

Digital Image Processing, 3rd ed.

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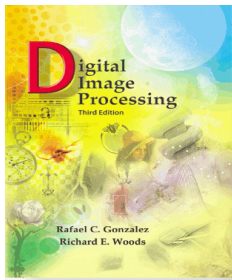
Woods

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Chapter 3

Intensity Transformations & Spatial Filtering

Intensity Transformations & Spatial Filtering



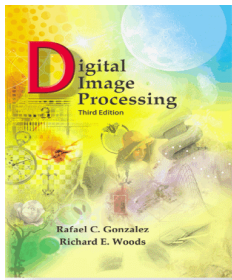
Chapter 3

Intensity Transformations & Spatial Filtering

- Spatial Domain
 - Apply spatial filters directly

$$g(x,y) = T[f(x,y)]$$

- Frequency / Transform Domain
 - Take the image to transform domain
 - Apply filter
 - Bring back image to spatial domain



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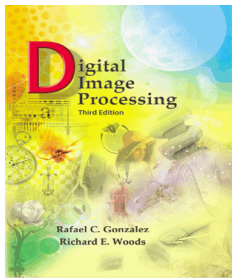
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Chapter 3

Intensity Transformations & Spatial Filtering

- Spatial Processing (Computationally efficient & less processing resources)
 - Intensity Transformation (point processing)
 - Contrast manipulation & Image thresholding
 - Spatial Filtering (neighbourhood processing)
 - Image sharpening



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Spatial Filtering

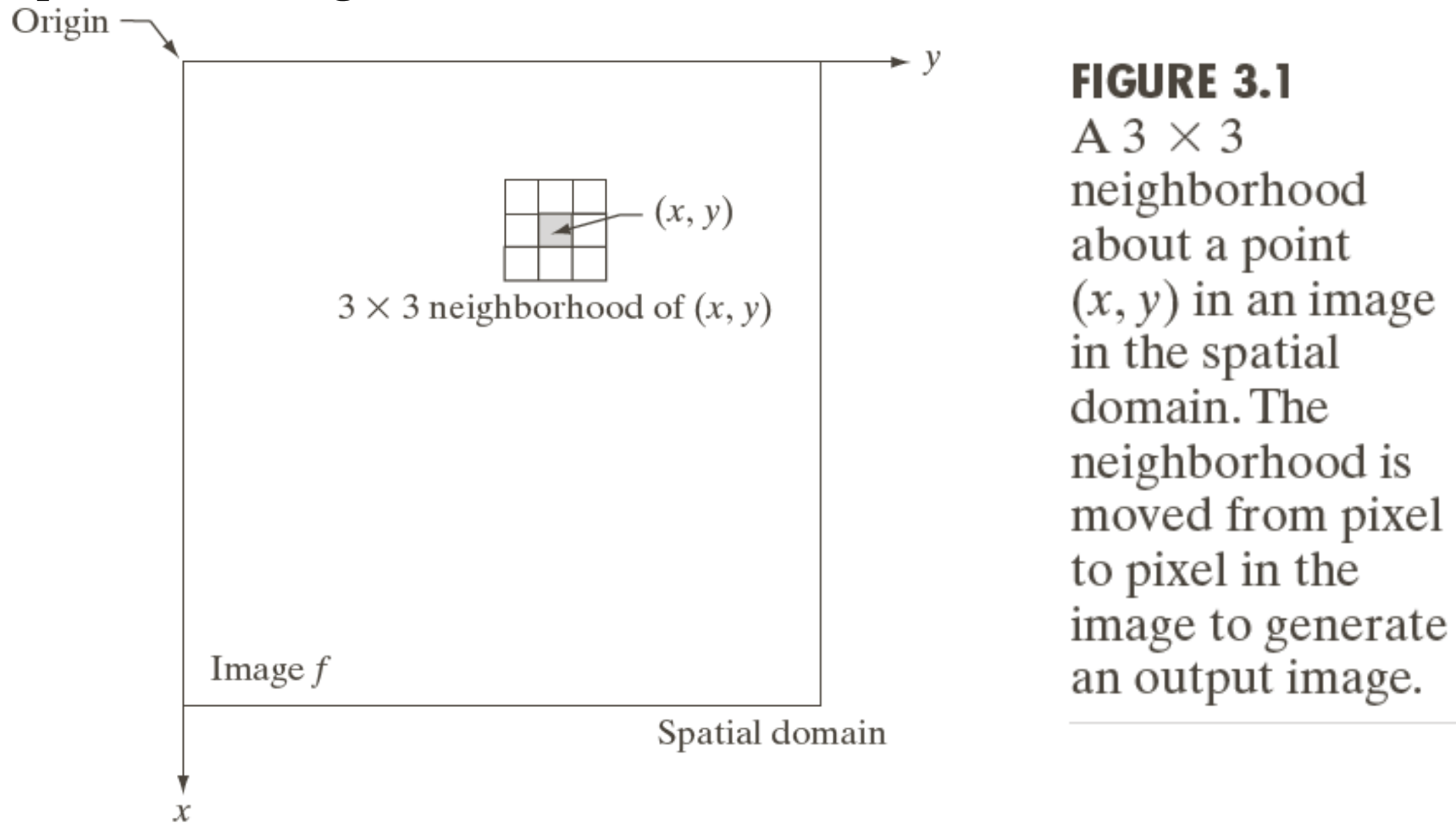
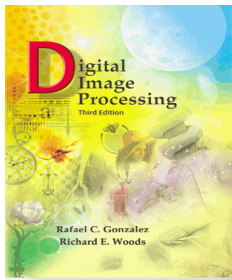


FIGURE 3.1

A 3×3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

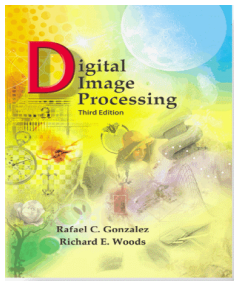


Chapter 3

Intensity Transformations & Spatial Filtering

- Spatial Filtering: 3x3
 - Spatial Mask
 - Kernel
 - Template
 - Window
- Intensity Transformation: 1x1
 - Grey-level Transformation
 - Mapping Transformation

$$s = T(r)$$



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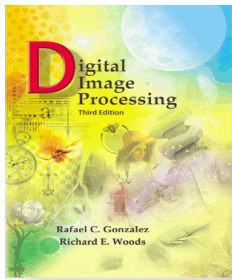
Intensity Transformations & Spatial Filtering

INTENSITY TRANSFORMATION

Applications: Enhancement & Segmentation

Contrast stretching function

Thresholding function

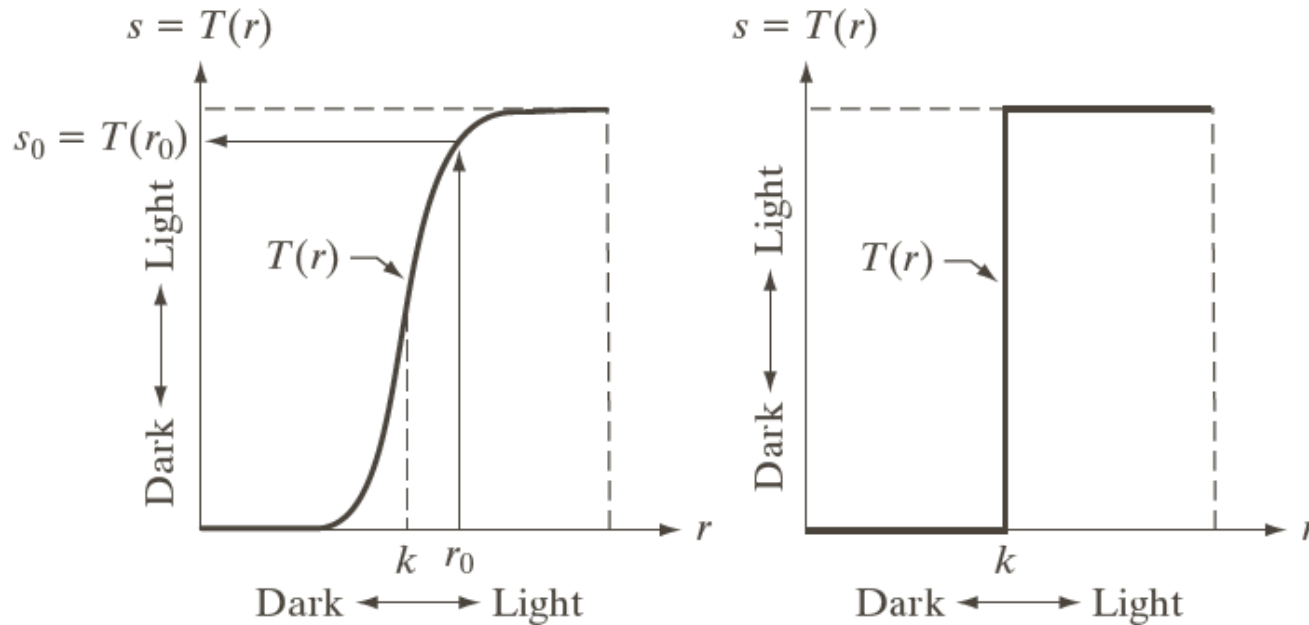


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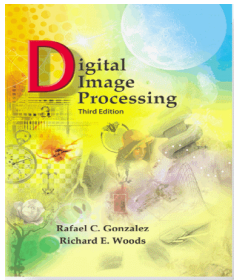
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Chapter 3 Intensity Transformations & Spatial Filtering



a b

FIGURE 3.2
Intensity transformation functions.
(a) Contrast-stretching function.
(b) Thresholding function.



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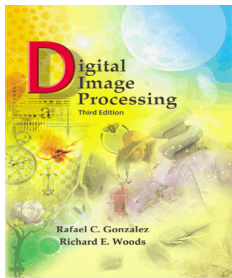
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- Image Enhancement
 - Process of manipulating an image so that the result is more suitable than the original for a specific application



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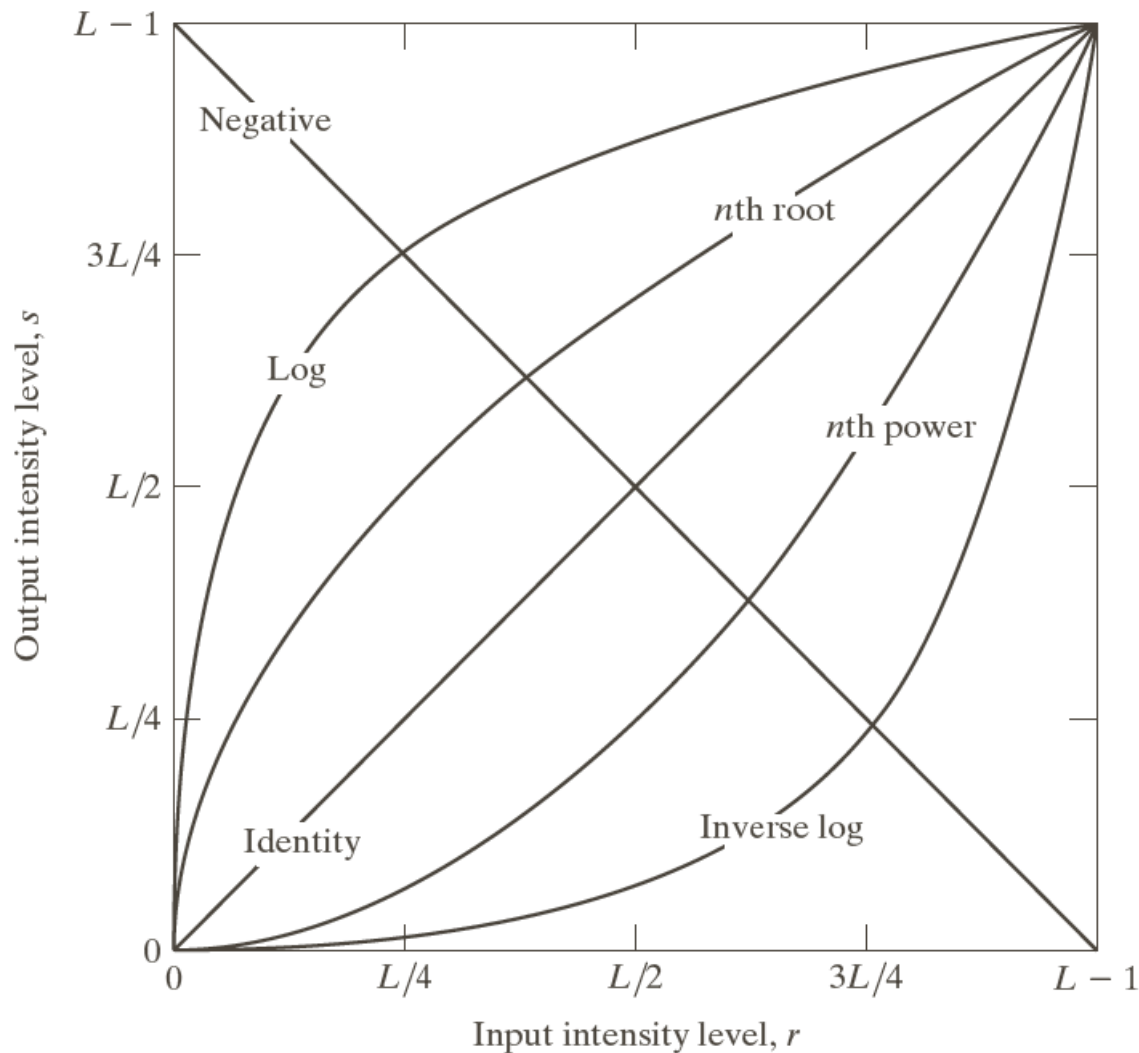
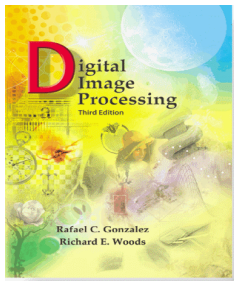


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.



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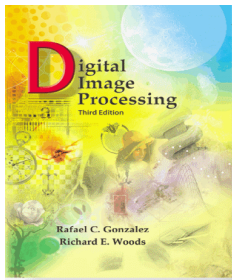
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- Image Negatives

$$s = L - 1 - r$$



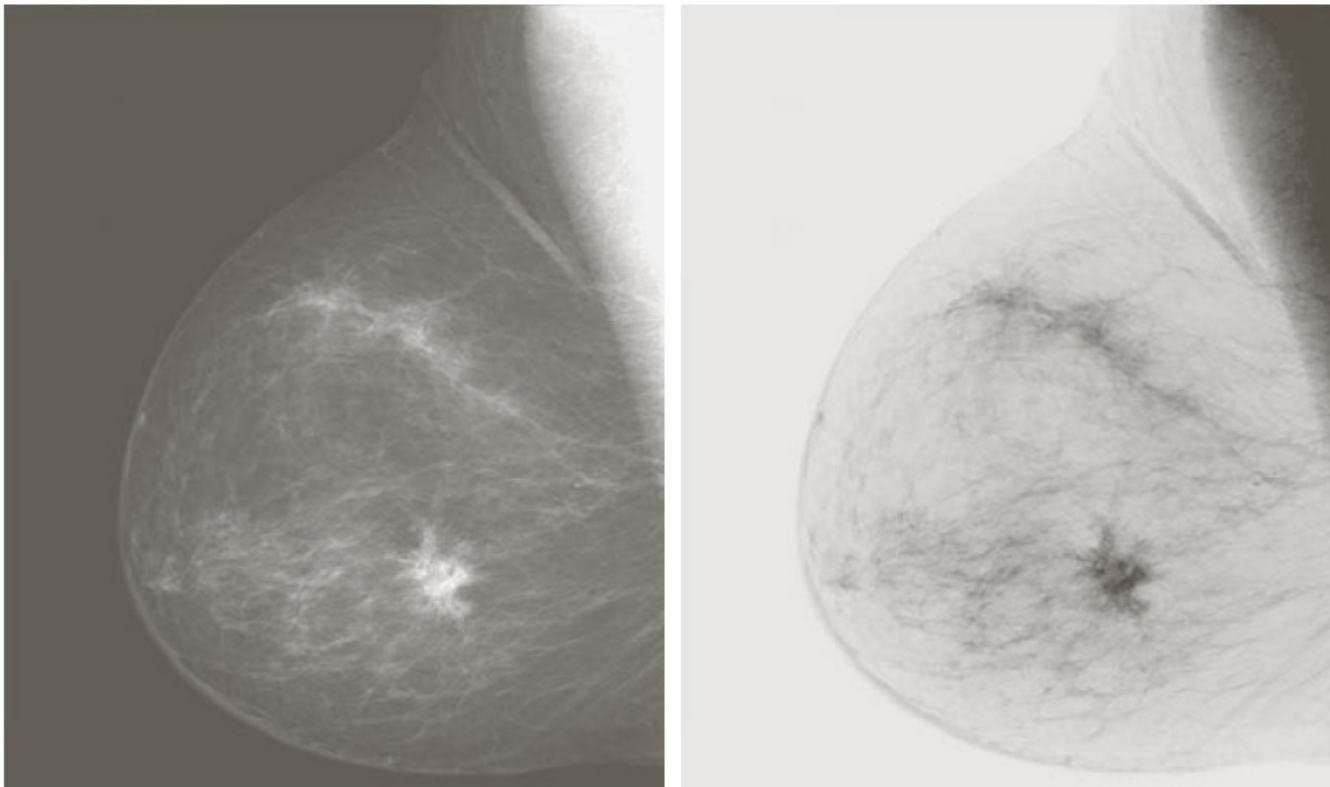
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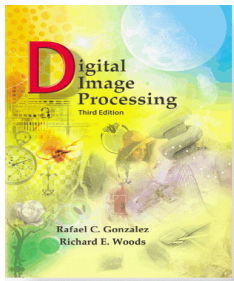
a b

FIGURE 3.4

(a) Original digital mammogram.

(b) Negative image obtained using the negative transformation in Eq. (3.2-1).

(Courtesy of G.E. Medical Systems.)



