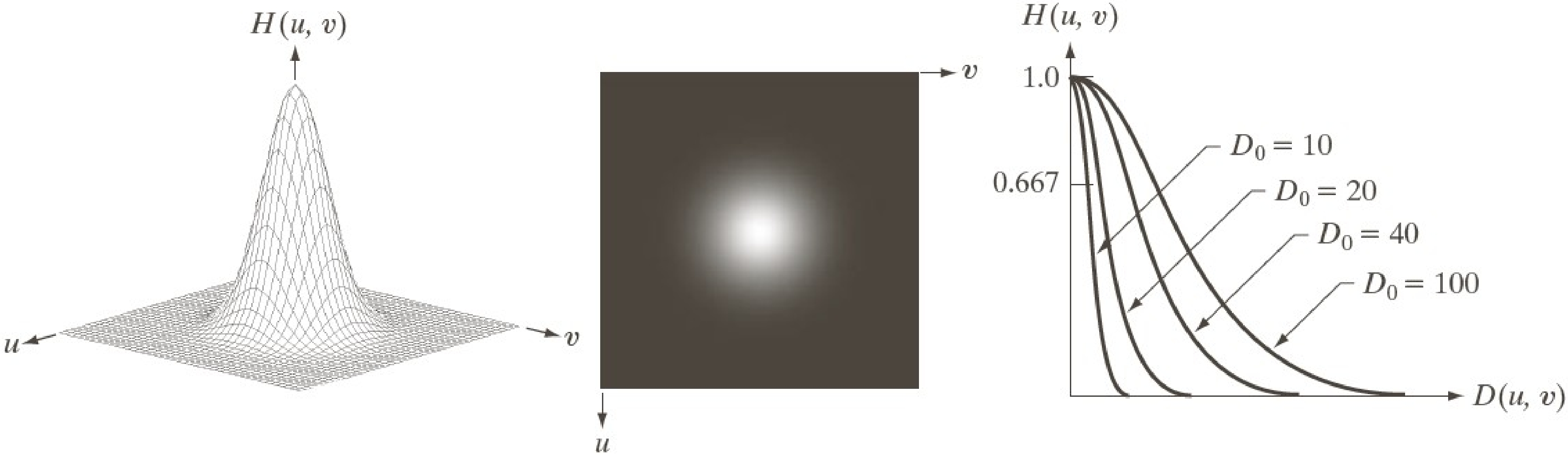
$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$$

σ : Measure of spread about center

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

$\sigma = D_0$: Cut-off Frequency

Gaussian Low Pass Filter (GLPF)