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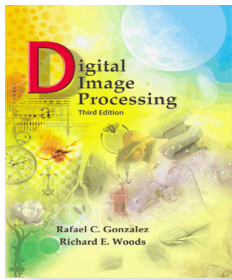
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- Log Transformations

$$s = c \log(1 + r)$$



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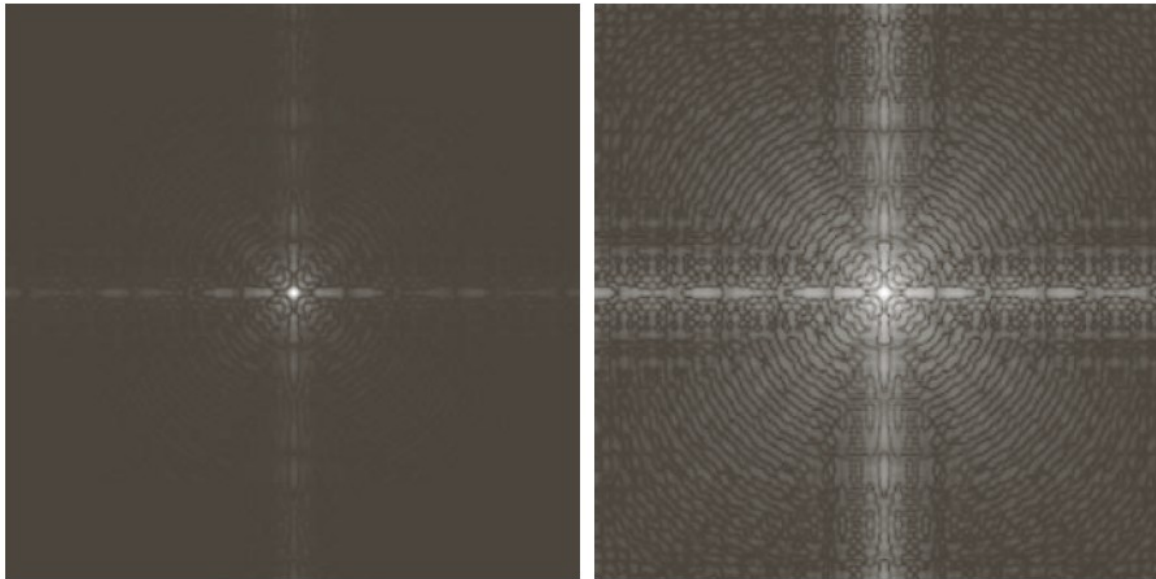
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a b

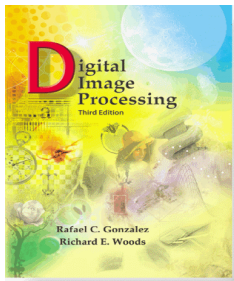
FIGURE 3.5

(a) Fourier spectrum.

(b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

0 to 1.5×10^6

0 to 6.2



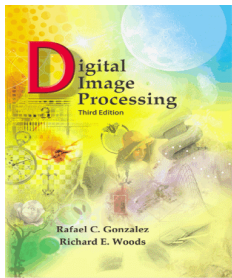
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Intensity Transformations & Spatial Filtering

- Power-Law (Gamma) Transformations

$$s = cr^{\gamma}$$

- Gamma Correction
 - The devices used to capture, print and display respond to power-law
 - The process to correct the power-law response phenomena



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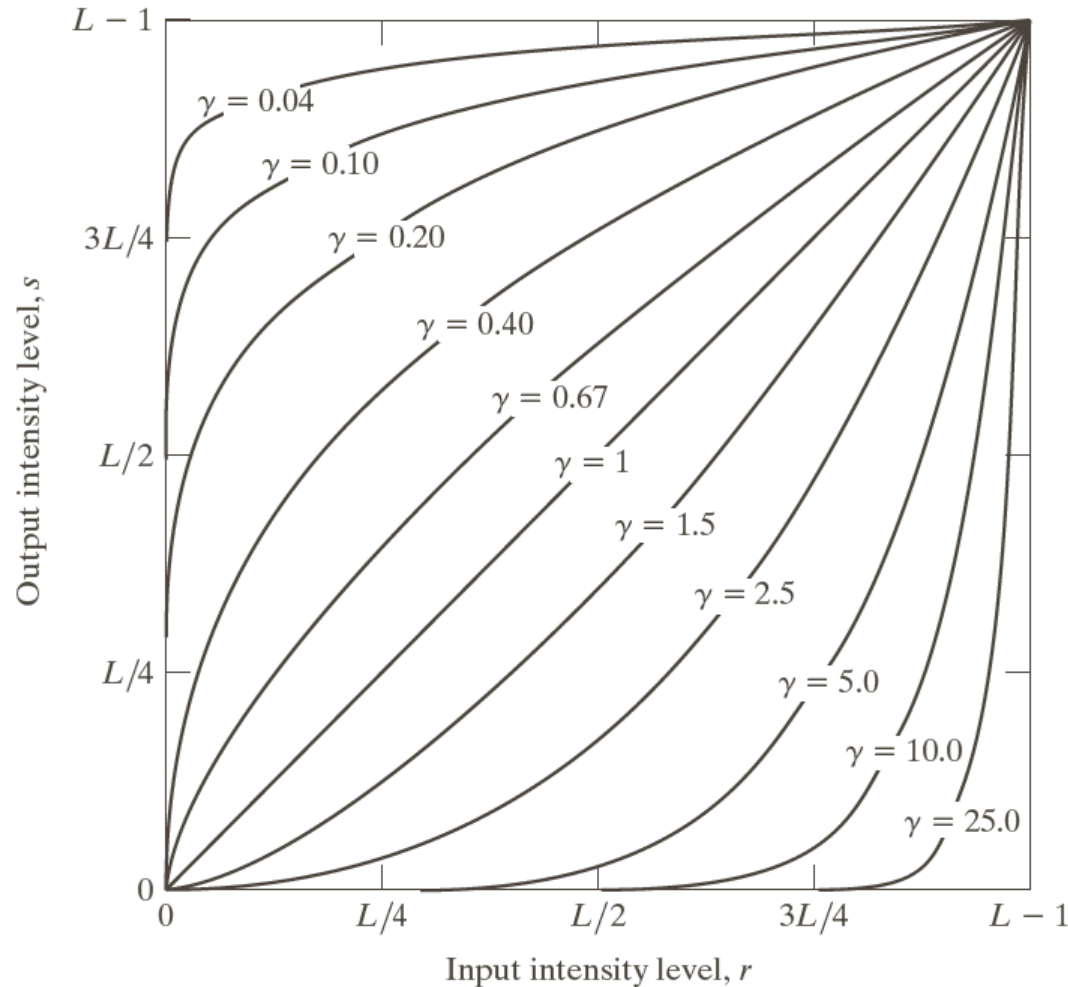
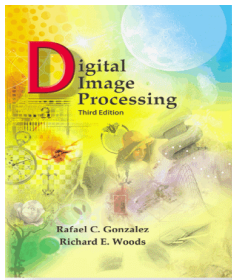


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.



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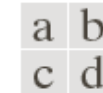
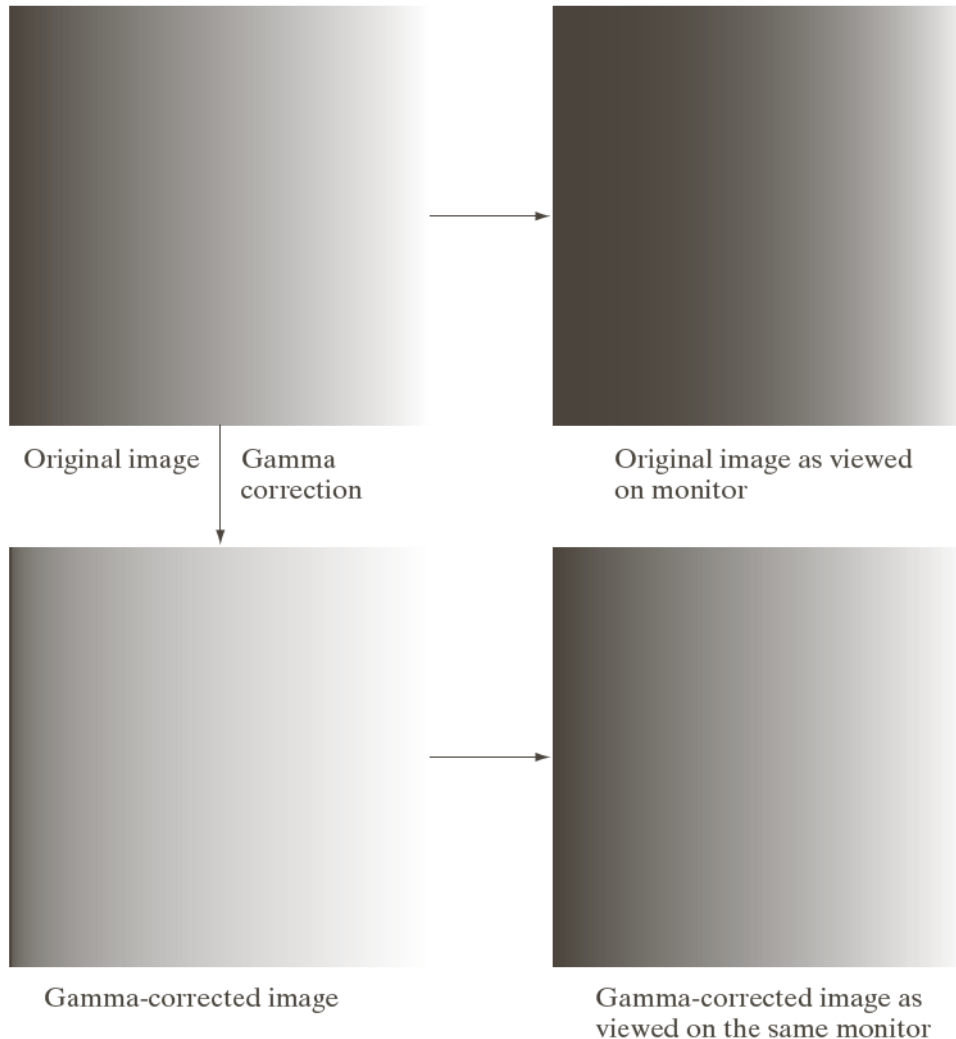
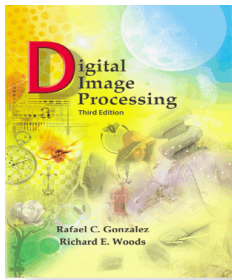


FIGURE 3.7

(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

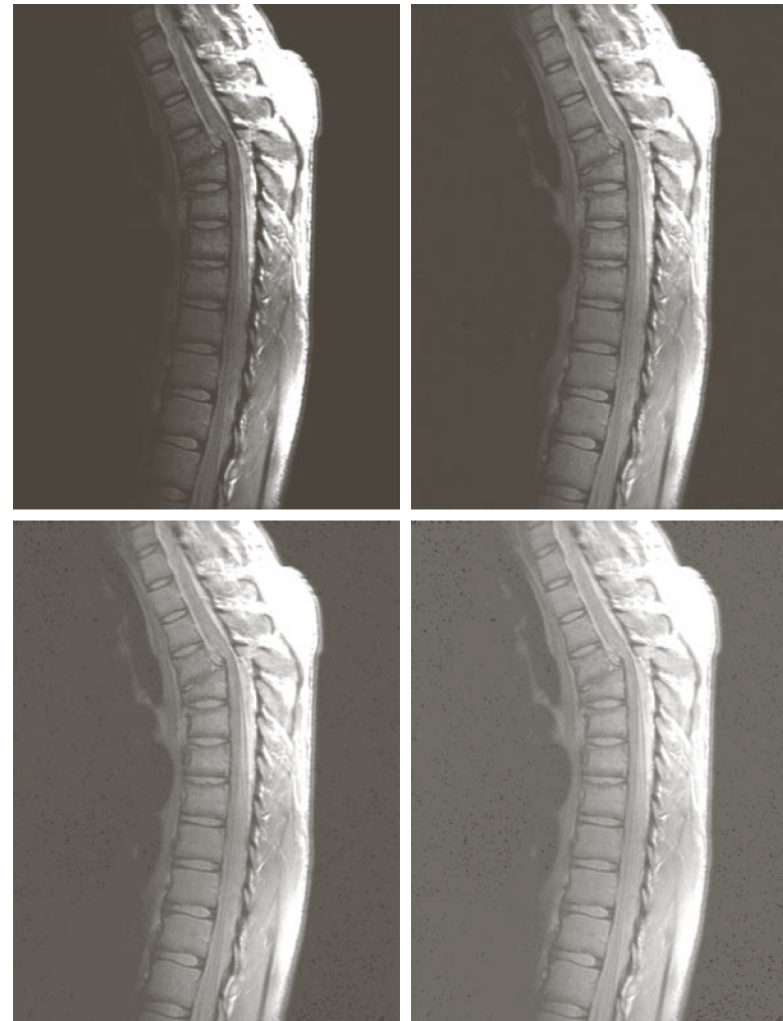


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a	b
c	d

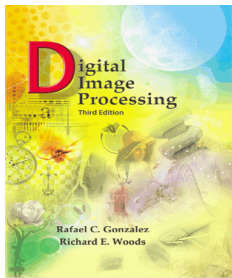
FIGURE 3.8

(a) Magnetic resonance image (MRI) of a fractured human spine.

(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and

$\gamma = 0.6, 0.4$, and 0.3 , respectively.

(Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



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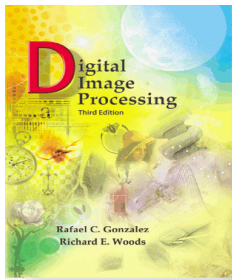
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a	b
c	d

FIGURE 3.9

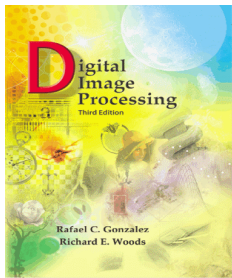
(a) Aerial image.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0$, and 5.0 , respectively. (Original image for this example courtesy of NASA.)



Chapter 3

Intensity Transformations & Spatial Filtering

- Piecewise Linear Transformation
 - Contrast Stretching
 - Poor illumination
 - Lack of dynamic range of image sensor
 - Wrong setting of lens aperture
 - Intensity Level Slicing
 - Bit-Plane Slicing



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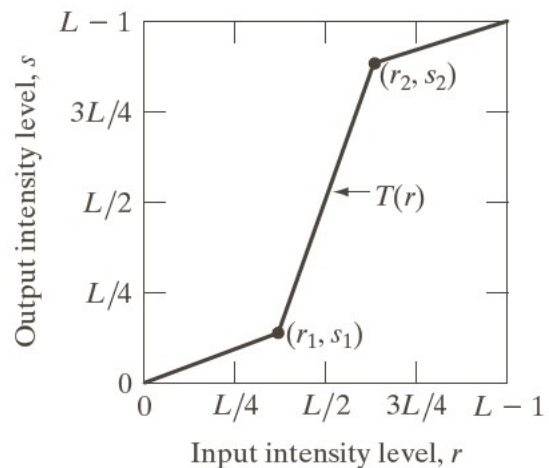


FIGURE 3.10

Contrast stretching.

(a) Form of transformation function.

(b) A low-contrast image.

(c) Result of contrast stretching.

(d) Result of thresholding.

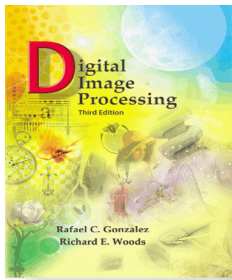
(Original image courtesy of Dr. Roger Heady,

Research School of Biological Sciences,

Australian National University,

Canberra,

Australia.)



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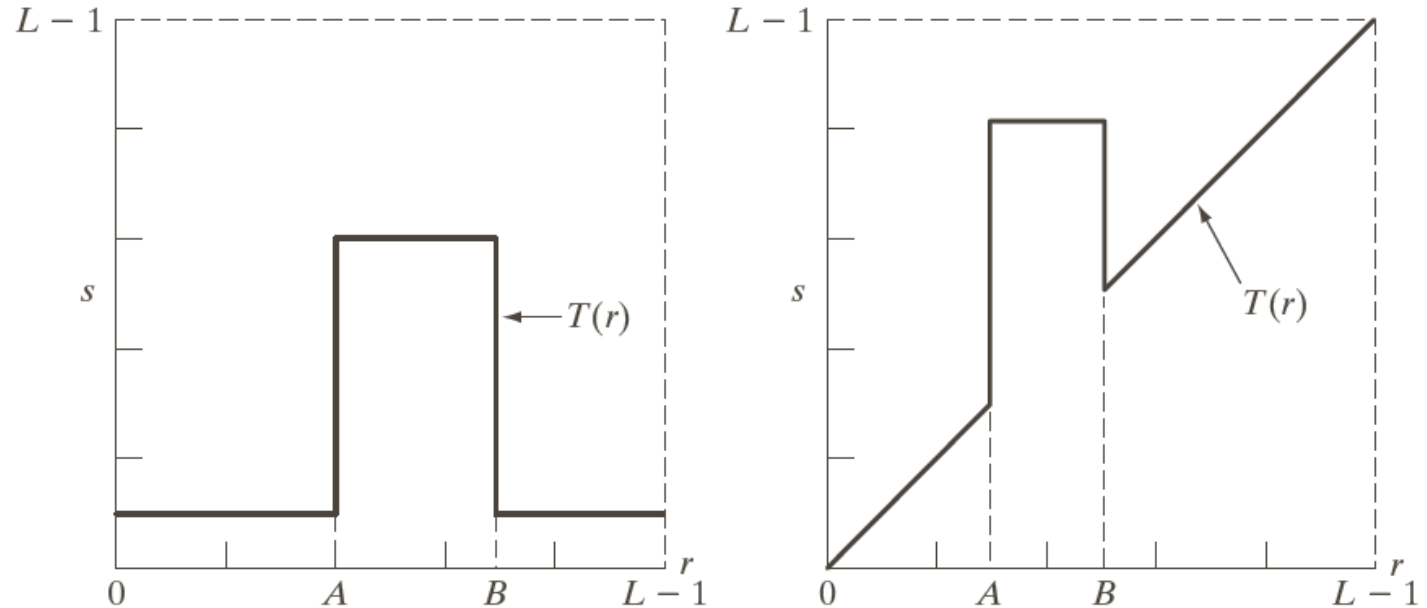
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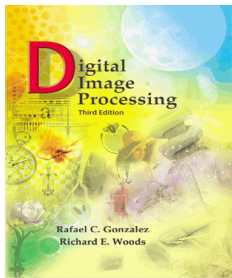
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a b

FIGURE 3.11 (a) This transformation highlights intensity range $[A, B]$ and reduces all other intensities to a lower level. (b) This transformation highlights range $[A, B]$ and preserves all other intensity levels.





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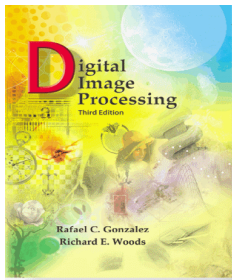
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a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)



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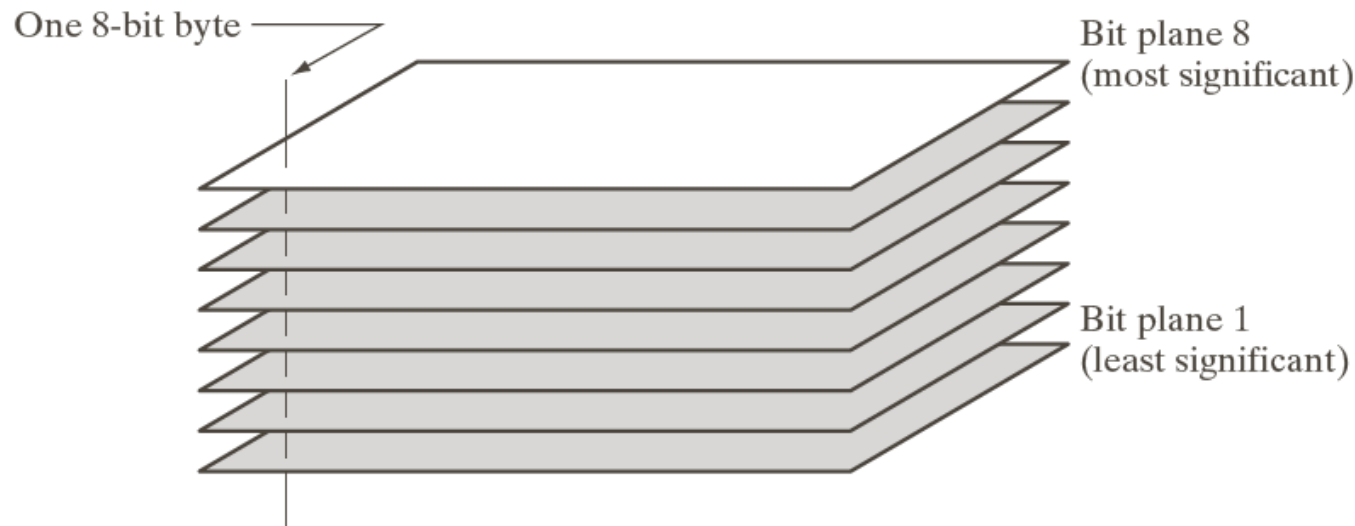
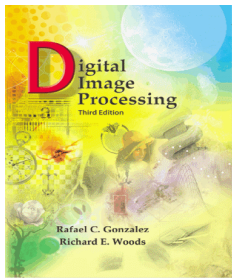


FIGURE 3.13
Bit-plane
representation of
an 8-bit image.



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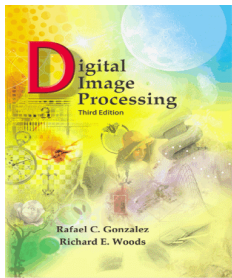
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a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500×1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.



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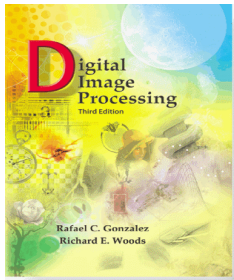
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a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).





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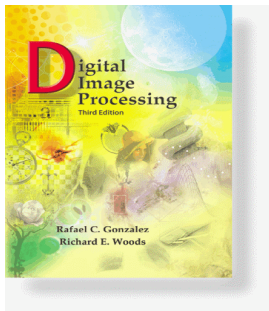
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Chapter 3

Intensity Transformations & Spatial Filtering

HISTOGRAM PROCESSING: EQUALIZATION



Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Processing

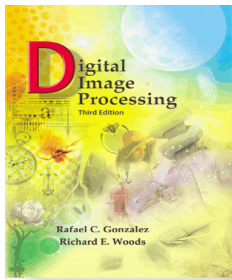
$$h(r_k) = n_k$$

□□□□□□□□□□□□□□□□□□□□□□□□ r_k : k th intensity value, $k \in [0, L-1]$

□□□□□□□□□□□□□□□□□□□□□□□□ n_k : # of pixels with value r_k

$$p(r_k) = n_k / MN$$

□□□□□□□□□□□□□□□□□□□□□□□□ $M \times N$: image dimension



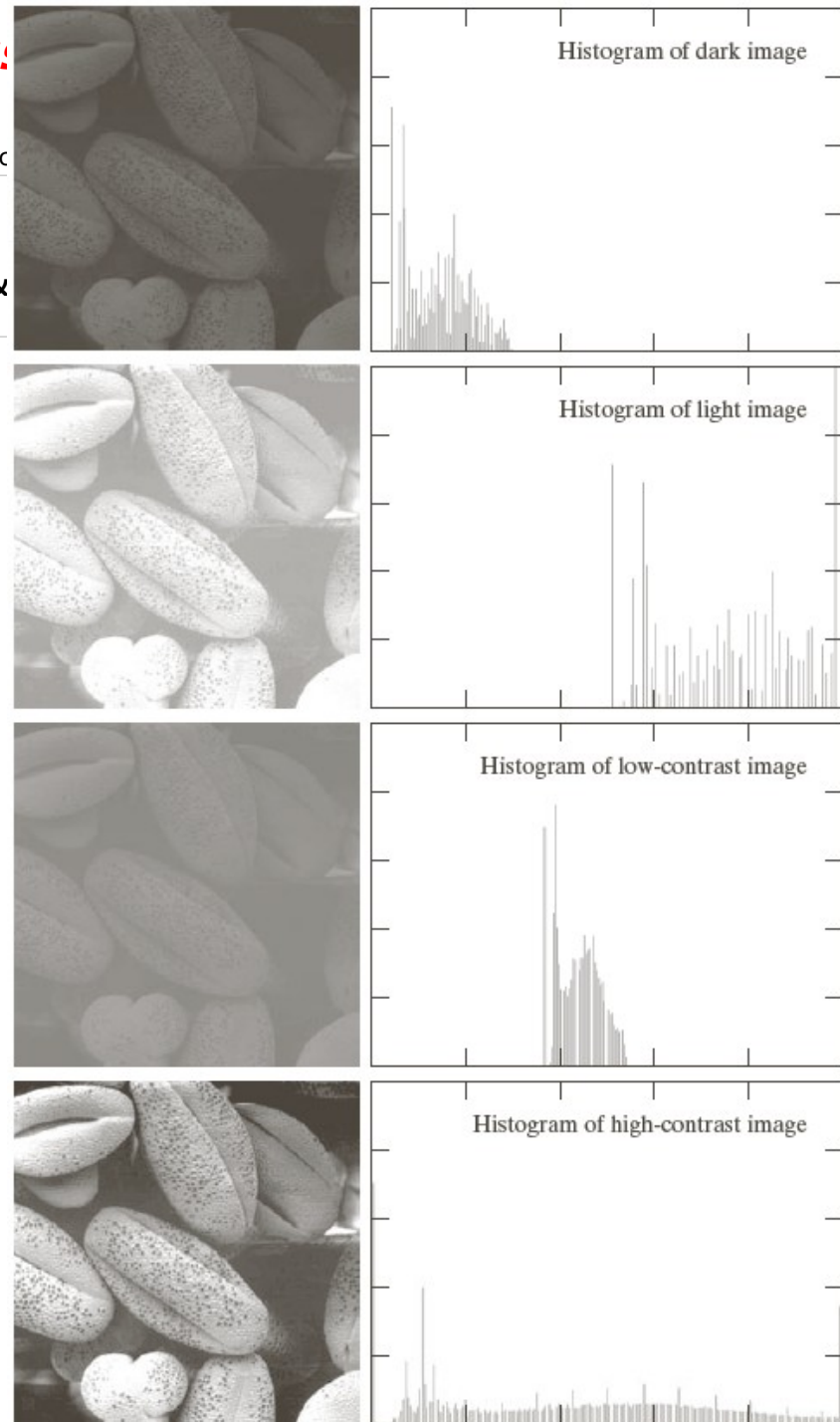
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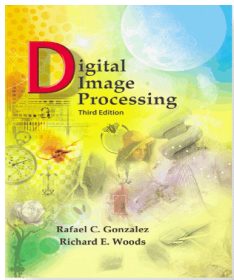
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FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.





Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Equalization

$$s = T(r), \quad 0 \leq r \leq L-1$$

- $T(r)$ should have the following properties:

- a) Monotonically increasing in $0 \leq r \leq L-1$

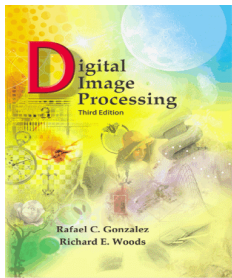
- Strictly monotonically increasing if

$$r = T^{-1}(s), \quad 0 \leq r \leq L-1$$

is to be used

- b) $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$

- c) **Desirable: Continuous and Differentiable**



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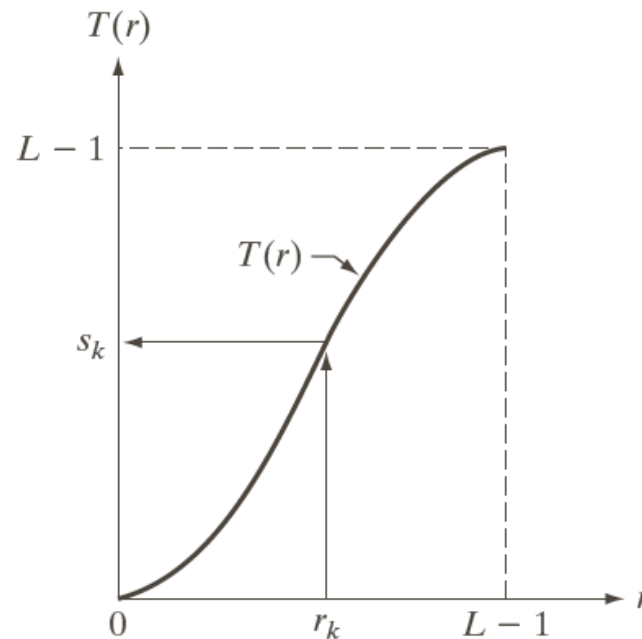
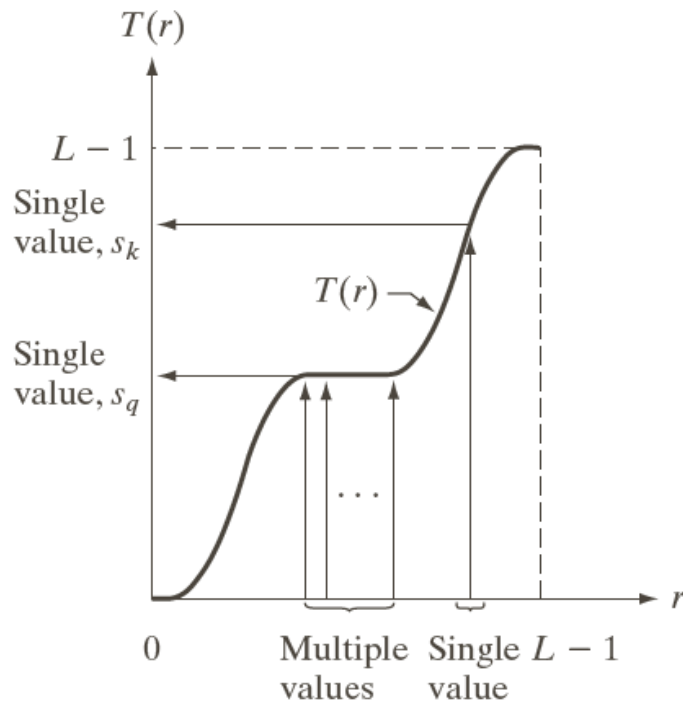
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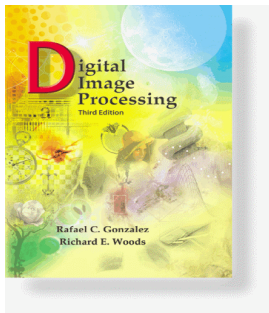
Intensity Transformations & Spatial Filtering



a b

FIGURE 3.17

(a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



Chapter 3

Intensity Transformations & Spatial Filtering

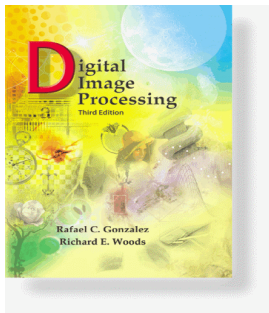
- Histogram Equalization

$$s = T(r)$$

$T(r)$ is continuous & differentiable

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

$p_r(r)$ and $p_s(s)$ are PDF of i/p & o/p values



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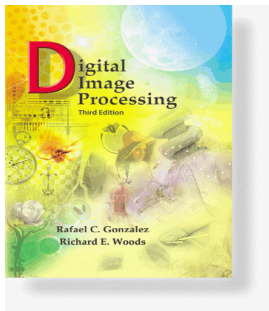
Intensity Transformations & Spatial Filtering

- Histogram Equalization

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

RHS is the CDF of r

$T(r)$ satisfies (a) & (b)



Chapter 3

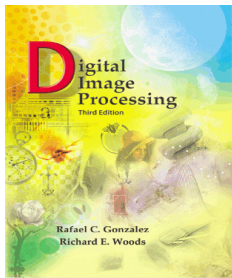
Intensity Transformations & Spatial Filtering

- Histogram Equalization

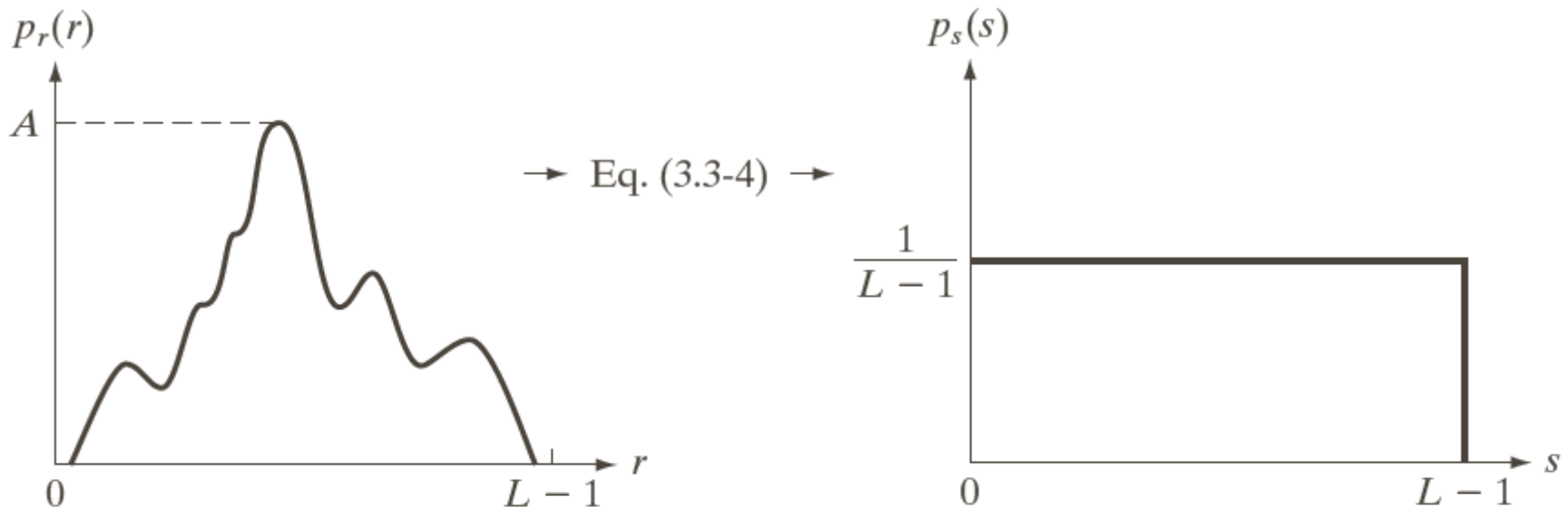
$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$s = (L-1) \frac{d}{dr} \left[\int_0^r p_r(w) dw \right]$$

$$s = (L-1) p_r(r)$$

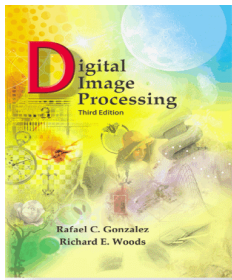


Chapter 3 Intensity Transformations & Spatial Filtering



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r . The resulting intensities, s , have a uniform PDF, independently of the form of the PDF of the r 's.

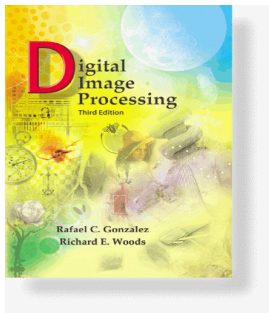


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Intensity Transformations & Spatial Filtering

- Histogram Equalization Example

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2} & \text{for } 0 \leq r \leq L-1 \\ 0 & \text{otherwise} \end{cases}$$

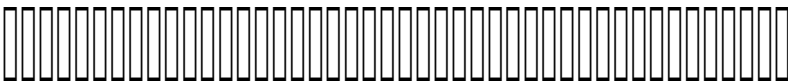


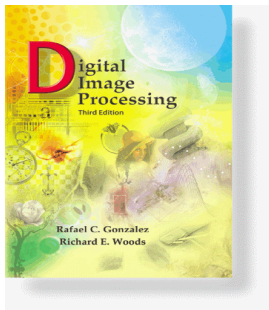
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- Histogram Equalization Example

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$


$$= \frac{2}{L-1} \int_0^r w dw = \frac{r^2}{L-1}$$

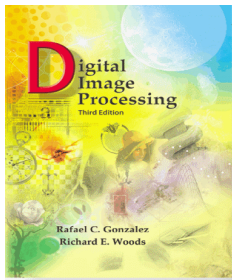


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Intensity Transformations & Spatial Filtering

- Histogram Equalization Example

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = \frac{2r}{(L-1)^2} \left| \left[\frac{ds}{dr} \right]^{-1} \right|$$
$$\frac{2r}{(L-1)^2} \left| \left[\frac{d}{dr} \frac{r^2}{L-1} \right]^{-1} \right| = \frac{2r}{(L-1)^2} \left| \frac{L-1}{2r} \right|$$
$$\frac{1}{L-1}, 0 \leq s \leq L-1$$



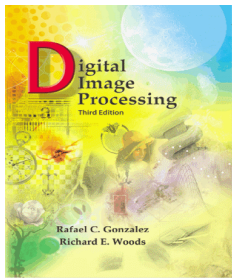
Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Equalization – Discrete Probability

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j)$$
$$= \frac{L-1}{MN} \sum_{j=0}^k n_j \text{ for } k=0,1,2,\dots,L-1$$

$$\text{where } p_r(r_k) = \frac{n_k}{MN}$$



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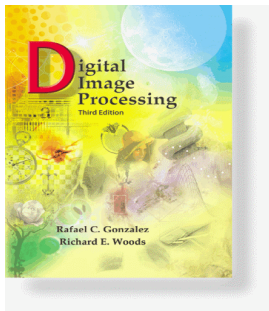
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Intensity Transformations & Spatial Filtering

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1
Intensity
distribution and
histogram values
for a 3-bit,
 64×64 digital
image.