a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

GLPF

Radius
10
30
60
160
460

- No ringing as IDFT of Gaussian is Gaussian

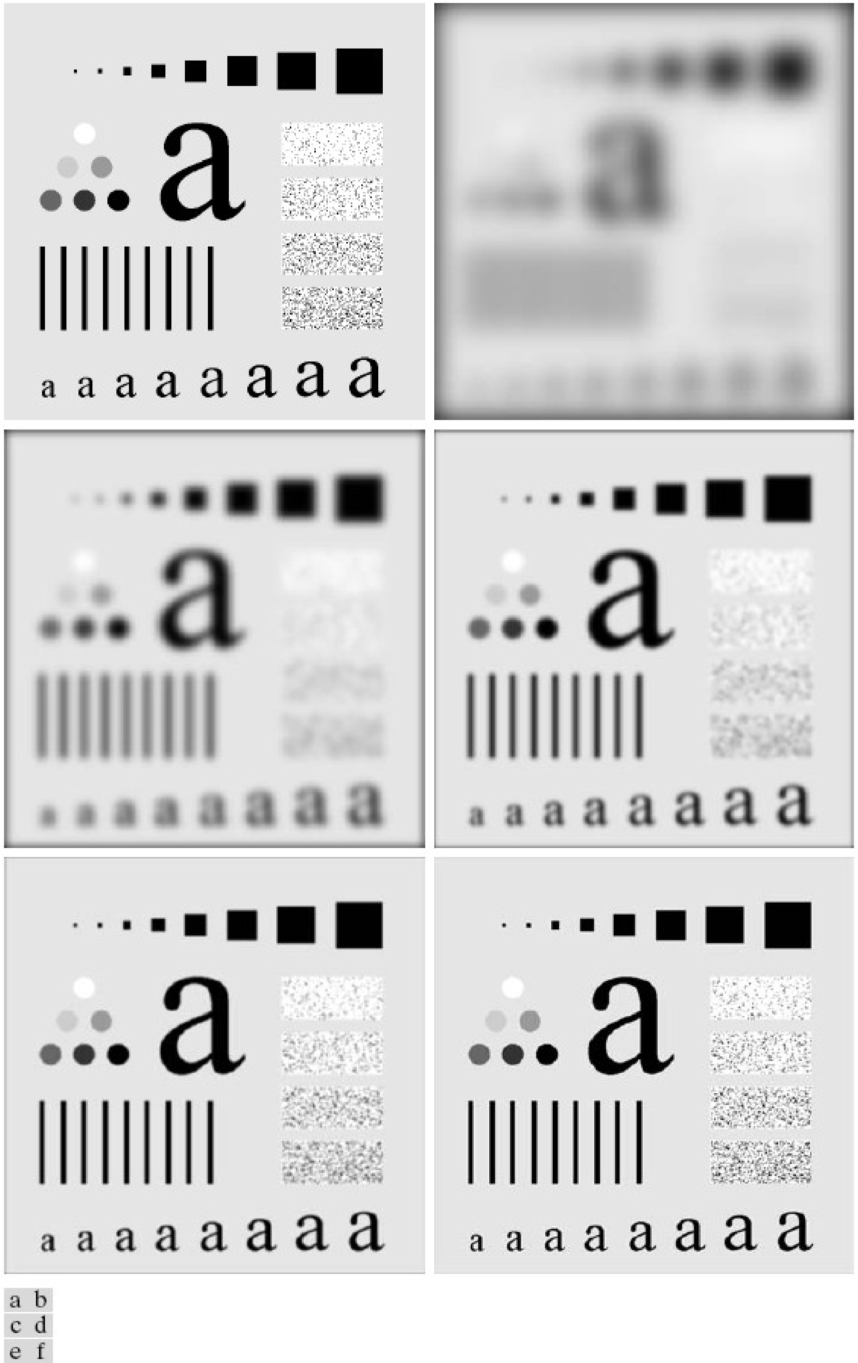


FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

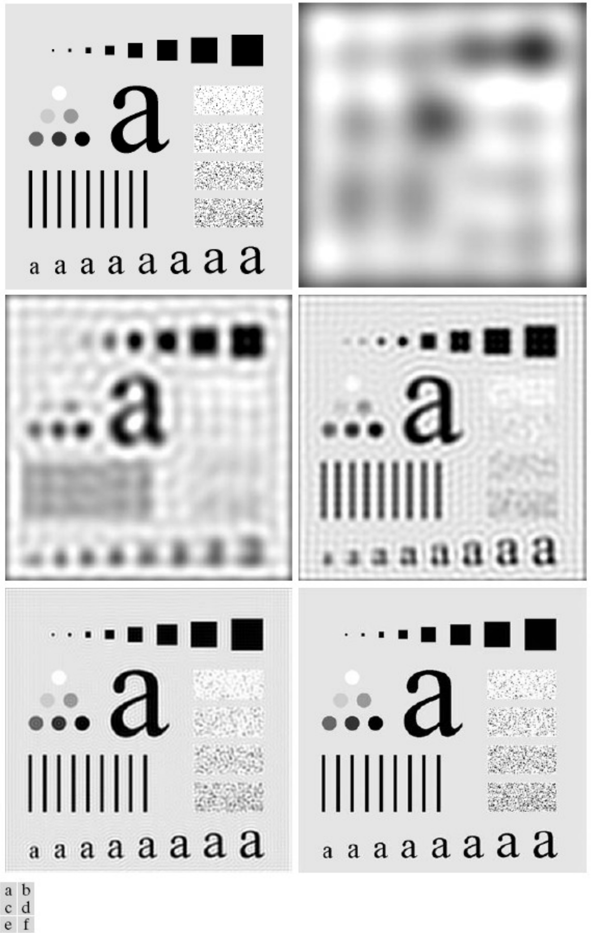


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

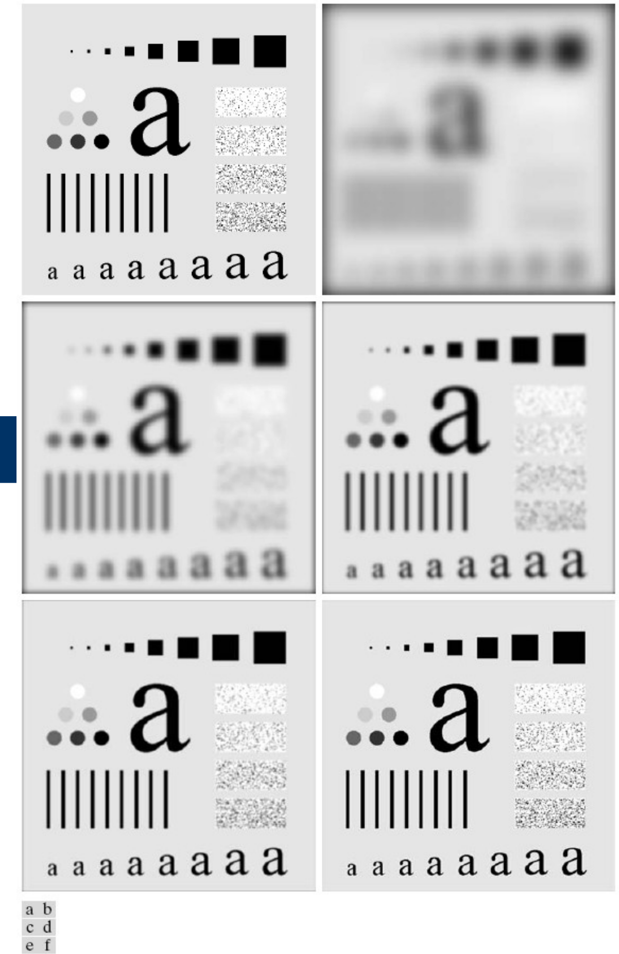


FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

ILPF

- Sharp transition in response causing heavy blurring & ringing

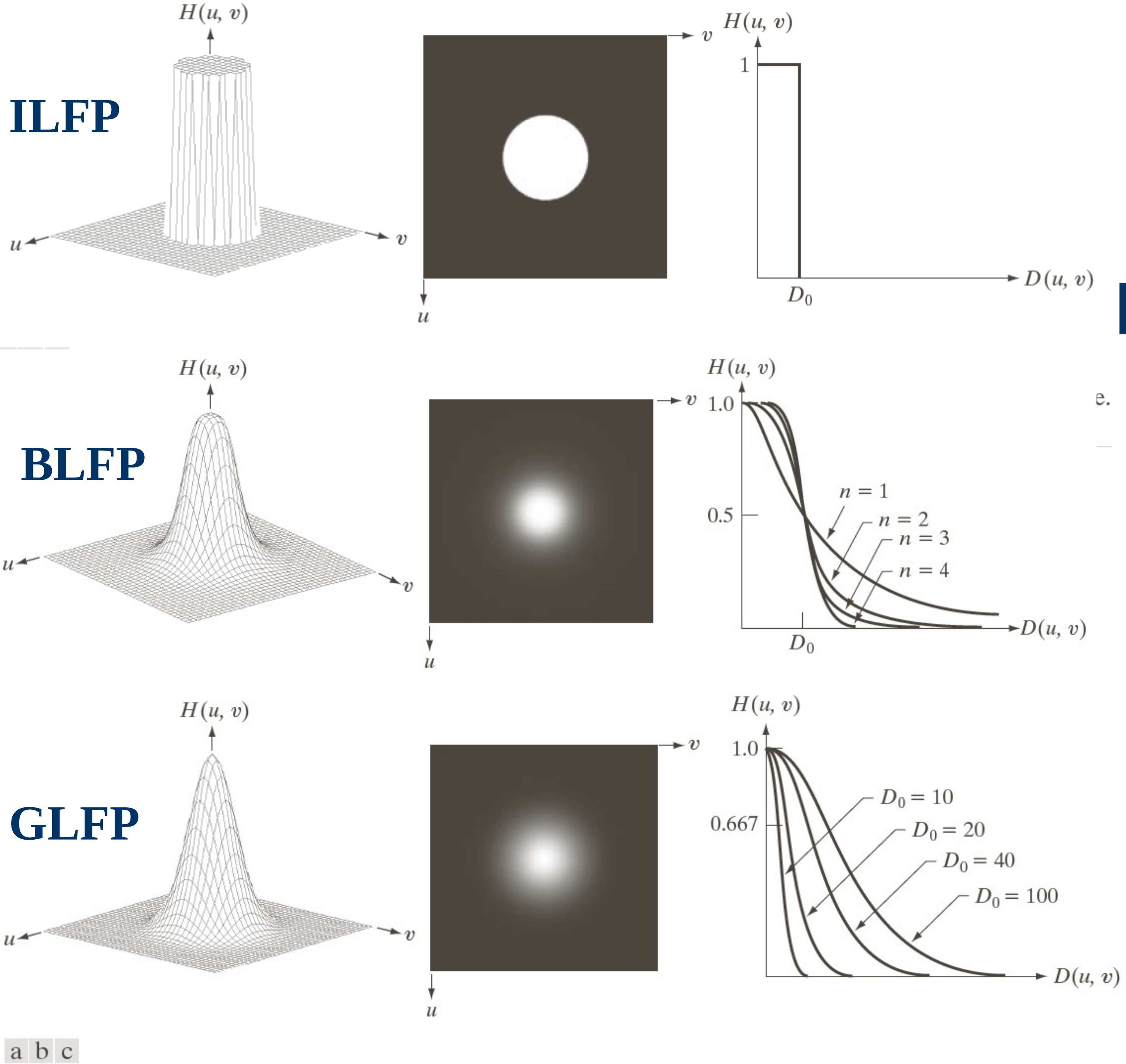
BLPF

-Smooth transition in blurring
- No ringing

GLPF

- No ringing

Radius: 10, 30, 60, 160, 460



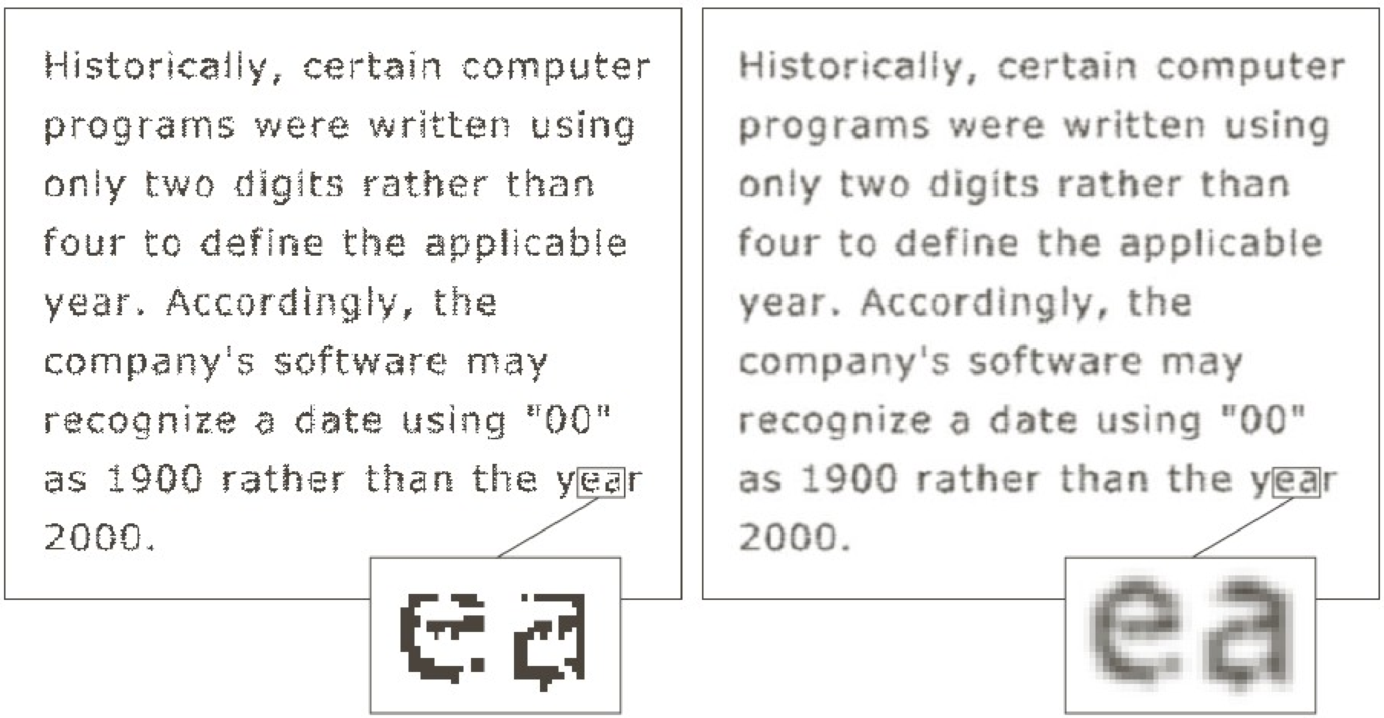
ILFP
BLFP
GLFP
H(u, v)
v
u
1
D(u, v)
D0
1.0
0.5
n = 1
n = 2
n = 3
n = 4
0.667
D0 = 10
D0 = 20
D0 = 40
D0 = 100
a b c

Image Smoothing: Low-Pass Filters

TABLE 4.4
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

LPF: Character Recognition



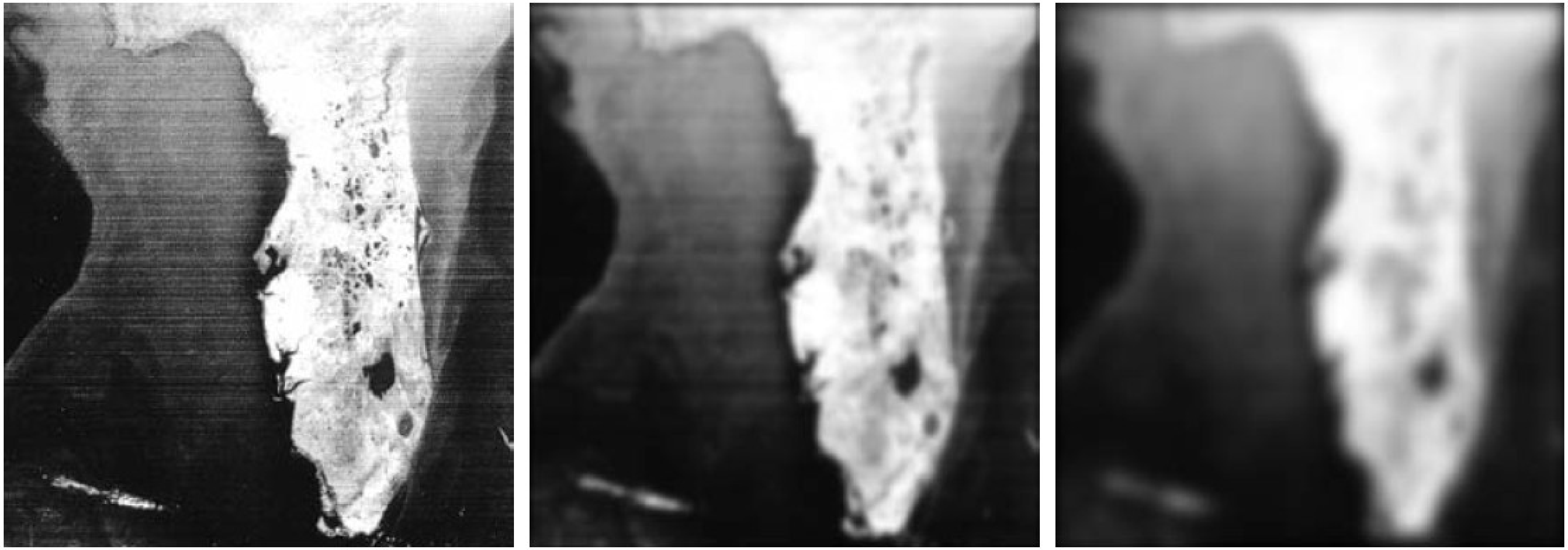
a b

FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

LFT: Printing & Publishing

FIGURE 4.50 (a) Original image (784×732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

LPF: Satellite Imagery



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Image Sharpening

Frequency Domain Filters

High-Pass Filter (HPF)

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

Image Sharpening – High-pass Filter

- High-pass Filtering
  - Ideal: Very sharp
  - Butterworth
  - Gaussian: Very Smooth
- Butterworth Filter is parameterized by Filter Order
  - High Order → Ideal
  - Low Order → Gaussian

Ideal High-Pass Filter (IHPF)

$$H(u,v) = \begin{cases} 0, & D(u,v) \le D_0 \\ 1, & D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

D_0 : Cut-off Frequency

IHFP

BHFP

GHFP

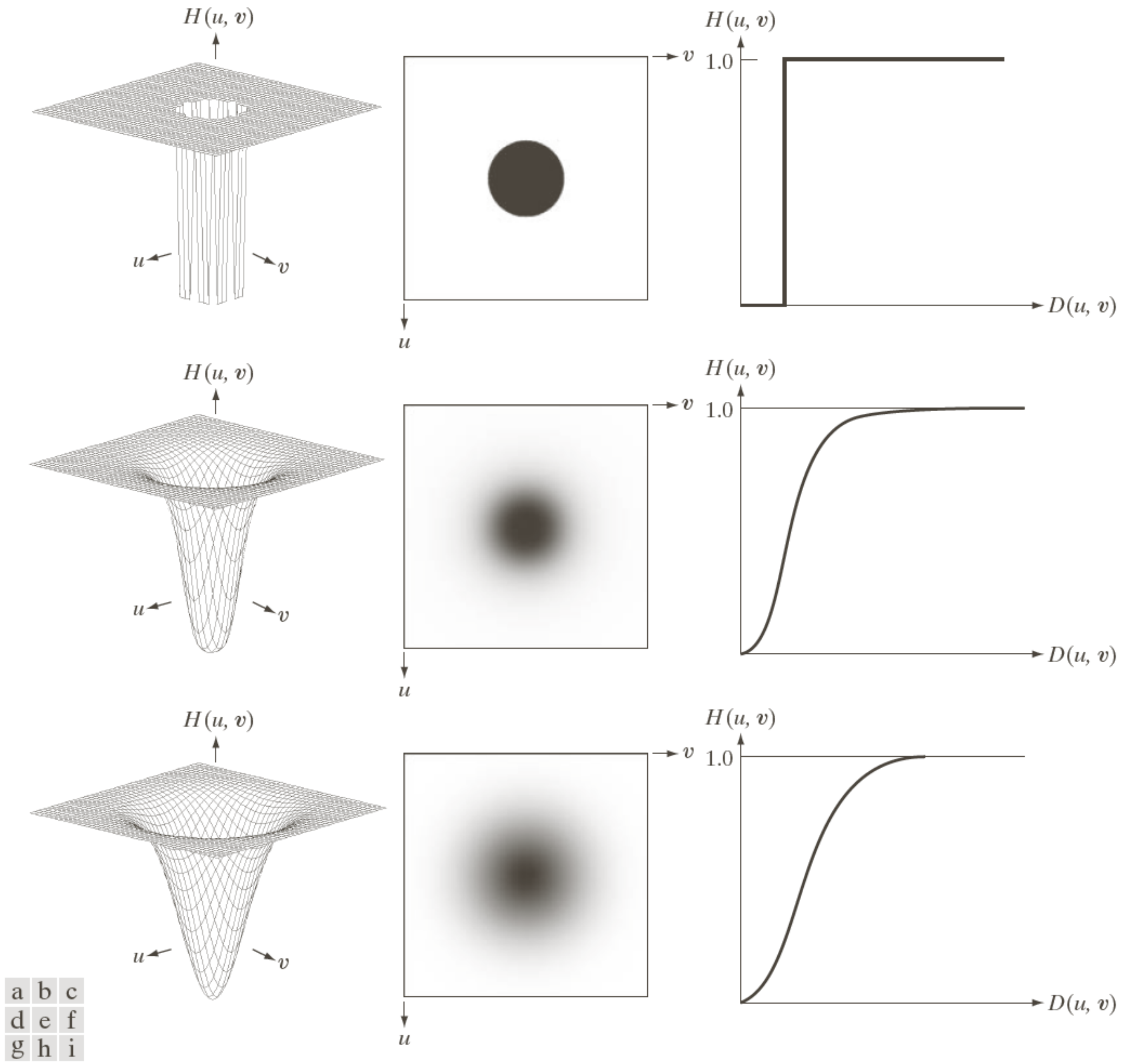


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Image Sharpening: High-Pass Filters

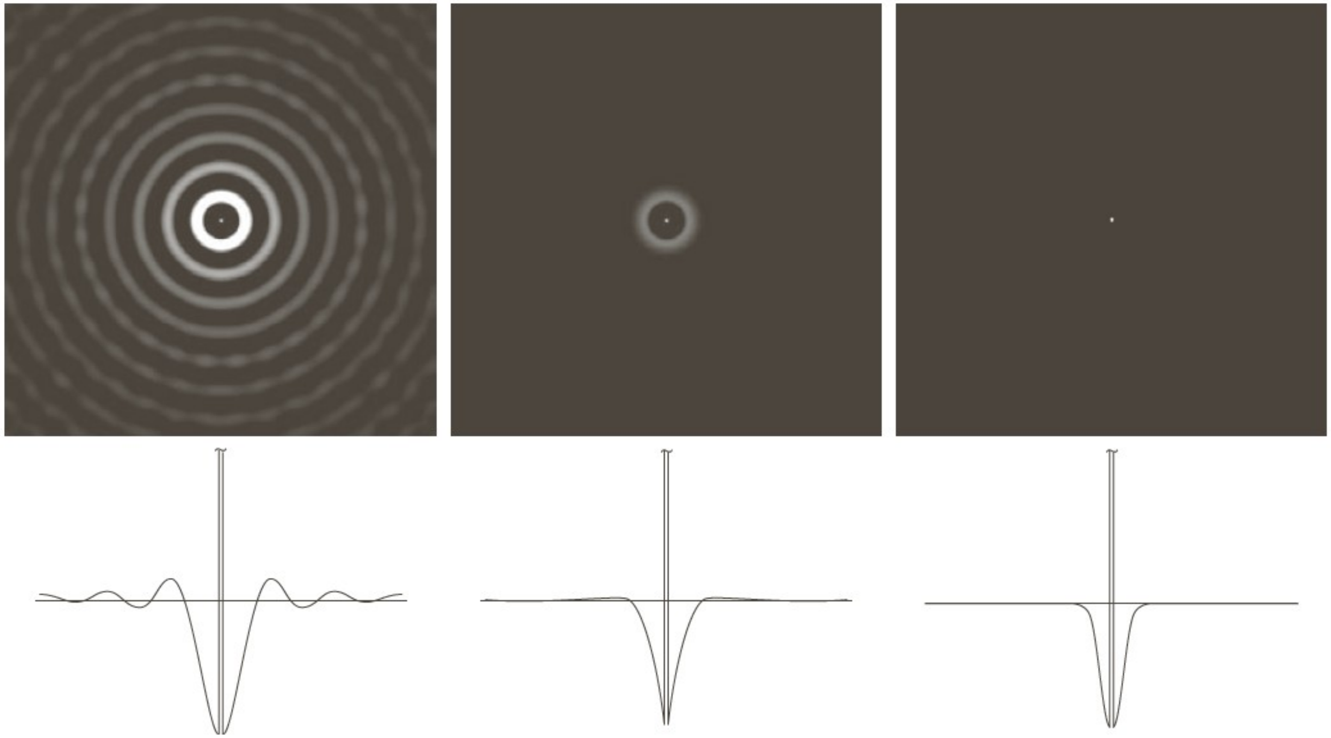
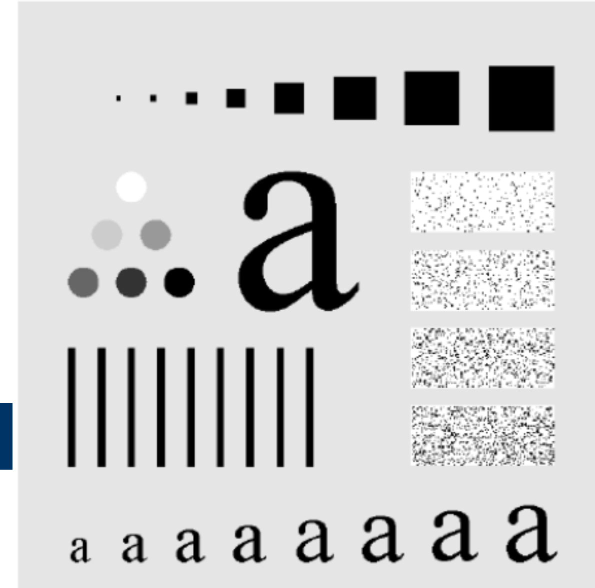


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Ideal High-Pass Filter (IHPF)



a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60,$ and 160.