Chapter 3

Intensity Transformations & Spatial Filtering

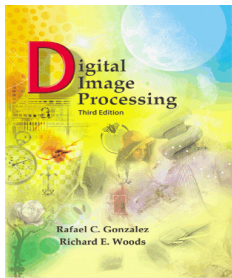
- Histogram Equalization – Example

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 p_r(r_0) = 1.33 [1]$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 p_r(r_0) + 7 p_r(r_1) = 3.08 [3]$$

$$s_2 = 4.55 [4], s_3 = 5.67 [5], s_4 = 6.23 [6],$$

$$s_5 = 6.65 [7], s_6 = 6.86 [7], s_7 = 7.00 [7]$$



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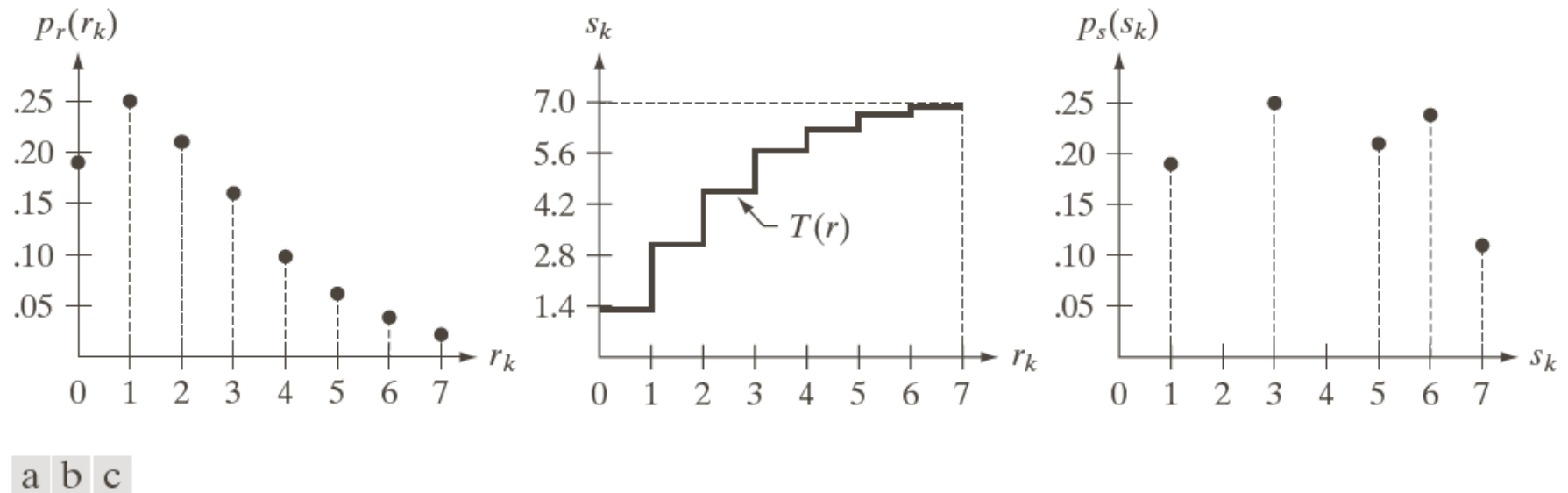


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

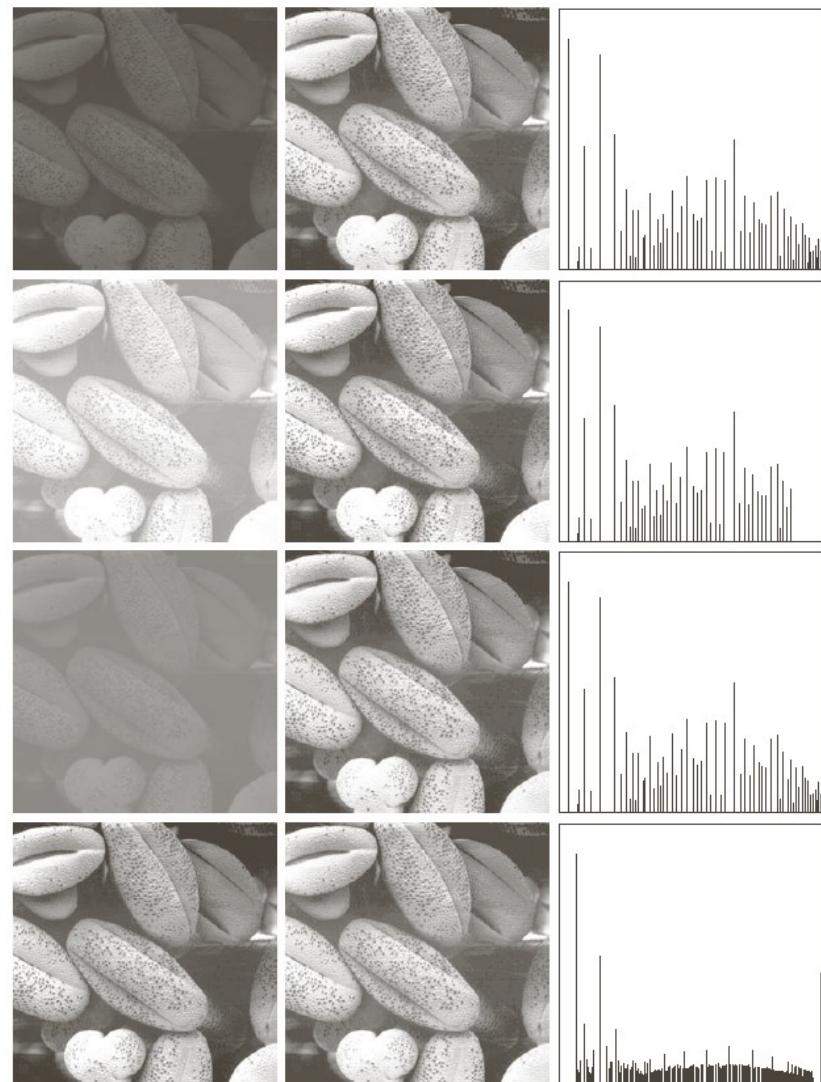
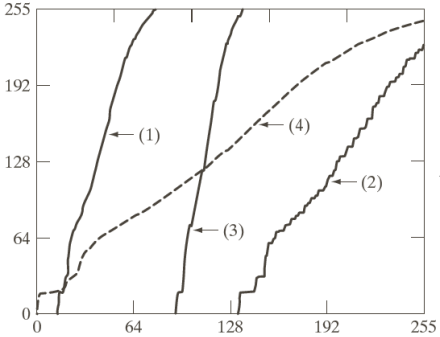
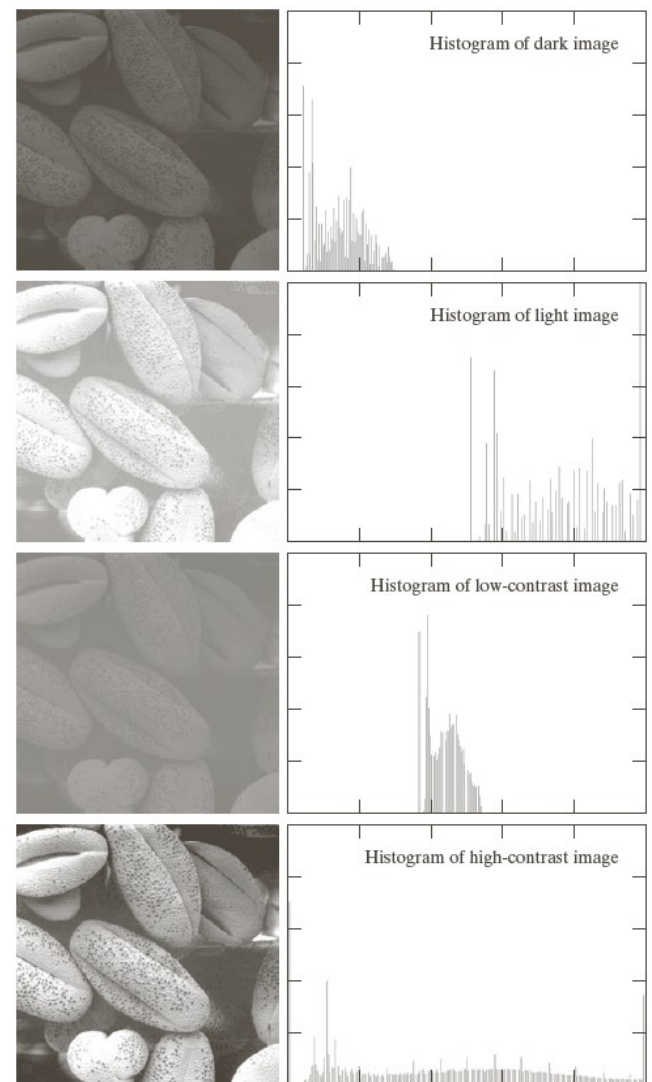
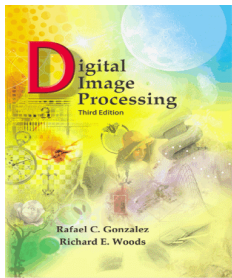


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.





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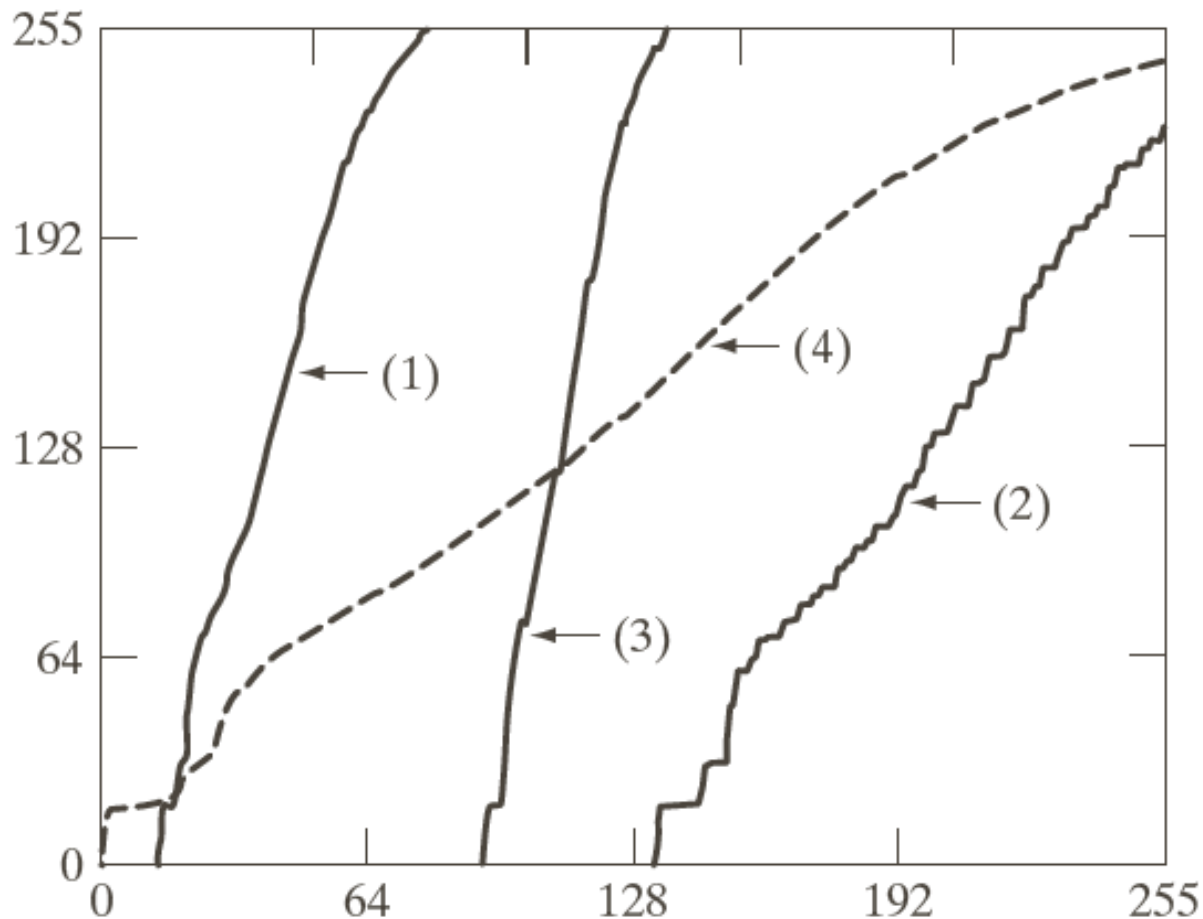
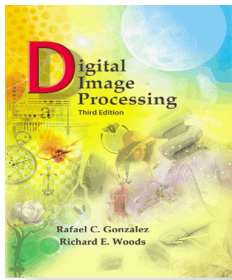


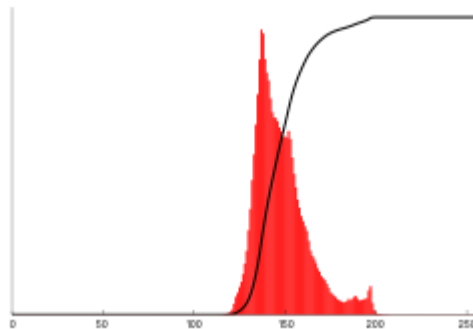
FIGURE 3.21
Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).



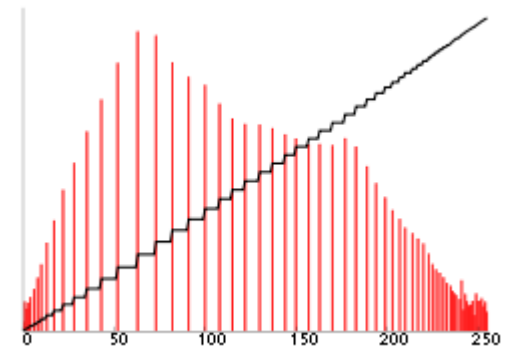
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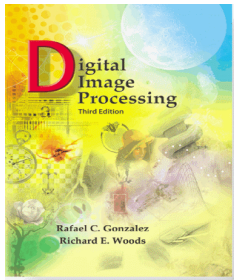
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Histogram & Cumulative Histogram



Before & After Histogram Equalization



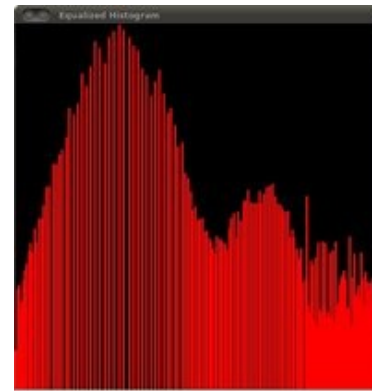
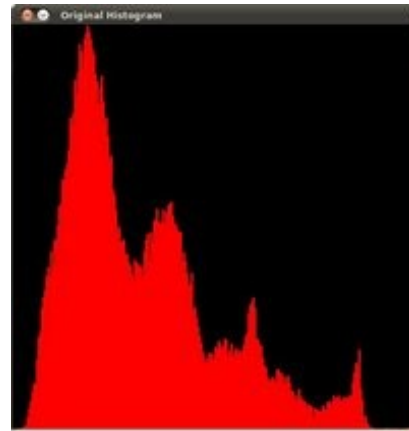
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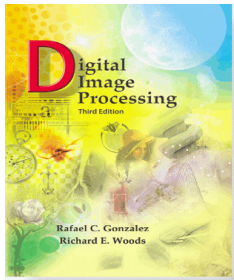
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Before & After Histogram Equalization

Source: http://docs.opencv.org/2.4/doc/tutorials/imgproc/histgrams/histogram_equalization/histogram_equalization.html



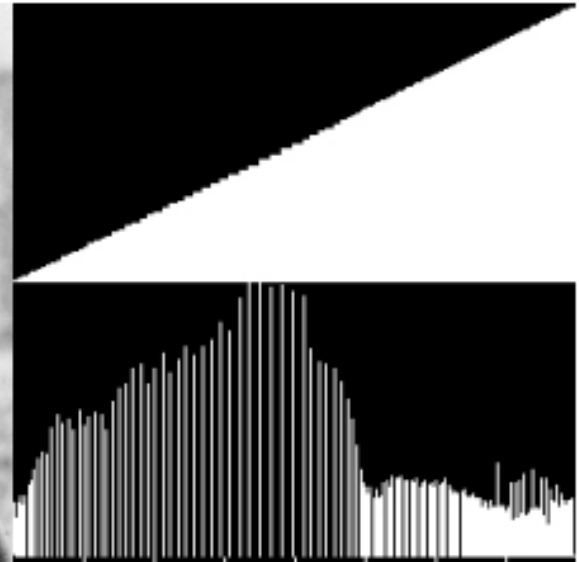
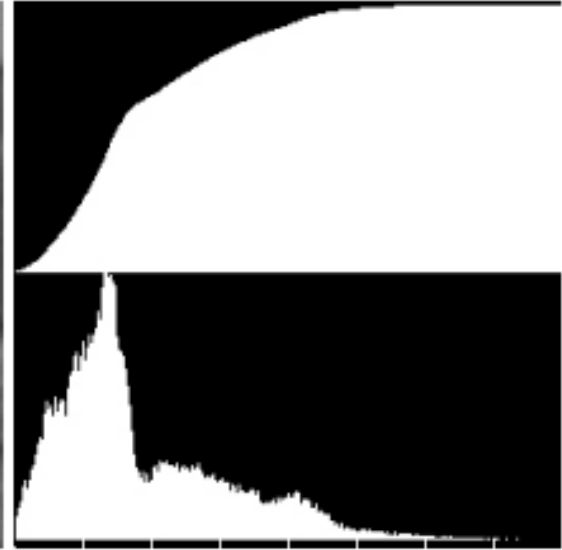
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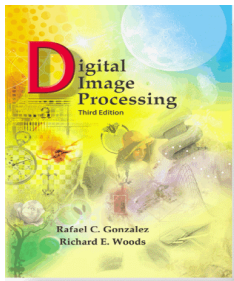
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Int

Before & After Histogram Equalization



Source: <http://fourier.eng.hmc.edu/e161/lectures/HistogramEqualization.pdf>



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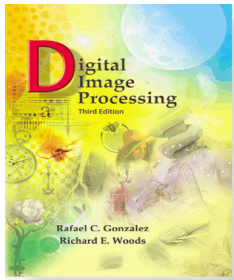
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HISTOGRAM PROCESSING: SPECIFICATION



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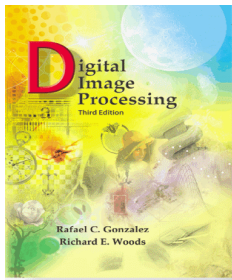
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Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Equalization – Failure Example
 - If the histogram is heavily skewed, equalization may not produce good result
 - Then we need to find transformation to a ‘desired’ histogram



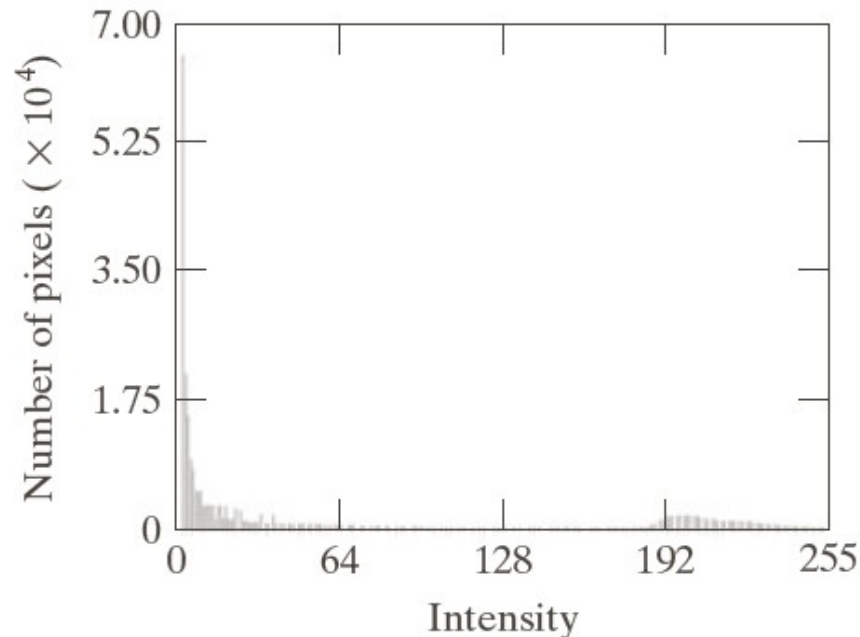
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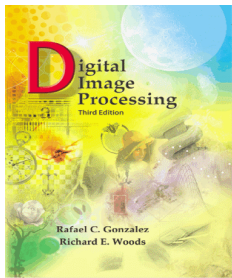
Chapter 3

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a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*.
(b) Histogram. (Original image courtesy of NASA.)