Butterworth High-Pass Filter (BHPF)

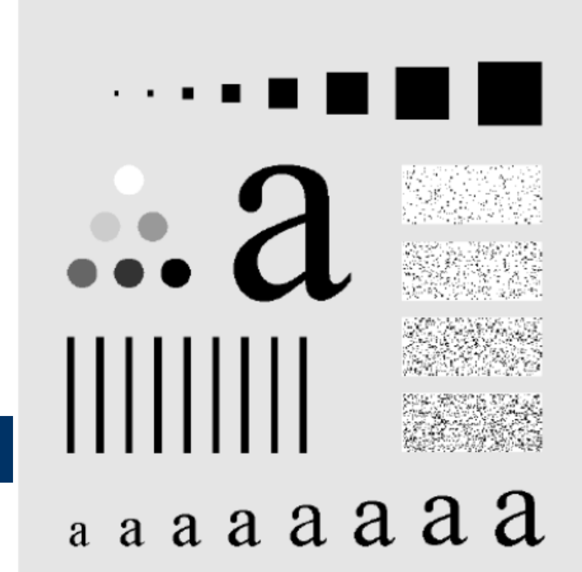
$$H(u,v) = \frac{1}{1 + \left[D_0 / D(u,v) \right]^{2n}}$$

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

D_0 : Cut-off Frequency

n : Order of BLPF

Butterworth High-Pass Filter (BHPF)



a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

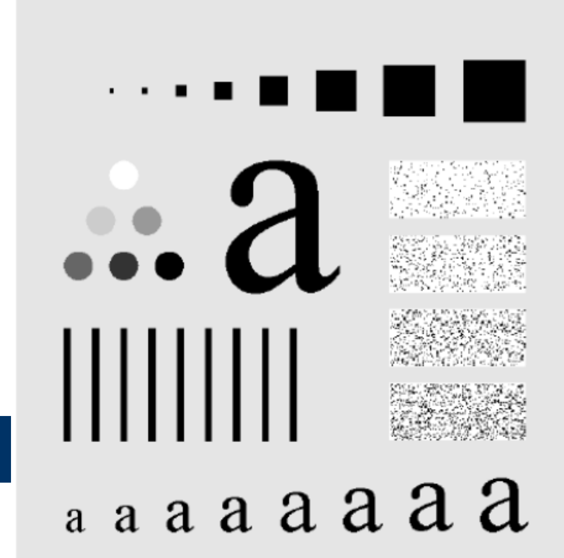
Gaussian High Pass Filter (GHPF)

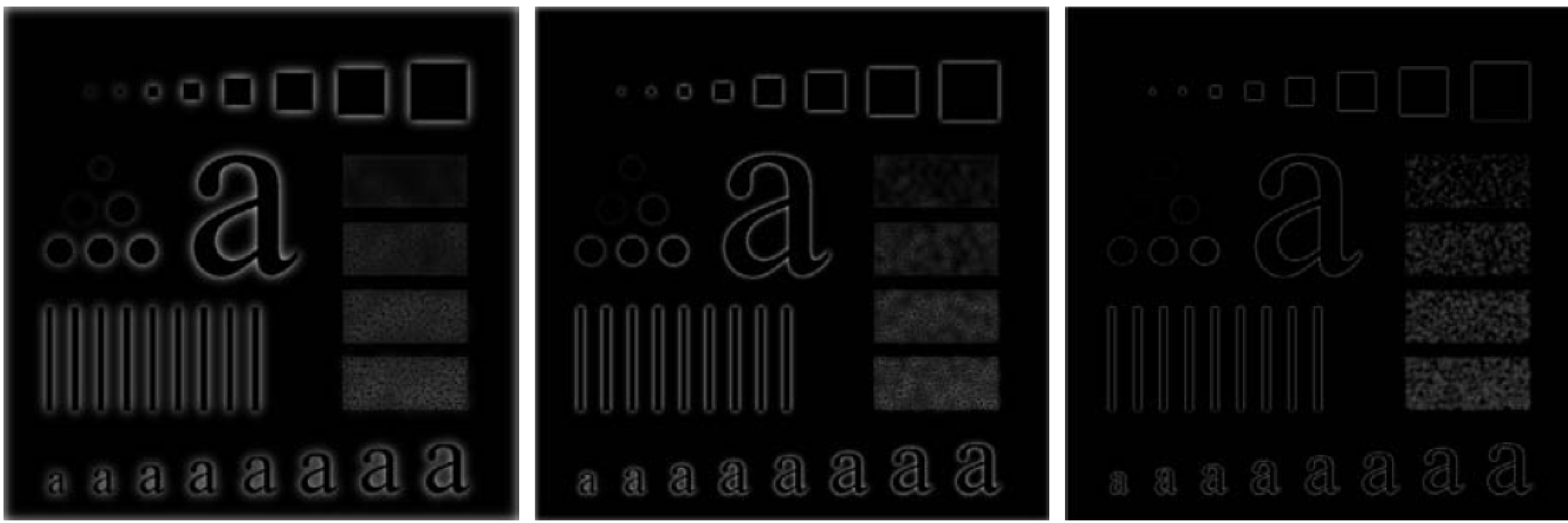
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$$

D_0 : Cutt - off Frequency

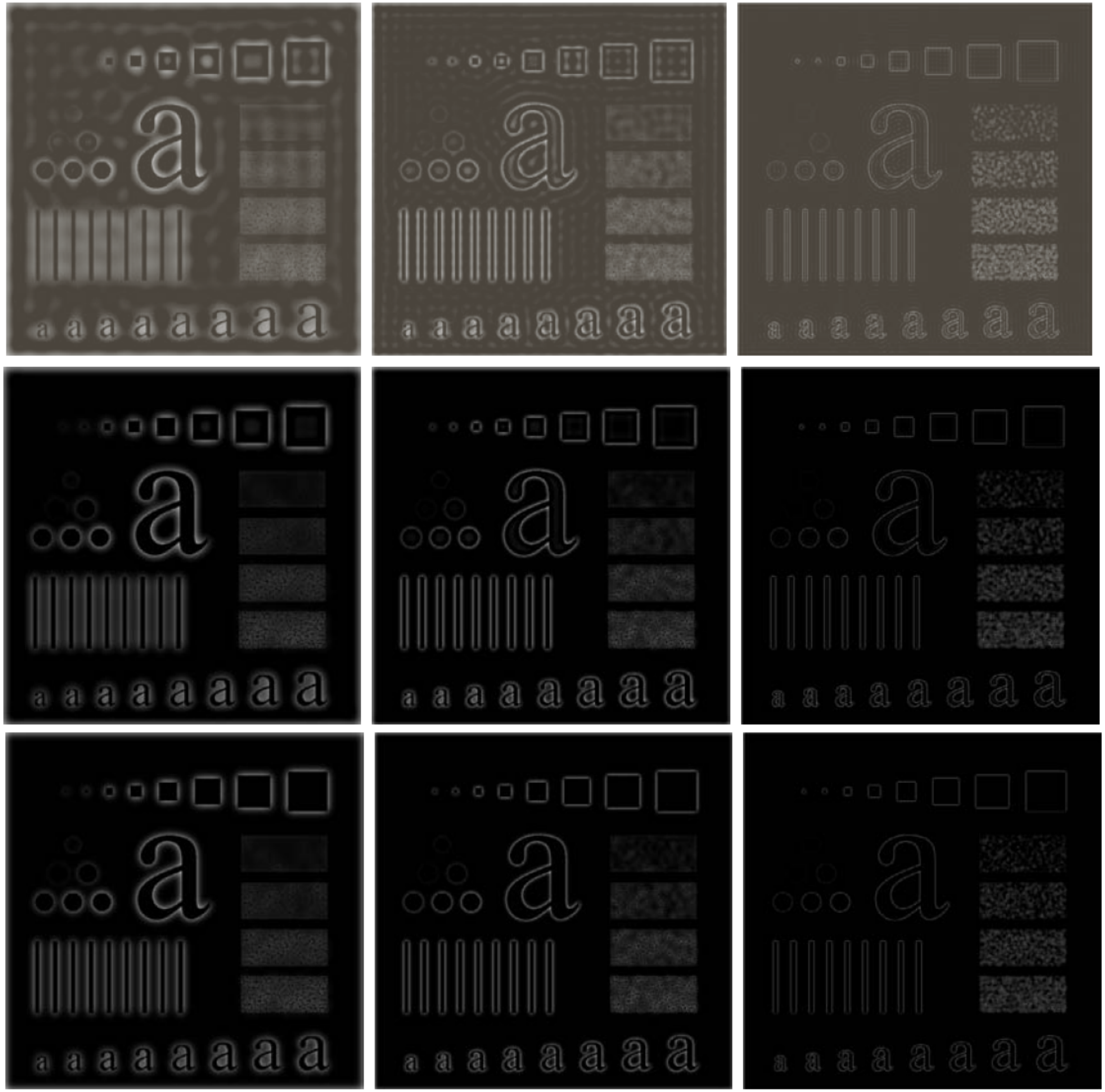
Gaussian High Pass Filter (GHPF)

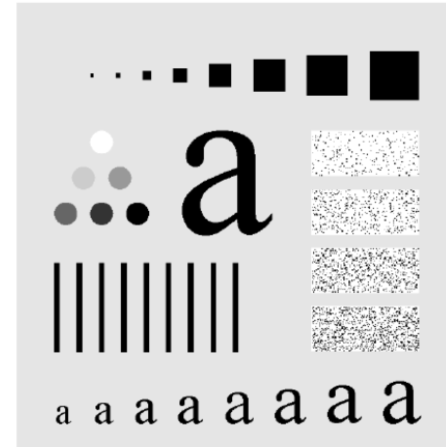




a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.





IHPF

BHPF

GHPF

Radius:

30, 60, 160

Image Sharpening: High-Pass Filters

TABLE 4.5

Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

BHPF: n=4, D0=50

HPF: Finger Print

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Thank you

Filter Design

Lecture 23: 17-Sep-12

Dr. P P Das

$$\nabla^2 f = \frac{\partial^2 f}{\partial t^2} + \frac{\partial^2 f}{\partial z^2}$$

Laplacian in Spatial Domain

- Laplacian
  - Isotropic
  - Rotation Invariant

0	1	0
1	−4	1
0	1	0

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right]$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial t^2} + \frac{\partial^2 f}{\partial z^2}$$

Laplacian in Frequency Domain

$$\Im\{f(t,z)\} = F(\mu,\nu) = \int_{\infty}^{\infty}\int_{\infty}^{\infty} f(t,z)e^{-j2\pi\mu t}e^{-j2\pi\mu u}dtdu$$

$$f(t,z) = \Im^{-1}\{F(\mu,\nu)\} = \int_{\infty}^{\infty}\int_{\infty}^{\infty} F(\mu,\nu)e^{j2\pi\mu t}e^{j2\pi\mu\nu}d\mu d\nu$$

$$\frac{\partial^2 f}{\partial t^2} = (j2\pi\mu)^2 \int_{\infty}^{\infty}\int_{\infty}^{\infty} F(\mu,\nu)e^{j2\pi\mu t}e^{j2\pi\mu\nu}d\mu d\nu$$

$$= -4\pi^2\mu^2\Im^{-1}\{F(\mu,\nu)\}$$

$$\Im\{\frac{\partial^2 f}{\partial t^2}\} = -4\pi^2\mu^2 F(\mu,\nu) \qquad \Im\{\frac{\partial^2 f}{\partial z^2}\} = -4\pi^2\nu^2 F(\mu,\nu)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial t^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\Im\{\nabla^2 f\} = -4\pi^2(\mu^2 + \nu^2)F(\mu,\nu)$$

Laplacian in Frequency Domain

$$H(u,v) = -4\pi^2(u^2 + v^2)$$

With respect to center of frequency rectangle:

$$H(u,v) = -4\pi^2\left[(u - P/2)^2 + (v - Q/2)^2\right]$$

$$= -4\pi^2 D^2(u,v)$$

Laplacian of an image:

$$\nabla^2 f(x,y) = \Im^{-1}\{H\{u,v)F(u,v)\}$$

Laplacian in Frequency Domain

- Enhancement Eq: $g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right]$

  $c = -1$
- Scales of $f(x,y)$ and $\nabla^2 f(x,y)$ as computed by DFT differ widely due to the DFT process
- Normalize $f(x,y)$ to [0,1] before DFT
- Normalize $\nabla^2 f(x,y)$ to [-1,1]

Laplacian in Frequency Domain

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Comparative Laplacian in Spatial & Frequency Domains

Unsharp Mask, Highboost Filtering & High-Frequency-Emphasis Filtering

- In spatial domain:

$$g_{mask}(x, y) = f(x, y) - \overline{f}(x, y)$$

$$g(x, y) = f(x, y) + k * g_{mask}(x, y)$$

$$k = 1 : \text{Unsharp Masking}$$

$$k > 1 : \text{Highboost Filtering}$$

$$k < 1 : \text{De-emphasized Unsharp Masking}$$

Unsharp Mask, Highboost Filtering & High-Frequency-Emphasis Filtering

- In frequency domain:

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

$$f_{LP}(x,y) = \Im^{-1}\left[H_{LP}(u,v)F(u,v)\right]$$

$$\begin{aligned} g(x,y) &= f(x,y) + k * g_{mask}(x,y) \\ &= \Im^{-1}\{\left[1 + k * \left[1 - H_{LP}(u,v)\right]\right]F(u,v)\} \\ &= \Im^{-1}\{\underbrace{\left[1 + k * H_{HP}(u,v)\right]}_{High-Frequency\ Emphasis\ Filter} F(u,v)\} \end{aligned}$$

Unsharp Mask, Highboost Filtering & High-Frequency-Emphasis Filtering

- In frequency domain:

$$g(x,y) = \Im^{-1}\{\underbrace{\left[k_1 + k_2 * H_{HP}(u,v)\right]}_{High\text{-}Frequency\ Emphasis\ Filter} F(u,v)\}$$

$k_1 \geq 0$: Controls the offset from origin

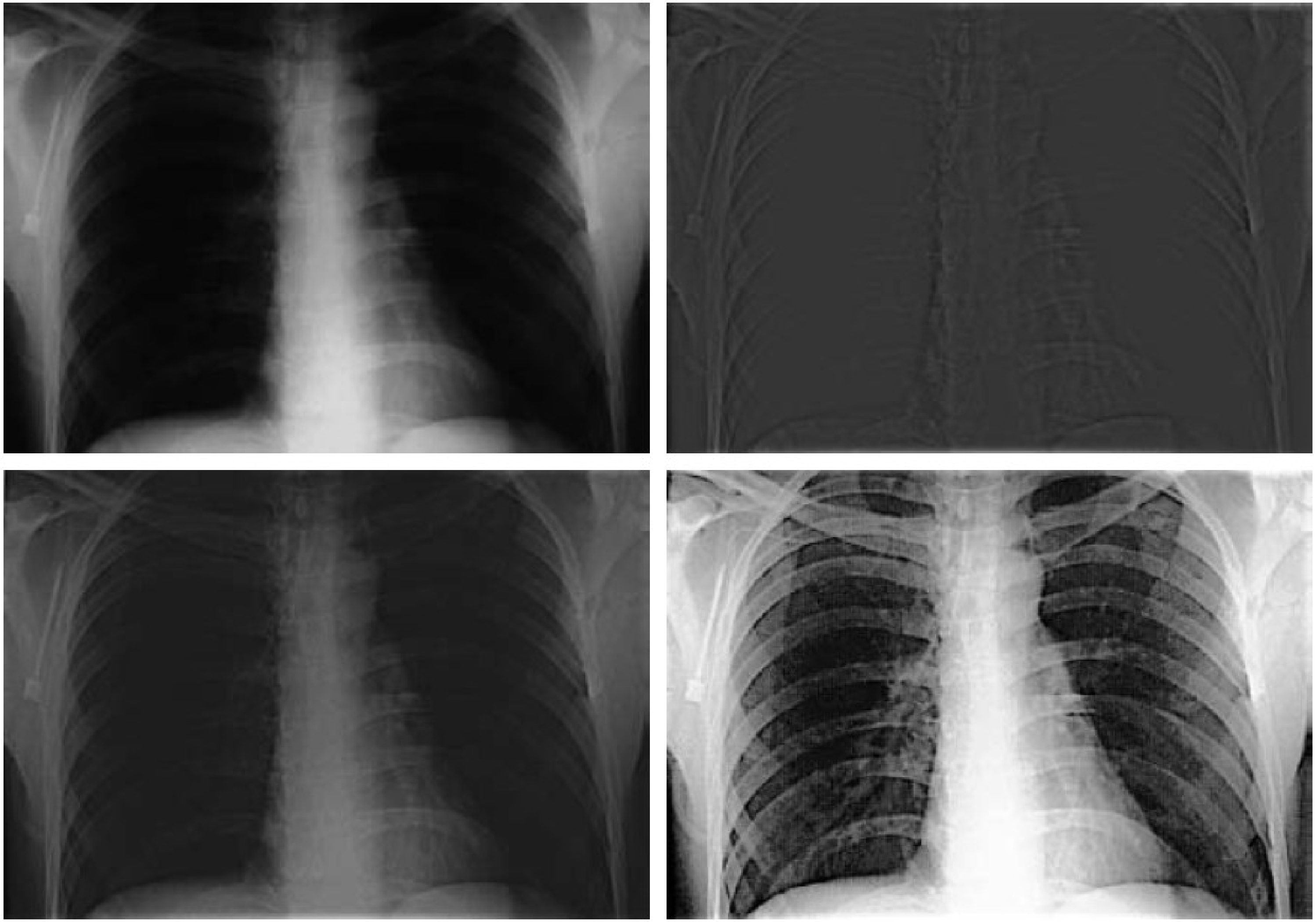
$k_2 \geq 0$: Controls the contribution of high frequencies

Image: 416x596

D0=40 (5% of short side of padded image)

k1=0.5

k2=0.75



a b
c d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

Homomorphic Filtering

- Illumination-Reflectance Model in frequency domain
- Illumination Component
  - Slow Spatial Variations
  - Attenuate contributions by illumination
- Reflectance Component
  - Varies abruptly – junctions of dissimilar objects
  - Amplify contributions by reflectance
- Simultaneous dynamic range compression & contrast enhancement

Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$$z(x,y) = \ln f(x,y)$$

$$= \ln i(x,y) + \ln r(x,y)$$

$$\Im\{z(x,y)\} = \Im\{\ln f(x,y)\}$$

$$= \Im\{\ln i(x,y)\} + \Im\{\ln r(x,y)\}$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Homomorphic Filtering

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

$$S(u,v) = H(u,v)Z(u,v)$$

$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

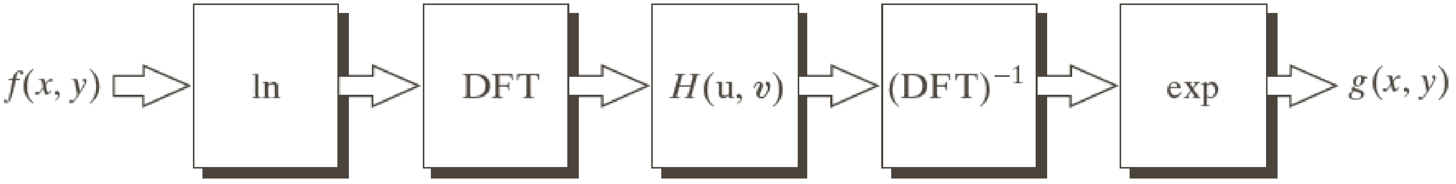
$$s(x,y) = \Im^{-1}\{S(u,v)\}$$

$$= \Im^{-1}\{H(u,v)F_i(u,v)\} + \Im^{-1}\{H(u,v)F_r(u,v)\}$$

$$= i'(x,y) + r'(x,y)$$

Homomorphic Filtering

FIGURE 4.60
Summary of steps in homomorphic filtering.