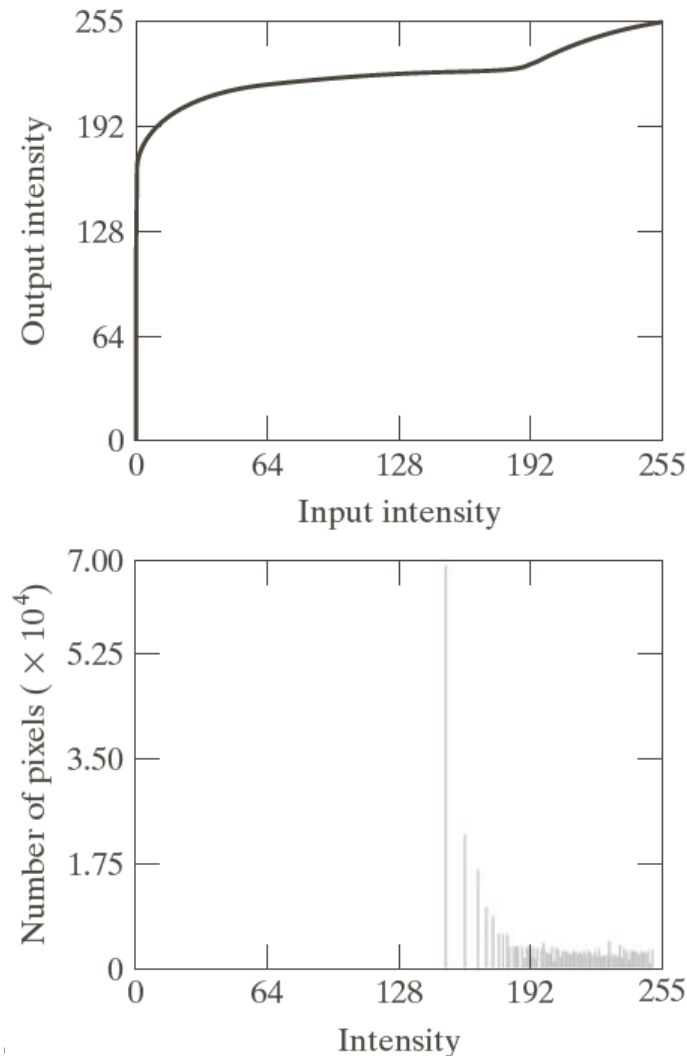


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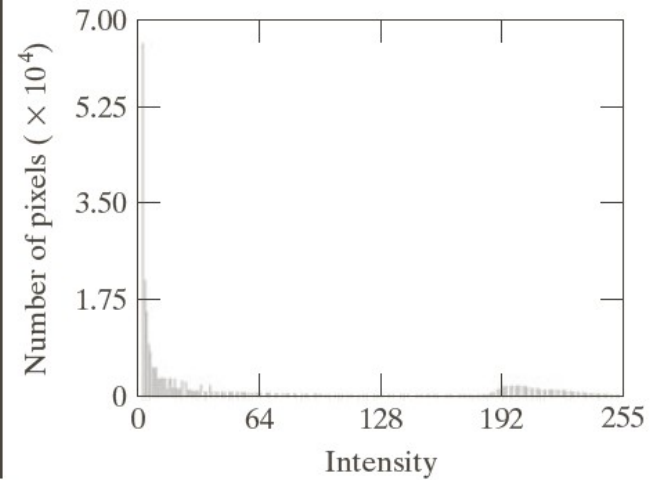
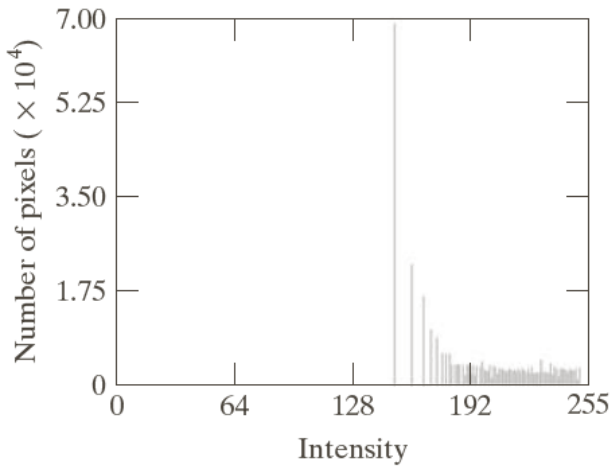
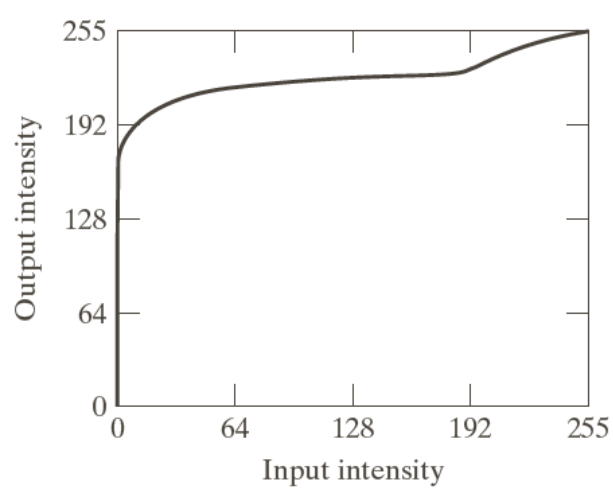
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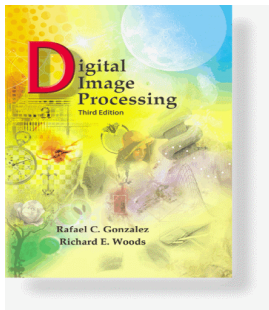


a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

& Spatial Filtering





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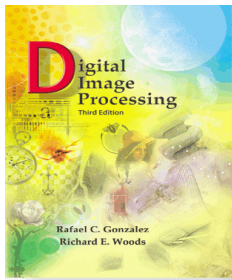
- Histogram Matching

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

$$G(z) = (L-1) \int_0^z p_z(t) dt = s$$

$$z = G^{-1}[T(r)] = G^{-1}(s)$$

where $p_r(r)$ and $p_z(z)$ are i/p & o/p PDFs



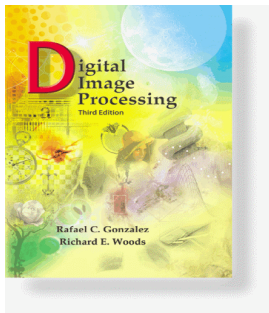
Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Matching Example

$$p_r(r) = \left\{ \begin{array}{ll} \frac{2r}{(L-1)^2} & \text{for } 0 \leq r \leq L-1 \\ 0 & \text{otherwise} \end{array} \right\}$$

$$p_z(z) = \left\{ \begin{array}{ll} \frac{3z^2}{(L-1)^3} & \text{for } 0 \leq r \leq L-1 \\ 0 & \text{otherwise} \end{array} \right\}$$

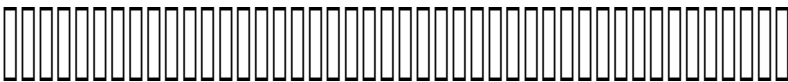


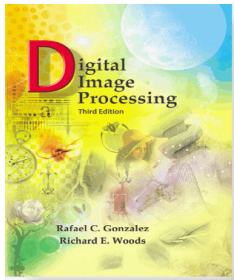
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- Histogram Matching Example

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$


$$= \frac{2}{L-1} \int_0^r w dw = \frac{r^2}{L-1}$$

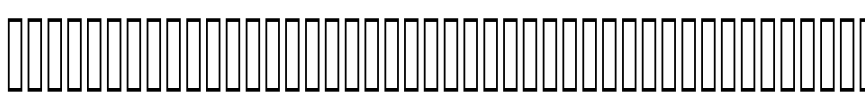


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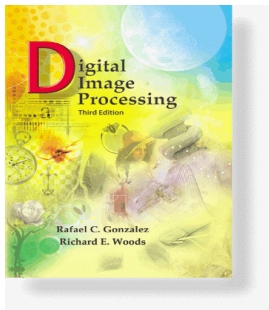
Intensity Transformations & Spatial Filtering

- Histogram Matching Example

$$G(z) = (L-1) \int_0^z p_z(w) dw$$



$$= \frac{3}{(L-1)^2} \int_0^z w^2 dw = \frac{z^3}{(L-1)^2}$$



Chapter 3

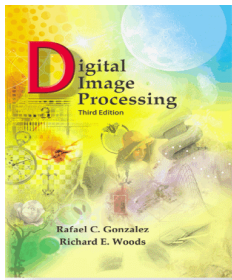
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- Histogram Matching Example

$$G(z) = s \quad \text{or} \quad \frac{z^3}{(L-1)^2} = s$$

$$z = \left((L-1)^2 s \right)^{1/3} = \left((L-1)^2 \frac{r^2}{(L-1)} \right)^{1/3}$$

$$= \left((L-1) r^2 \right)^{1/3}$$



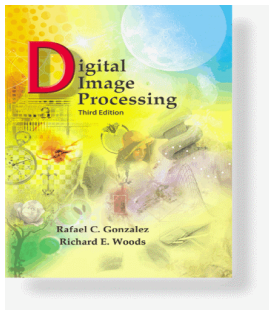
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Intensity Transformations & Spatial Filtering

- Histogram Matching – Discrete Probability

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j)$$
$$= \frac{L-1}{MN} \sum_{j=0}^k n_j \text{ for } k=0,1,2,\dots,L-1$$

$$\text{where } p_r(r_k) = \frac{n_k}{MN}$$



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Intensity Transformations & Spatial Filtering

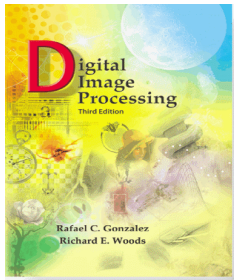
- Histogram Matching – Discrete Probability

$$G(z_q) = (L-1) \sum_{j=0}^q p_z(z_j)$$

for a value of q such that

$$G(z_q) = s_k \text{ or}$$

$$z_q = G^{-1}(s_k)$$



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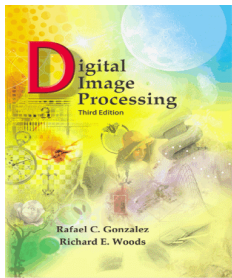
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Intensity Transformations & Spatial Filtering

- Histogram Matching – Discrete Probability
- Home Work
 - Computational Algorithm
 - Numerical Example



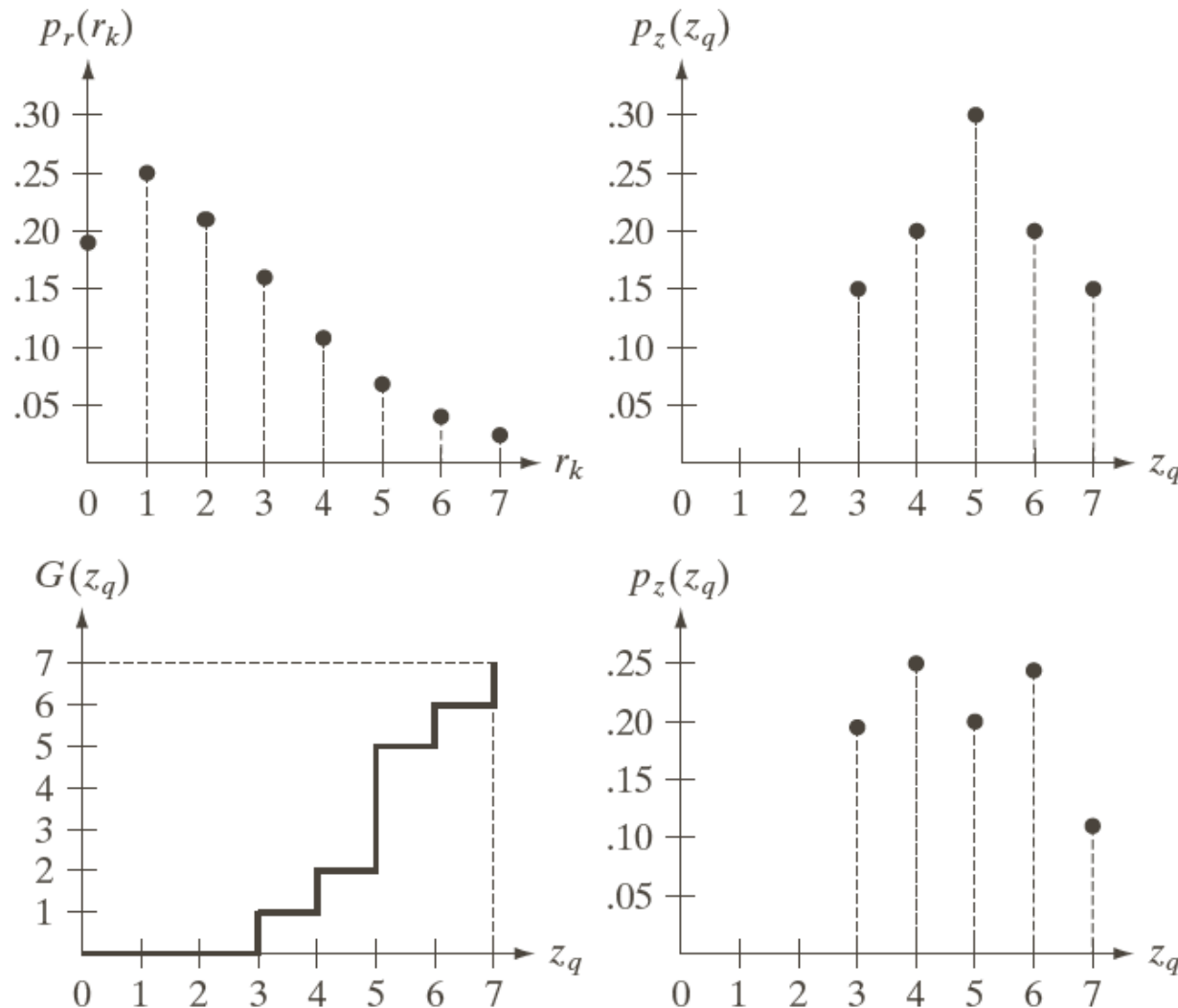
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a b
c d

FIGURE 3.22

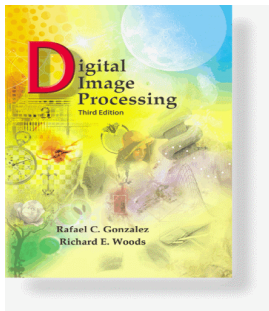
(a) Histogram of a 3-bit image. (b)

Specified histogram.

(c) Transformation function obtained from the specified histogram.

(d) Result of performing histogram specification.

Compare (b) and (d).



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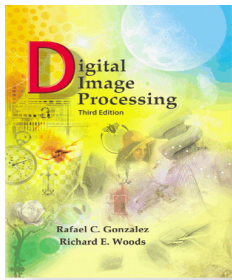
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Intensity Transformations & Spatial Filtering

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2

Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).



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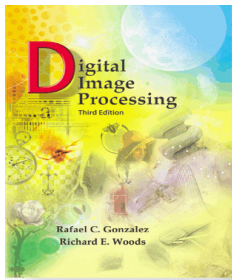
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z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

TABLE 3.3

All possible values of the transformation function G scaled, rounded, and ordered with respect to z .



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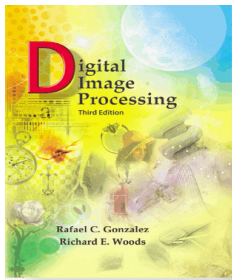
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s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

TABLE 3.4

Mappings of all the values of s_k into corresponding values of z_q .



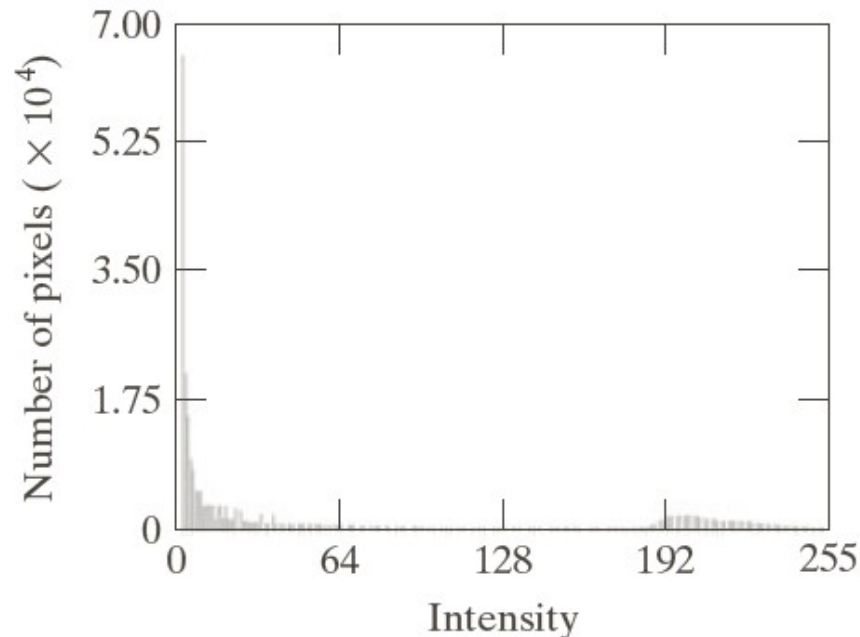
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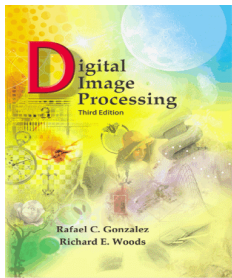
Chapter 3

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a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*.
(b) Histogram. (Original image courtesy of NASA.)