$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)} e^{r'(x,y)} = i_0(x,y) r_0(x,y)$$

Homomorphic Filtering

- Illumination Component
  - Slow Spatial Variations
  - Low Frequencies → log of illumination
  - $\gamma_L < 1$ attenuate contributions by illumination
- Reflectance Component
  - Varies abruptly – junctions of dissimilar objects
  - High frequencies → log of reflectance
  - $\gamma_H > 1$ amplify contributions by reflectance
- Simultaneous dynamic range compression & contrast enhancement

Homomorphic Filtering

$$H(u,v) = (\gamma_L - \gamma_H)\left[1 - e^{-c\left[D^2(u,v)/D_0^2\right]}\right] + \gamma_L$$

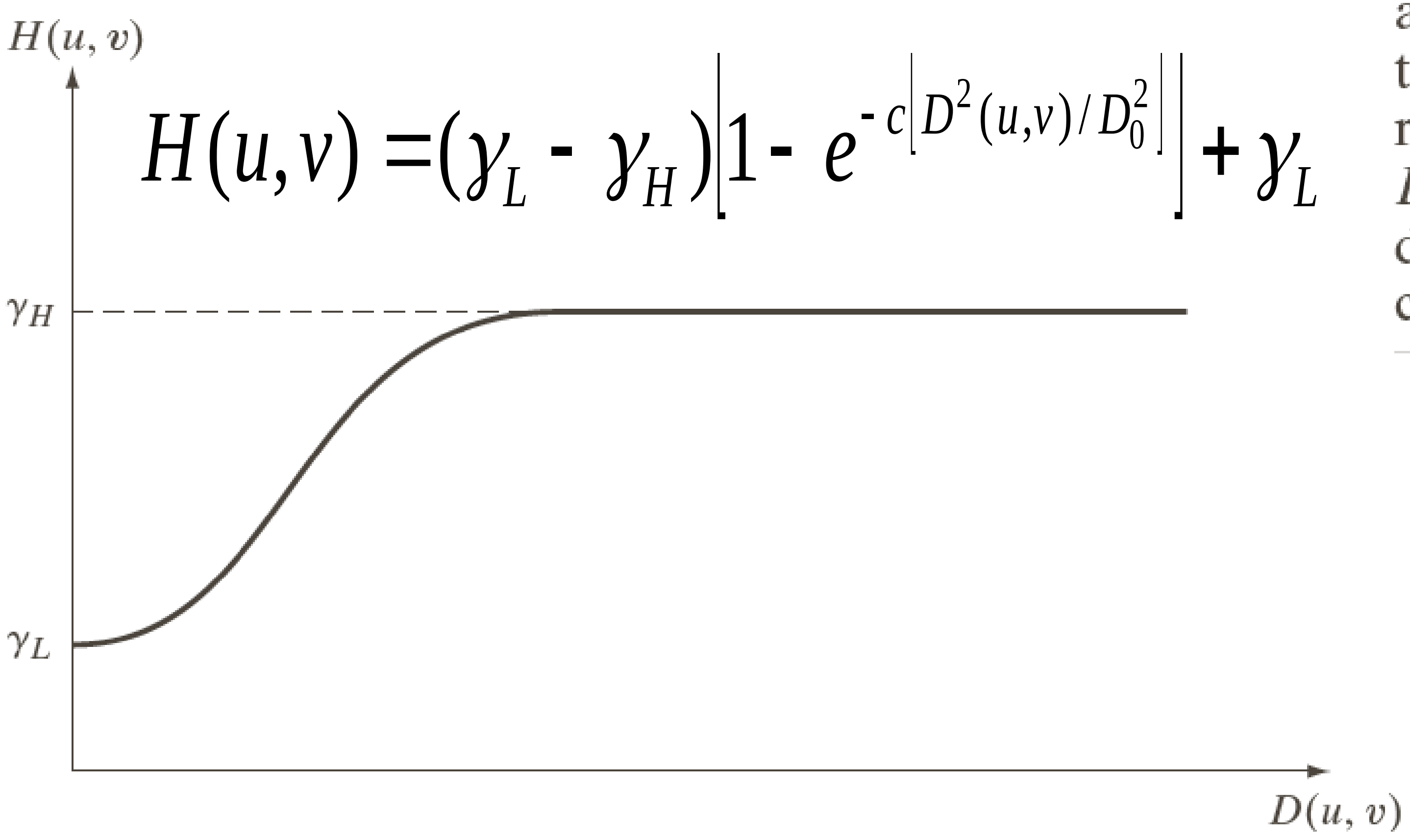
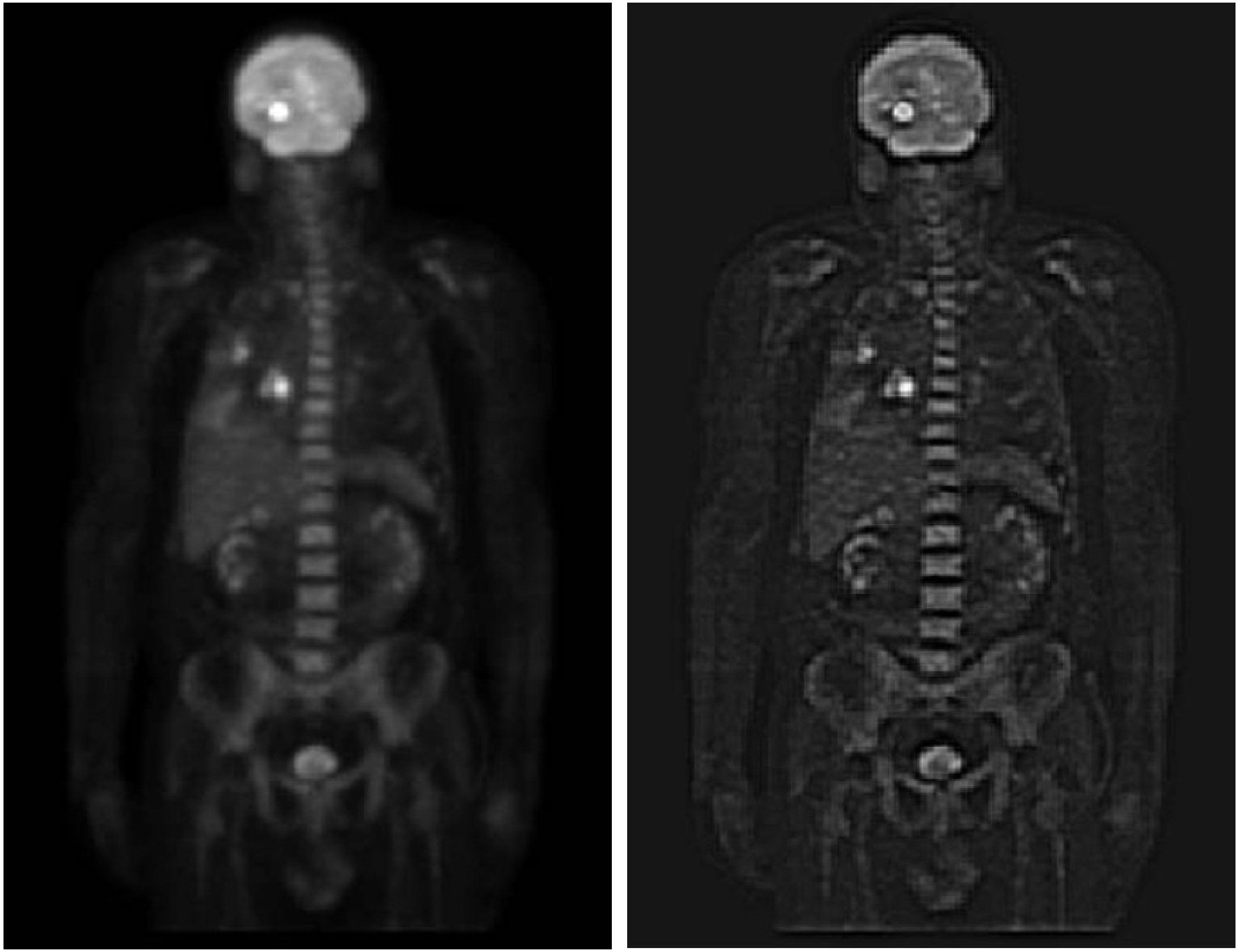


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.



a b

FIGURE 4.62 (a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

Image: 1162x746
γL=0.25, γH=2, c=1, D0=80

Band-reject & Band-pass Filters

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Band-reject & Band-pass Filters

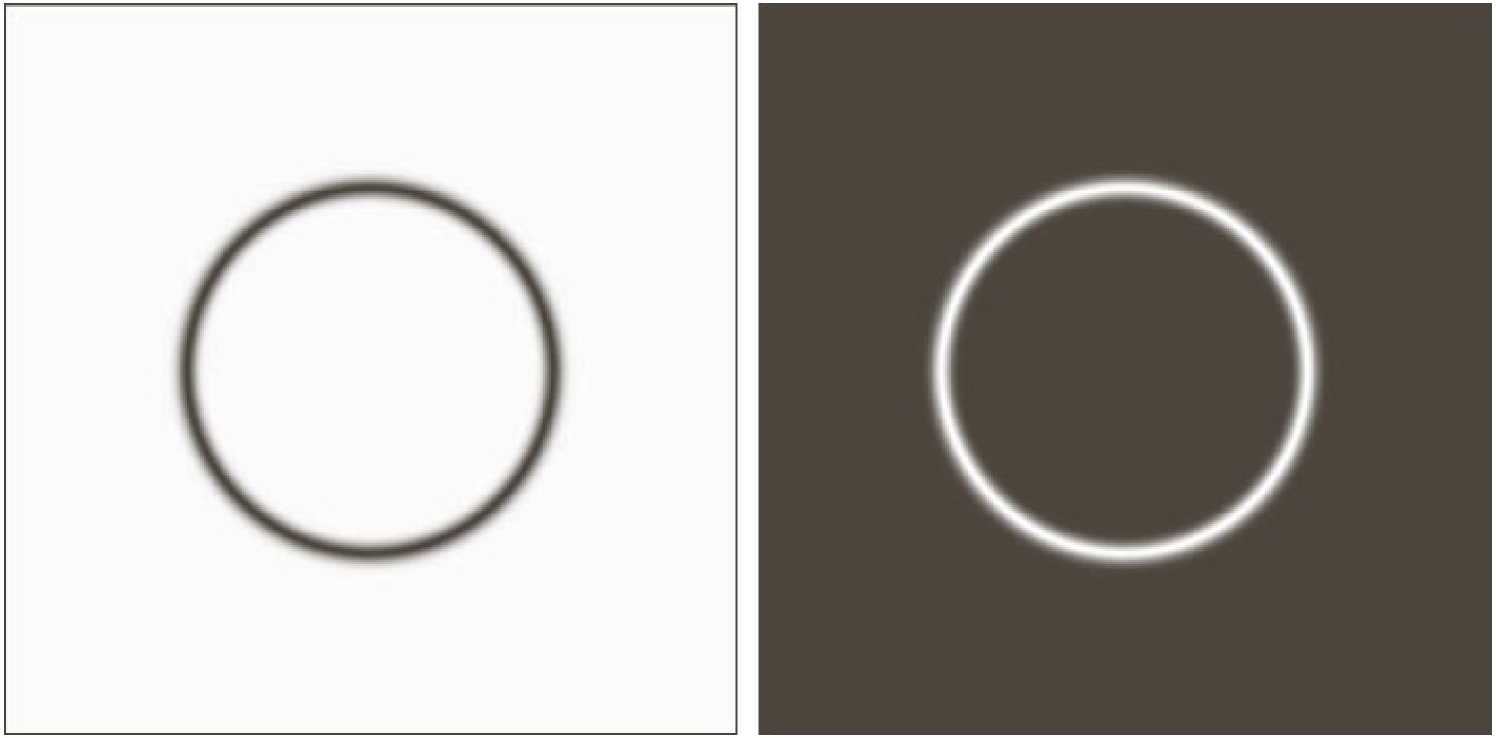
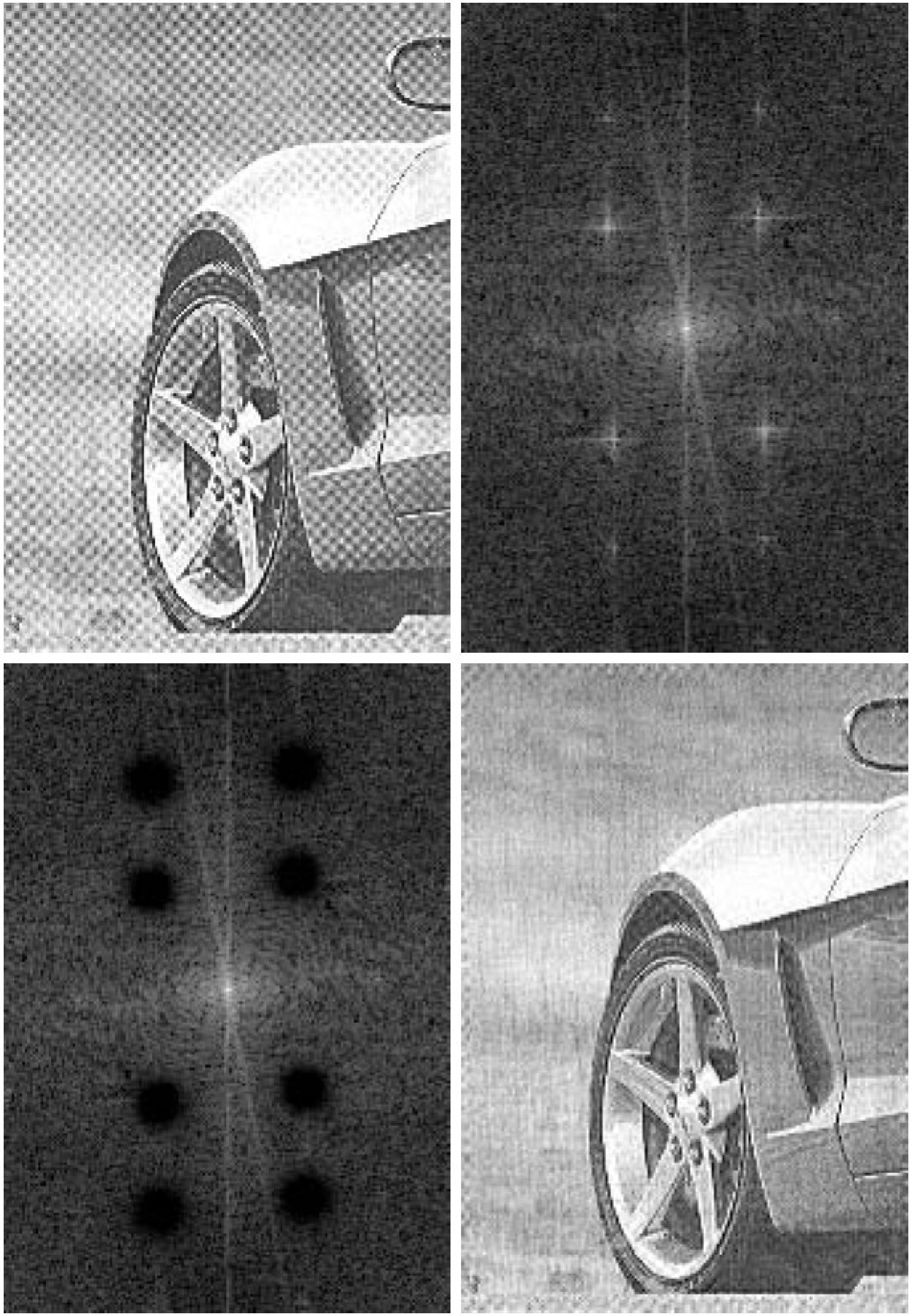


FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

D0=80, n=4

Notch Filters – Narrow Filtering



a	b
c	d

FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

Notch Filters – Narrow Filtering

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$

$$D_k(u,v) = \left[(u - M/2 - u_k)^2 + (v - N/2 - v_k)^2 \right]^{1/2}$$

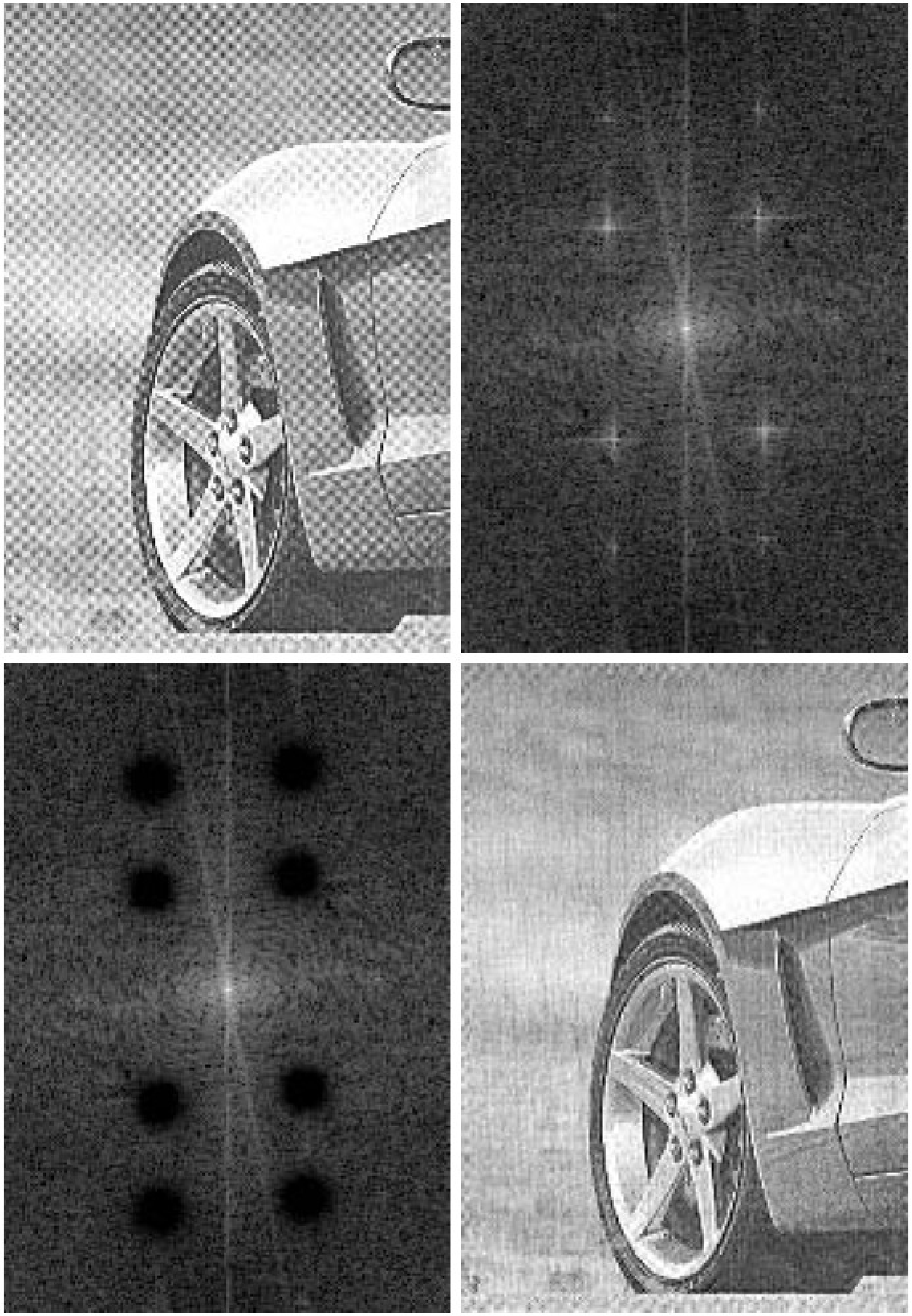
$$D_{-k}(u,v) = \left[(u - M/2 + u_k)^2 + (v - N/2 + v_k)^2 \right]^{1/2}$$