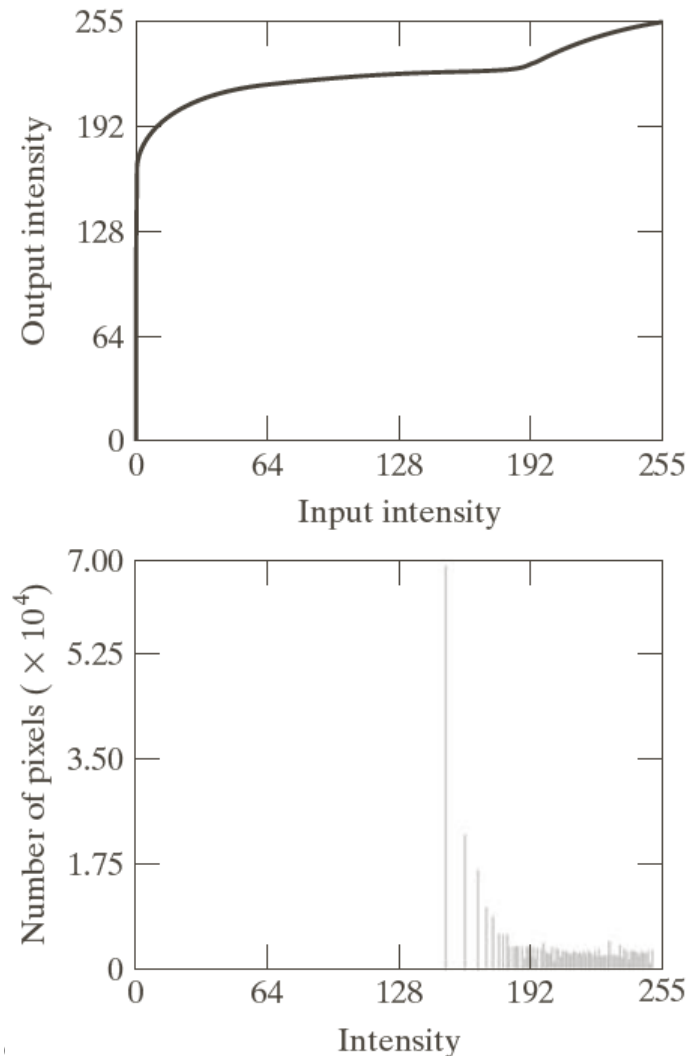


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a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

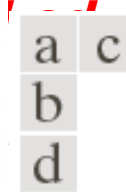
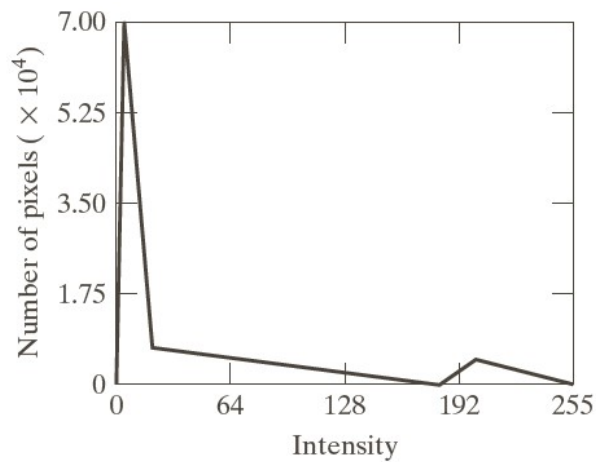
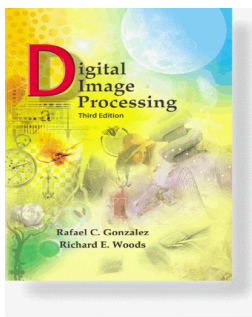
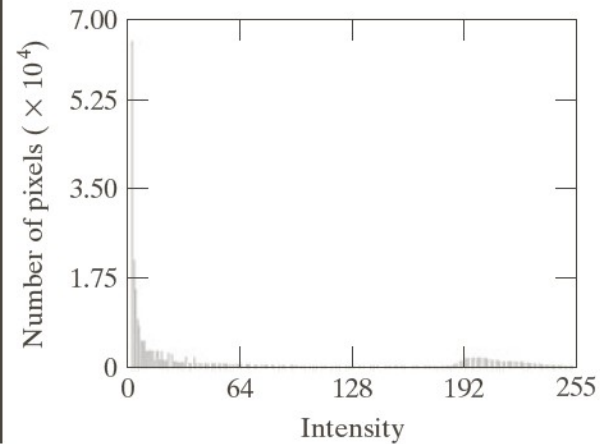
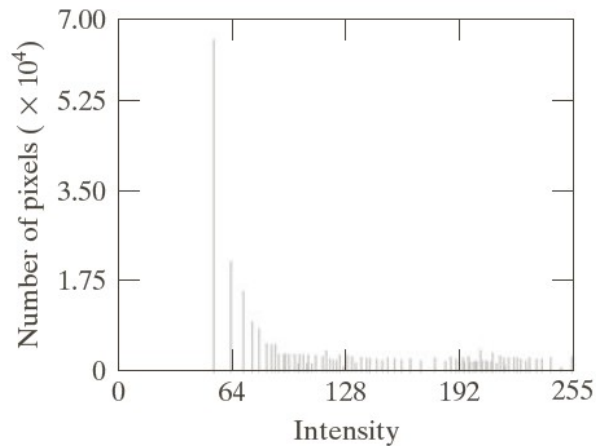
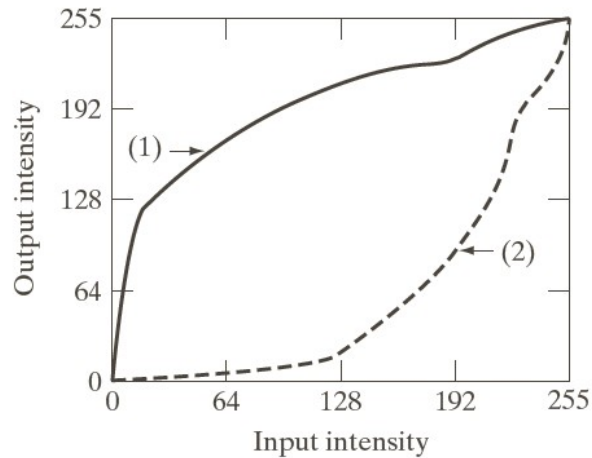
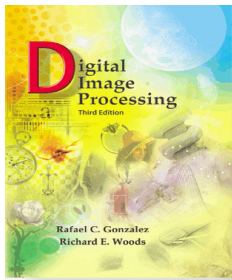


FIGURE 3.25
 (a) Specified histogram.
 (b) Transformations.
 (c) Enhanced image using mappings from curve (2).
 (d) Histogram of (c).





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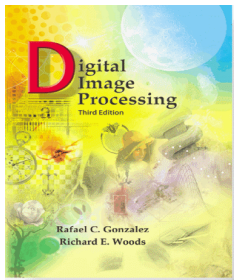
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Intensity Transformations & Spatial Filtering

- Local Histogram Processing
 - Enhance details in local areas
- Local Processing Steps
 - Define a Neighborhood
 - Move its center from pixel to pixel
 - Apply histogram equalization / matching @ center
 - Non-overlapping computation is fast but blocky



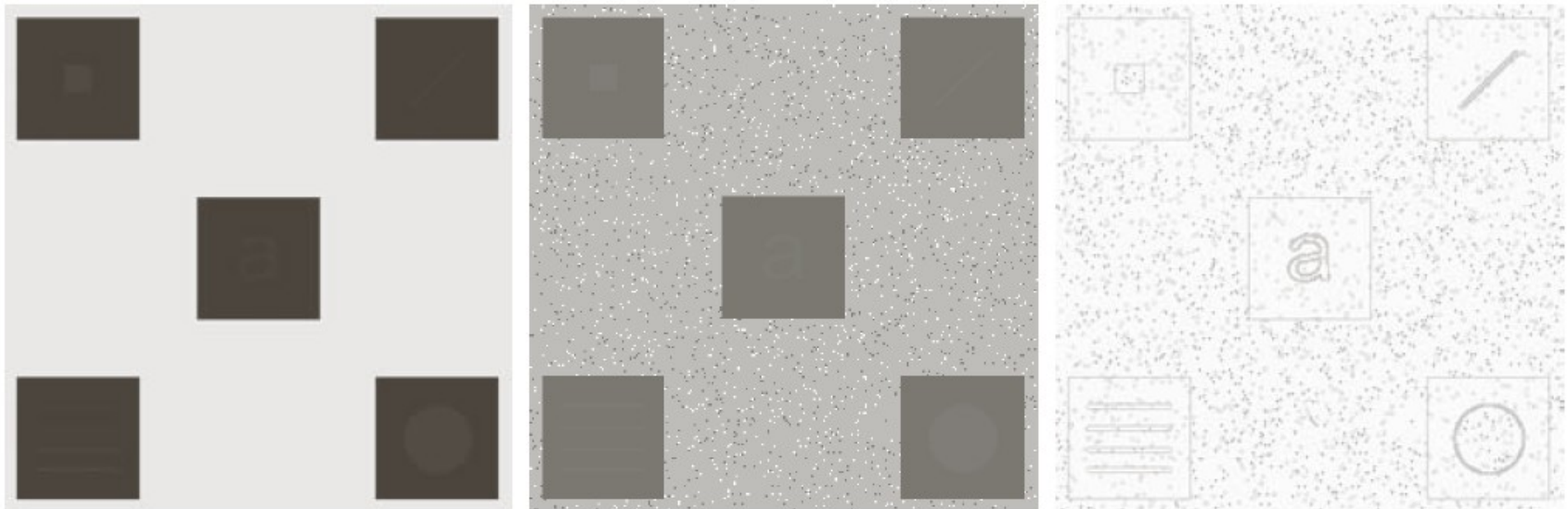
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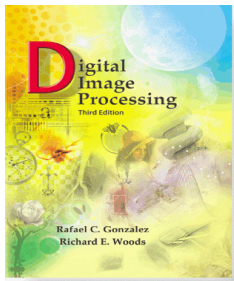
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Chapter 3 Intensity Transformations & Spatial Filtering



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3 .



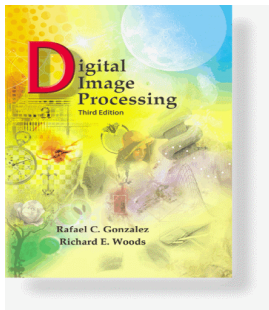
Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Statistics (Continuous)

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

$$\mu_2(r) = \sigma^2 = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$



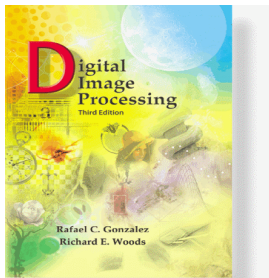
Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Statistics (Discrete)

$$m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

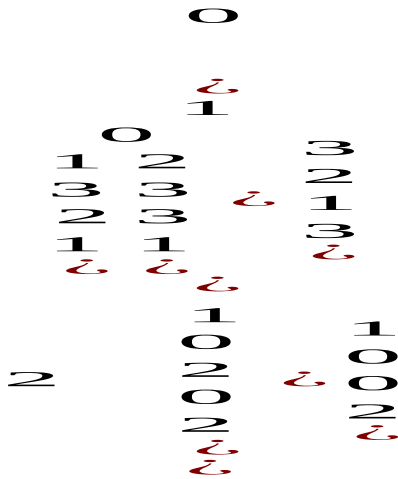
$$\mu_2(r) = \sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (f(x, y) - m)^2$$



Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Statistics (Discrete): Example



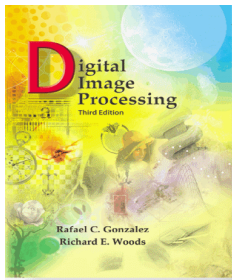
$$p(r_0) = \frac{6}{25} = 0.24, \quad p(r_1) = \frac{7}{25} = 0.28,$$

$$p(r_2) = \frac{7}{25} = 0.28, \quad p(r_3) = \frac{5}{25} = 0.20.$$

$$m = \sum_{i=0}^3 r_i p(r_i)$$

$$0 * 0.24 + 1 * 0.28 + 2 * 0.28 + 3 * 0.20$$

$$1.44$$



Chapter 3

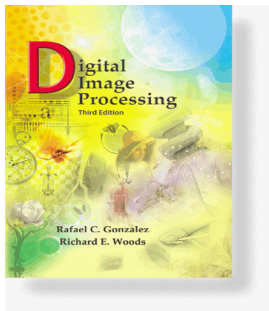
Intensity Transformations & Spatial Filtering

- Histogram Statistics – Local Mean & Variance

$$m_{S_{xy}} = \sum_{i=0}^{L-1} r_i p_{S_{xy}}(r_i)$$

$$\sigma_{S_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{S_{xy}})^2 p_{S_{xy}}(r_i)$$

where S_{xy} is a sub-image at (x,y)



Chapter 3

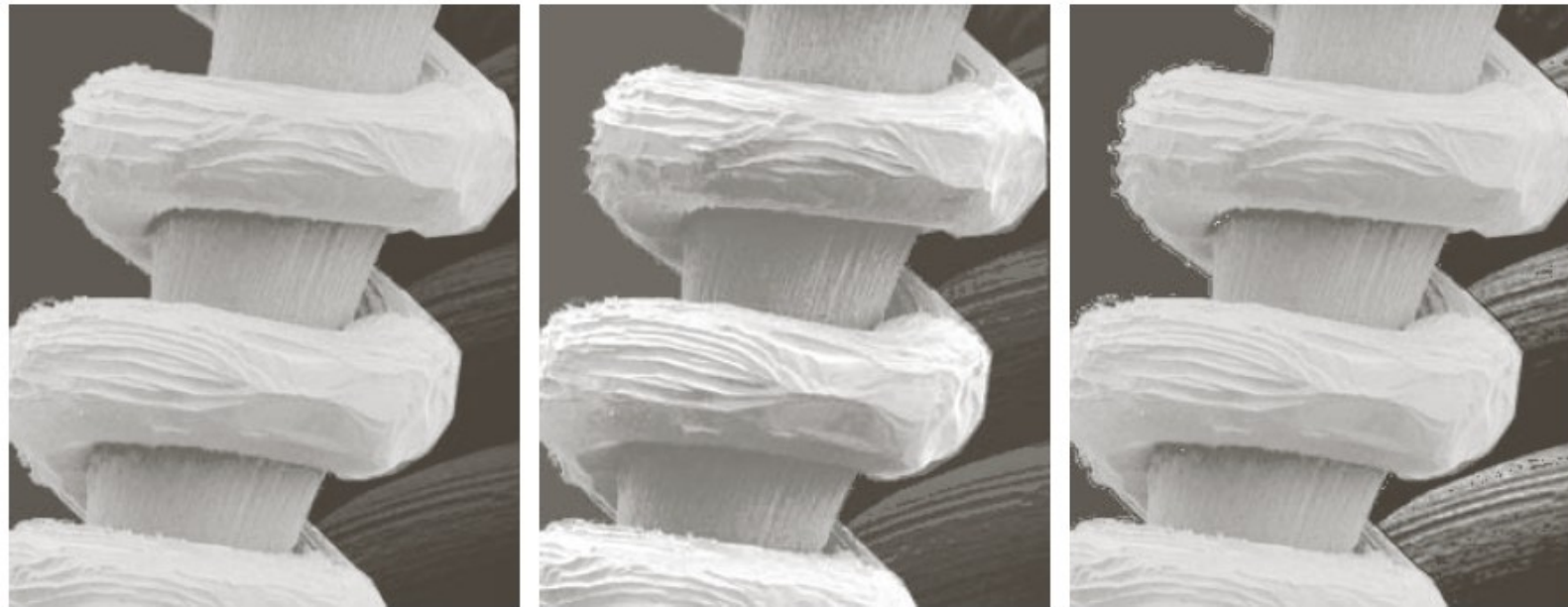
Intensity Transformations & Spatial Filtering

- Enhancement with Local Mean & Variance

$$g(x,y) = \left\{ \begin{array}{l} E.f(x,y), \text{ if } m_{S_{xy}} \leq k_0 m_G \text{ \& } \\ \quad \quad \quad k_1 \sigma_G \sigma_{S_{xy}} k_2 \sigma_G \\ f(x,y), \quad \text{ otherwise} \end{array} \right\}$$

where m_G is the global mean and

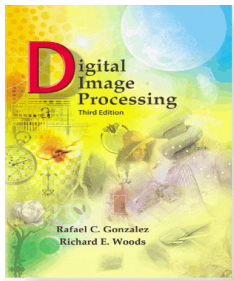
σ_G is the global standard deviation



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately $130\times$. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

$$E = 4.0, k_0 = 0.4, k_1 = 0.02, k_2 = 0.4$$



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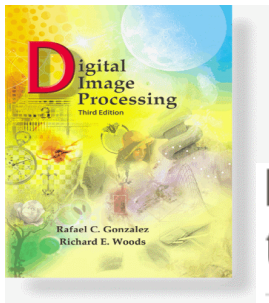
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Chapter 3

Intensity Transformations & Spatial Filtering

SPATIAL FILTERING



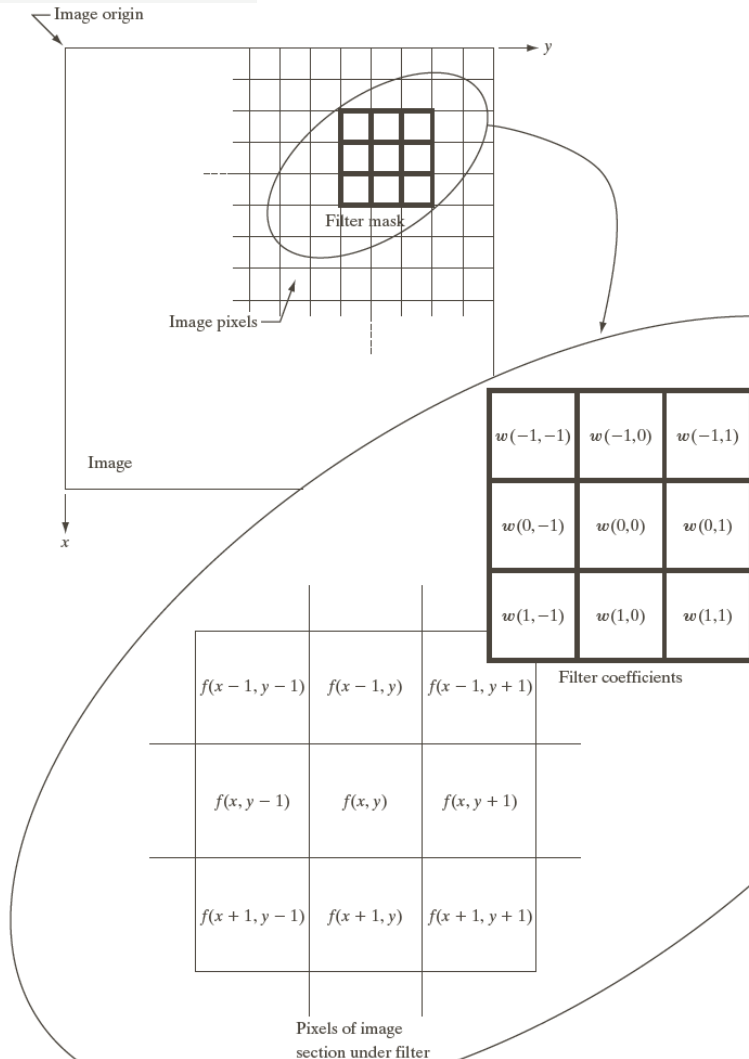
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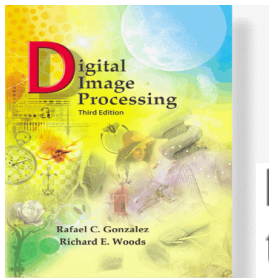
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FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.