Butterworth Notch Reject Filters

$$H_{NR}(u,v) = \prod_{k=1}^{3}\left[\frac{1}{1+\left[D_{0k}/D_k(u,v)\right]^{2n}}\right]\left[\frac{1}{1+\left[D_{0k}/D_{-k}(u,v)\right]^{2n}}\right]$$

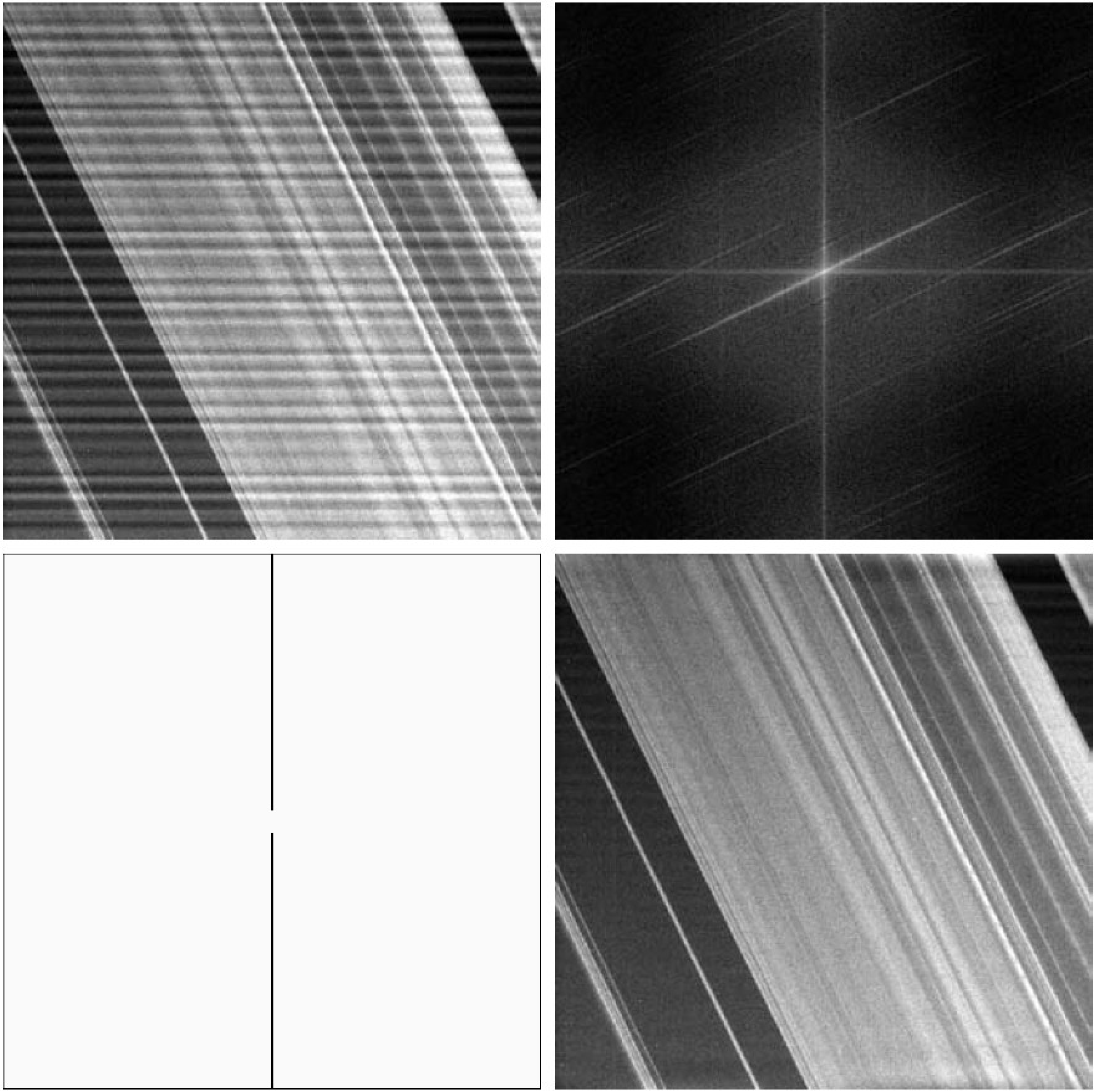
$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

D0=80, n=4



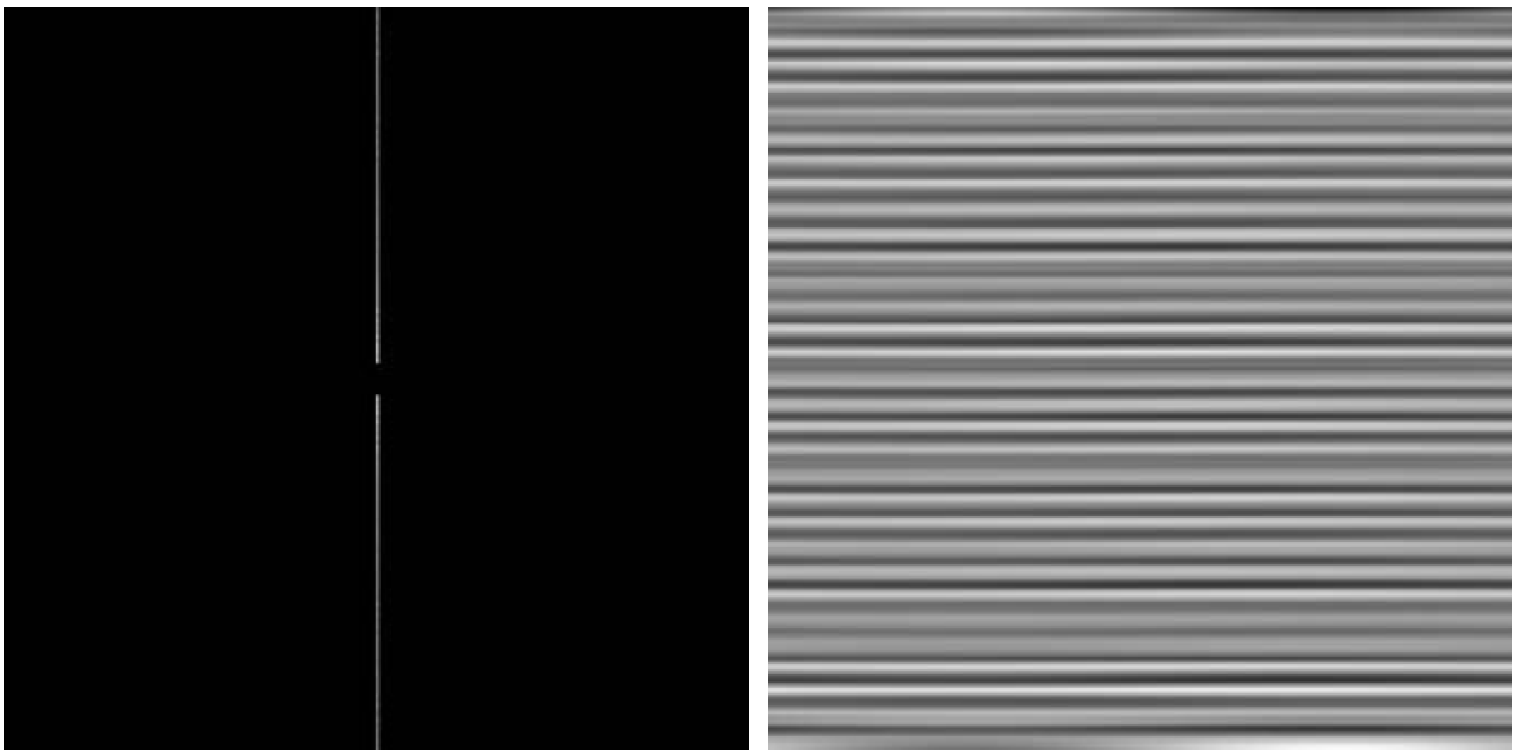
a	b
c	d

FIGURE 4.64
(a) Sampled newspaper image showing a moiré pattern.
(b) Spectrum.
(c) Butterworth notch reject filter multiplied by the Fourier transform.
(d) Filtered image.



a b
c d

FIGURE 4.65
(a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)



a b

FIGURE 4.66 (a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a). (b) Spatial pattern obtained by computing the IDFT of (a).

Thank you