• Spatial Filtering

$$\begin{aligned}
 g(x, y) &= w(-1, -1)f(x-1, y-1) + \\
 &w(-1, 0)f(x-1, y) + \dots + \\
 &w(0, 0)f(x, y) + \dots + \\
 &w(+1, +1)f(x+1, y+1) \\
 &= \sum_{\delta x=-1}^{+1} \sum_{\delta y=-1}^{+1} w(\delta x, \delta y)f(x+\delta x, y+\delta y)
 \end{aligned}$$



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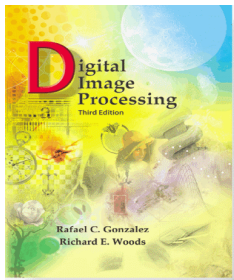
FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

- Vector Notation
 - Consider a 3X3 mask
 - Compute the response

$$R = w_1 z_1 + w_2 z_2 + \dots + w_9 z_9$$

$$\begin{bmatrix} \end{bmatrix} = \sum_{k=1}^9 w_k z_k$$



Chapter 3

Intensity Transformations & Spatial Filtering

- **Spatial Filtering**

- **Correlation**

- Process of moving the filter mask over the image and compute the sum-of-products at every location

- **Convolution**

- Same as correlation except the filter is first rotated by 180°

Appropriately pad enough 0's on all sides so that the mask can cover the function (image) at every position

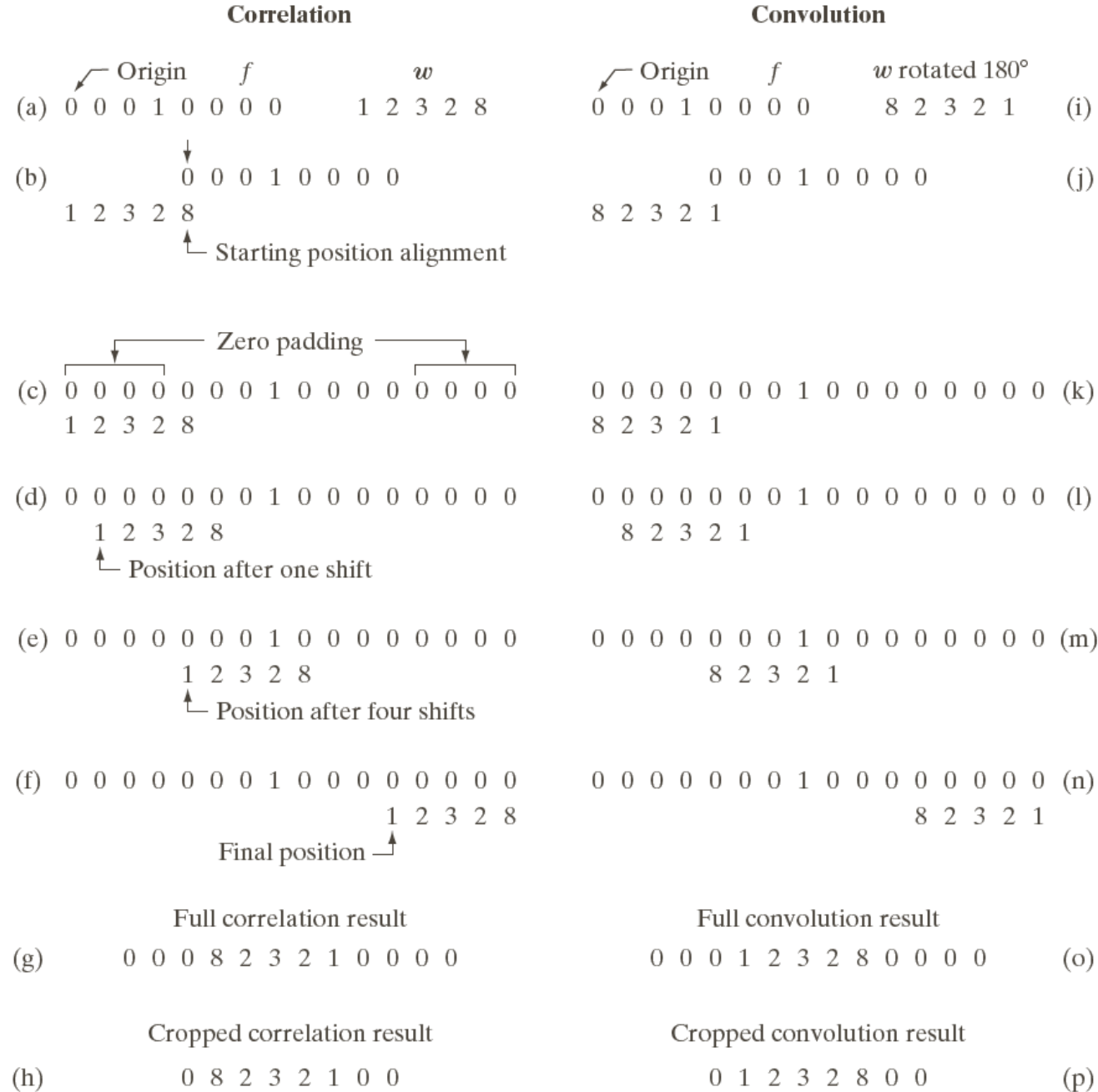
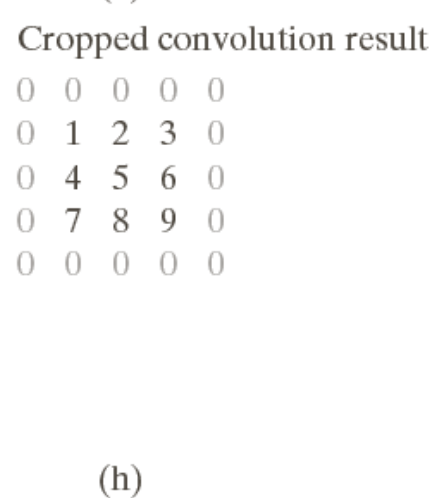
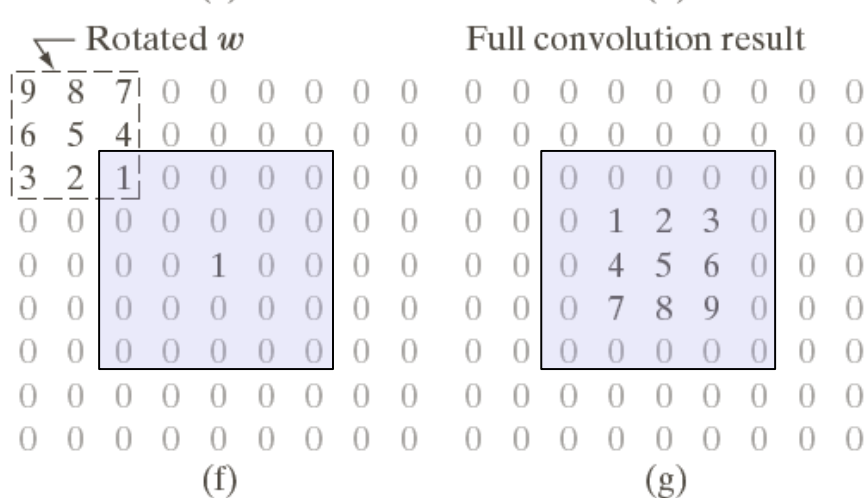
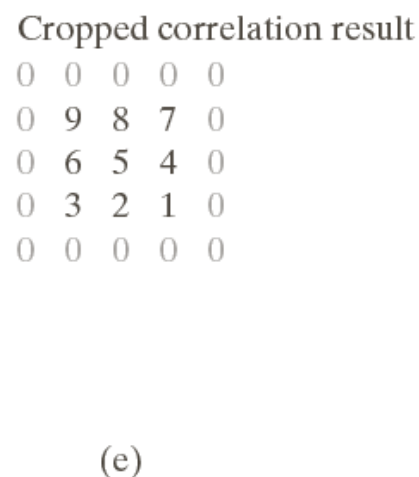
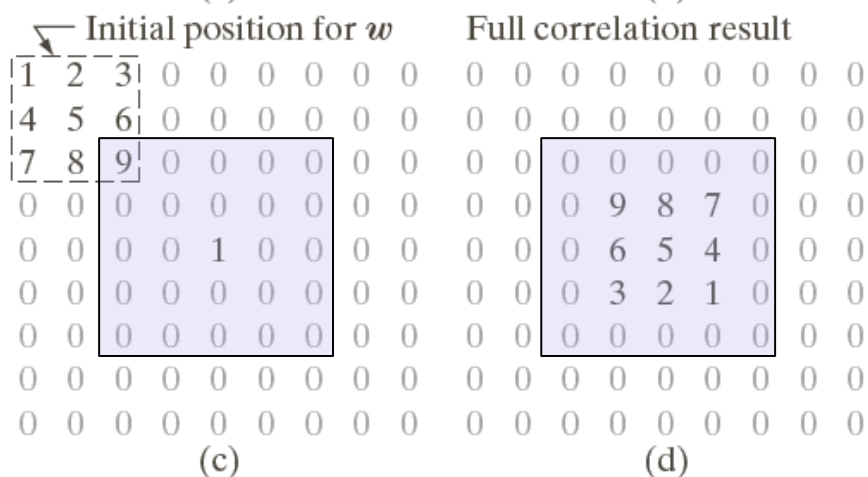
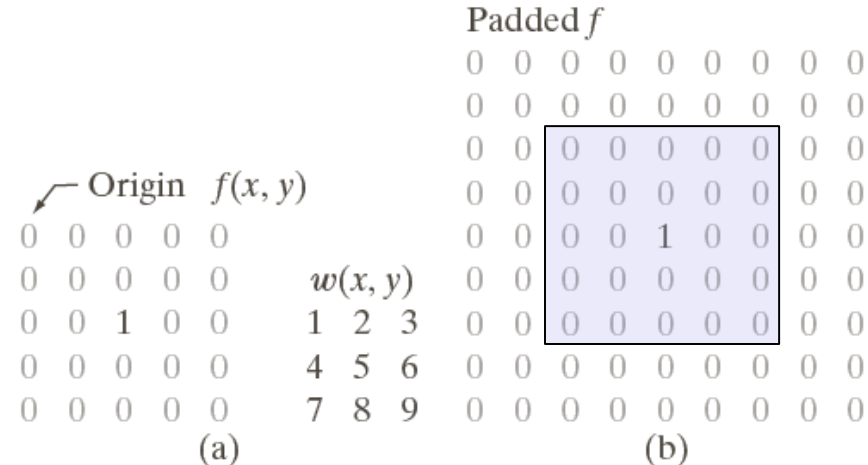
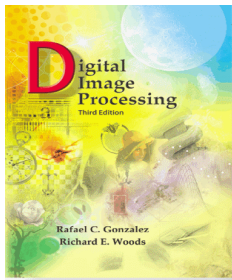


FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.



d.

FIGURE 3.30
Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.



Chapter 3

Intensity Transformations & Spatial Filtering

- Spatial Filtering

$m \times n$ mask

$$a = (m-1)/2, b = (n-1)/2$$

- Correlation

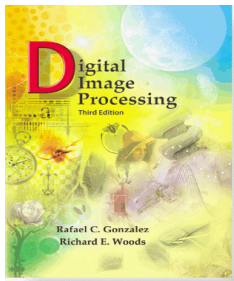
$$w(x, y) \circ f(x, y) = \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t) f(x+s, y+t)$$

$$f(x, y) \circ w(x, y)$$

- Convolution

$$w(x, y) \cdot f(x, y) = \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t) f(x-s, y-t)$$

$$f(x, y) \cdot w(x, y)$$



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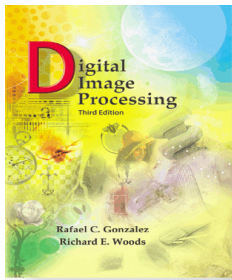
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Chapter 3

Intensity Transformations & Spatial Filtering

Box / Median Filter

SMOOTHING FILTER



Chapter 3

Intensity Transformations & Spatial Filtering

- Smoothing Filters

- Average

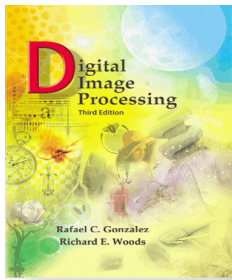
$$R = \frac{1}{9} \sum_{i=1}^9 z_i$$

$m \times n$ mask

- Weighted Average

$$a = (m-1)/2, b = (n-1)/2$$

$$g(x, y) = \frac{\sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t) f(x+s, y+t)}{\sum_{s=-a}^{+a} \sum_{t=-b}^{+b} w(s, t)}$$



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Chapter 3

Intensity Transformations & Spatial Filtering

$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

Box Filter

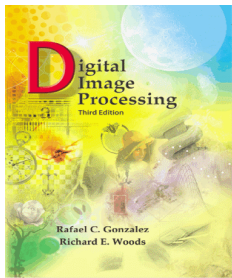
$$\frac{1}{16} \times$$

1	2	1
2	4	2
1	2	1

Gaussian Filter

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.



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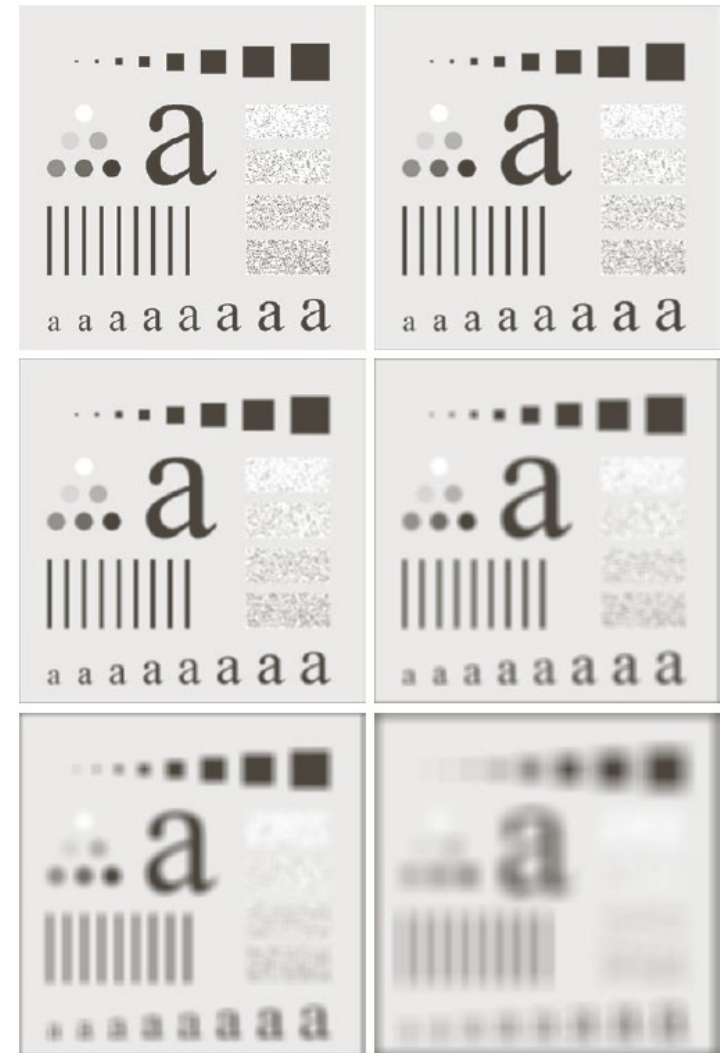
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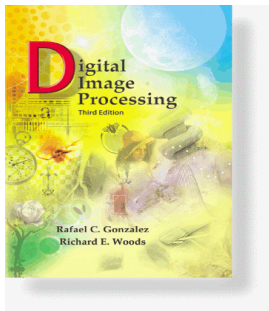
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Chapter 3 Intensity Transformations & Spatial Filtering

FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35 , respectively. The black squares at the top are of sizes $3, 5, 9, 15, 25, 35, 45$, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20% . The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

a b
c d
e f





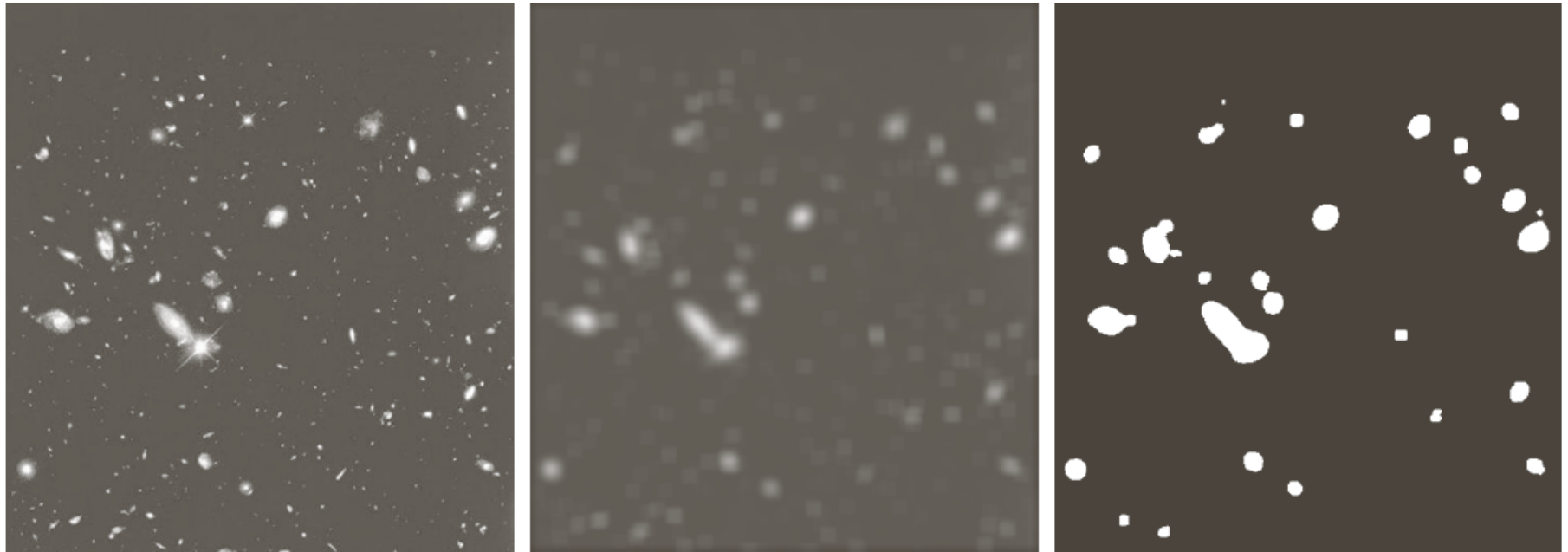
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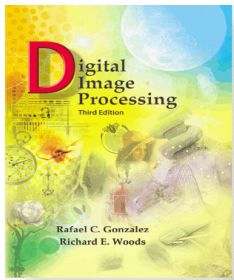
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Chapter 3 Intensity Transformations & Spatial Filtering



a b c

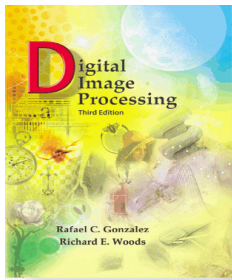
FIGURE 3.34 (a) Image of size 528×485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15×15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)



Chapter 3

Intensity Transformations & Spatial Filtering

- Order Statistics Filters
 - Median Filter
 - Replace center value by the median of values in the mask
 - Good for Impulse / Salt-and-Pepper Noise
 - Max Filter
 - To detect brightest points
 - Min Filter

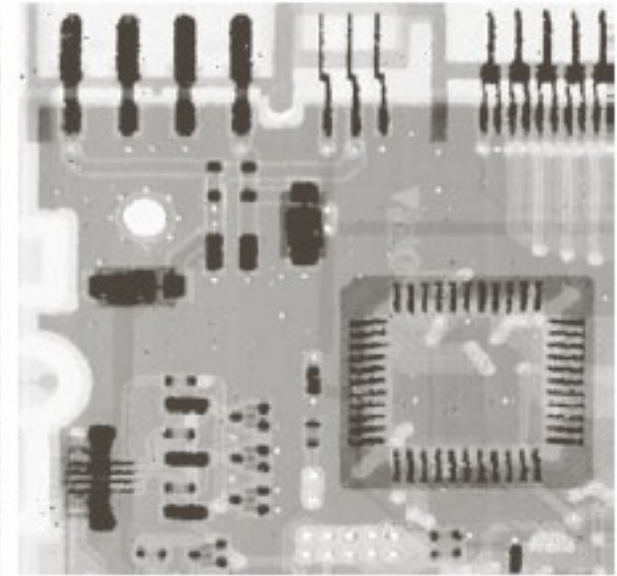
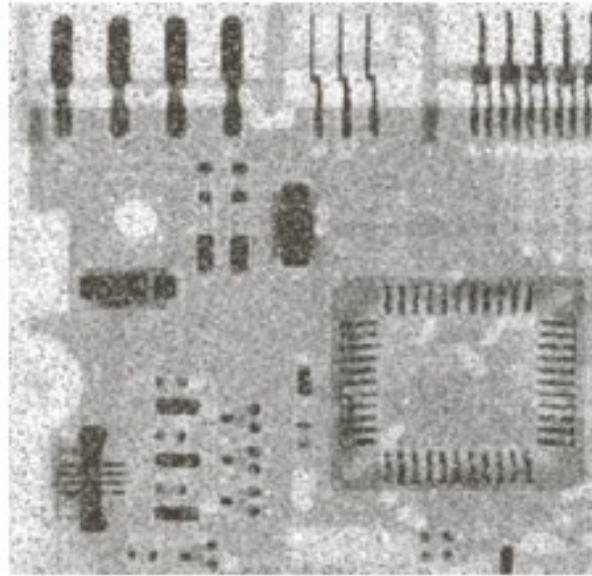
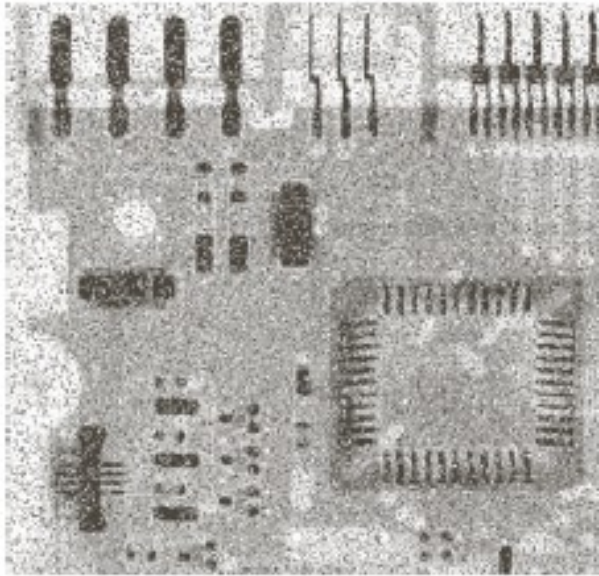


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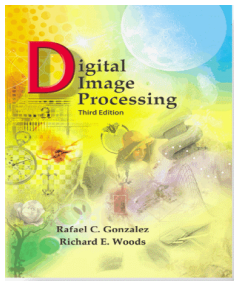
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Chapter 3 Intensity Transformations & Spatial Filtering



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



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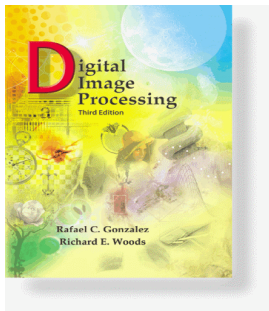
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Chapter 3

Intensity Transformations & Spatial Filtering

Gaussian Filter

SMOOTHING FILTER



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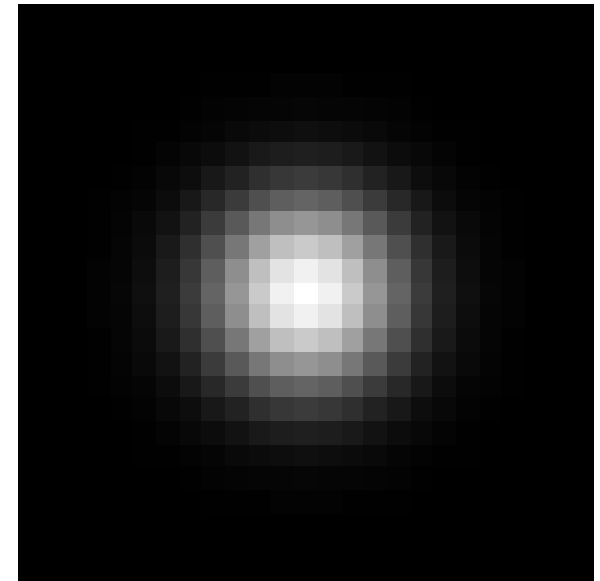
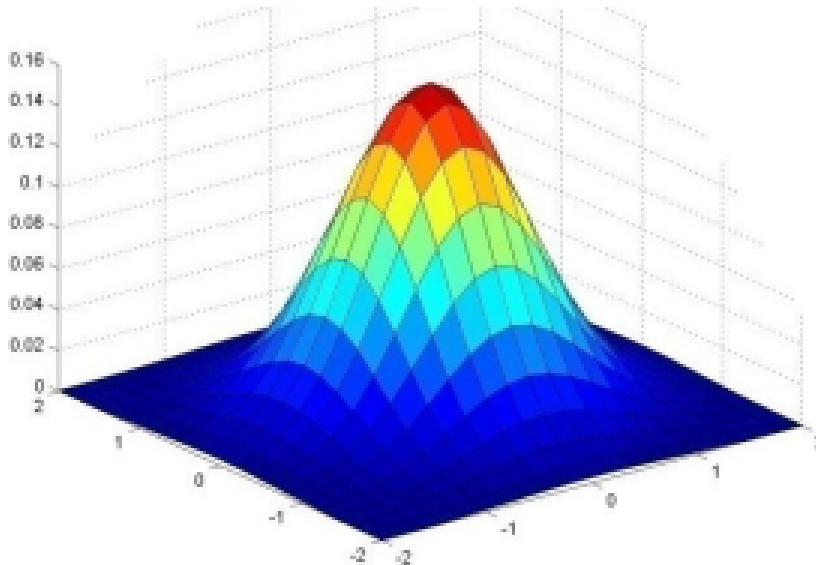
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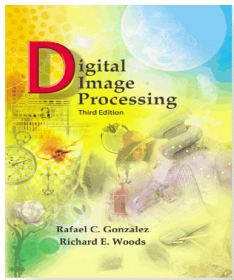
Intensity Transformations & Spatial Filtering

- Gaussian Kernel

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Source: http://www.cs.cornell.edu/courses/cs6670/2011sp/lectures/lec02_filter.pdf



Chapter 3

Intensity Transformations & Spatial Filtering

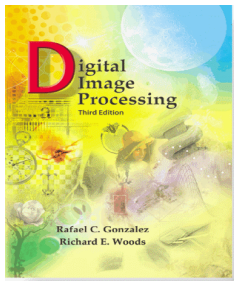
- Gaussian Filter

- Removes “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian



- Convolving two times with Gaussian kernel of width σ
= convolving once with kernel of width $\sigma\sqrt{2}$

Source: http://www.cs.cornell.edu/courses/cs6670/2011sp/lectures/lec02_filter.pdf

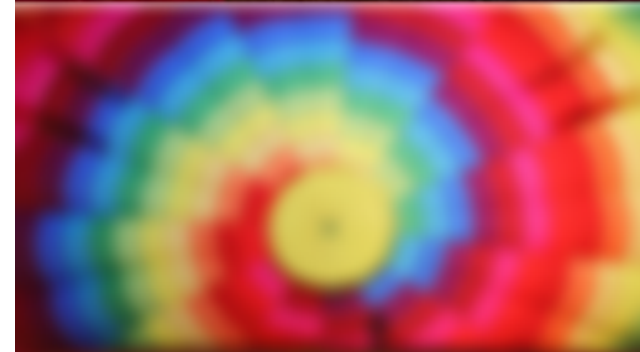


Chapter 3
Intensity Transformations & Spatial Filtering

- Gaussian Blur



Original

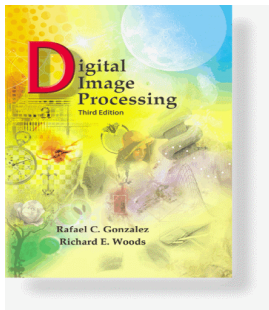


StDev = 3



StDev = 10

Source: https://en.wikipedia.org/wiki/Gaussian_blur



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Intensity Transformations & Spatial Filtering

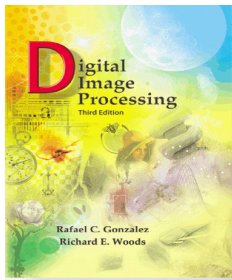
- Gaussian Kernels

$$\frac{1}{16} \times$$

1	2	1
2	4	2
1	2	1

$$\frac{1}{273}$$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1



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- Gaussian Filters

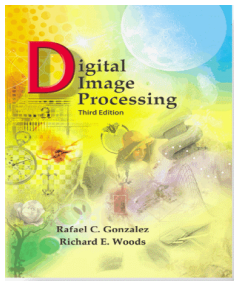
Original Image



Gaussian filtered image, $\sigma = 2$



Source: <https://in.mathworks.com/help/images/ref/imgaussfilt.html>



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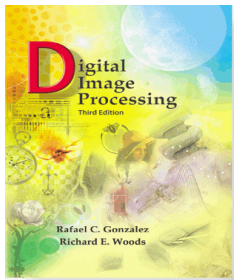
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- Gaussian Filters ($\sigma = 3$)



Source: <http://www.cse.psu.edu/~rtc12/CSE486/lecture04.pdf>



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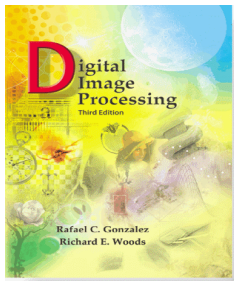
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- **Box Filter (3 x 3)**



Source: <http://www.cse.psu.edu/~rtc12/CSE486/lecture04.pdf>



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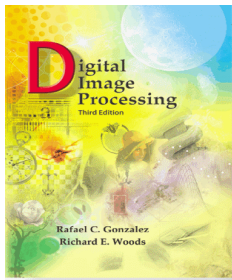
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- **Box vs. Gaussian Filters**



Source: <http://www.cse.psu.edu/~rtc12/CSE486/lecture04.pdf>



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- Gaussian is a true low-pass filter – little high frequency artifacts

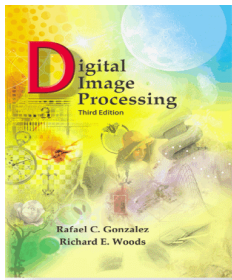


box filter



gaussian

Source: <http://www.cse.psu.edu/~rtc12/CSE486/lecture04.pdf>



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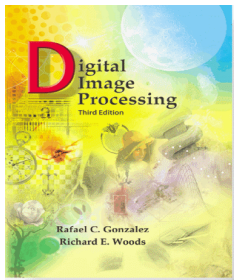
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- Gaussian at Different Scales ($\sigma = 1, 3, 10$)



Source: <http://www.cse.psu.edu/~rtc12/CSE486/lecture04.pdf>



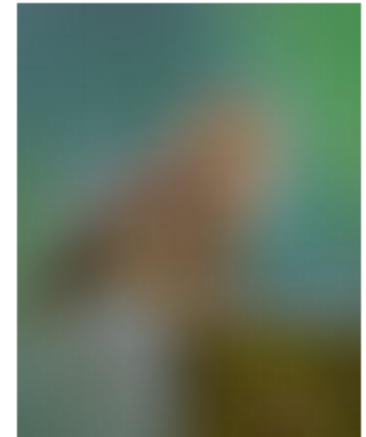
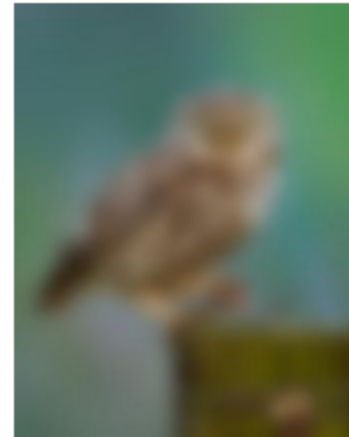
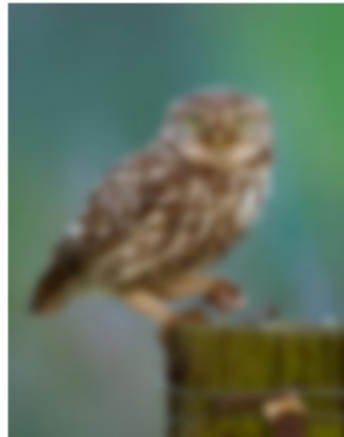
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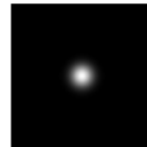
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Chapter 3 Intensity Transformations & Spatial Filtering

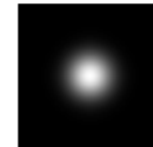
- Gaussian Filters



$\sigma = 1$ pixel



$\sigma = 5$ pixels

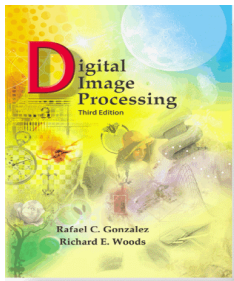


$\sigma = 10$ pixels



$\sigma = 30$ pixels

Source: http://www.cs.cornell.edu/courses/cs6670/2011sp/lectures/lec02_filter.pdf



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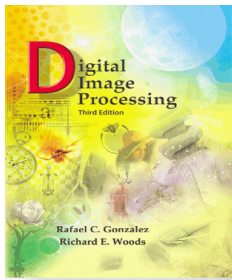
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Chapter 3

Intensity Transformations & Spatial Filtering

SHARPENING FILTER



Chapter 3

Intensity Transformations & Spatial Filtering

- Sharpening Filters

- First Derivative

$$f(x) = f(a) + (x-a) * f'(a)/1! + \dots = f(x+\Delta x) - \Delta x * f'(x+\Delta x)$$

$$\partial f / \partial x = f' = f(x+1) - f(x)$$

- Second Derivative

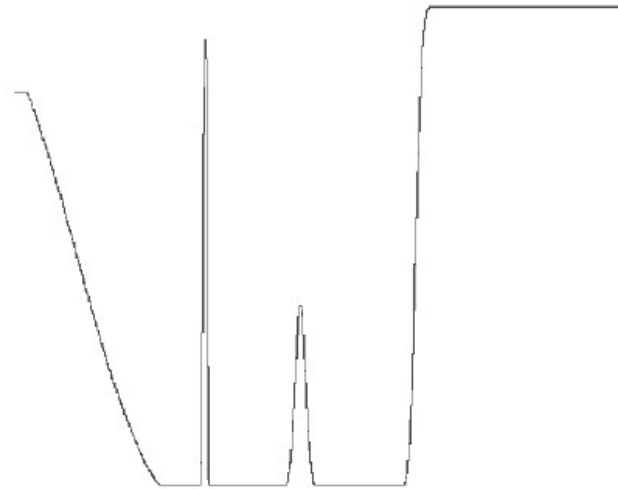
$$\partial^2 f / \partial x^2 = f'' = f'(x+1) - f'(x)$$

$$\text{[Diagram: A series of 10 small squares representing a 1D grid. The second square from the left is highlighted with a thicker border.]}$$

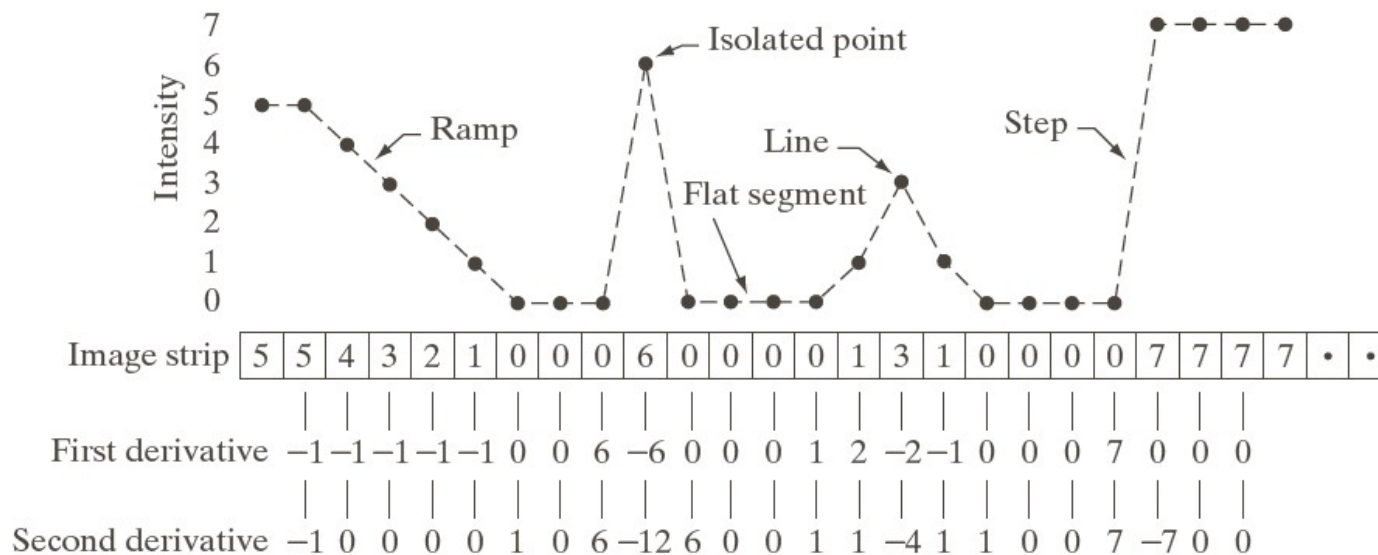
$$= f(x+2) - f(x+1) - f(x+1) + f(x)$$

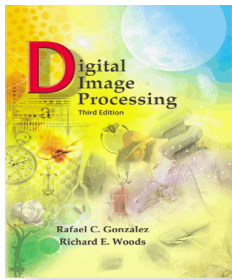
$$\partial^2 f / \partial x^2 = f(x+1) + f(x-1) - 2f(x)$$

Example



- Isolated Point
- Flat Segments
- Edges
 - Step
 - Ramp
 - Roof (line)





Chapter 3

Intensity Transformations & Spatial Filtering

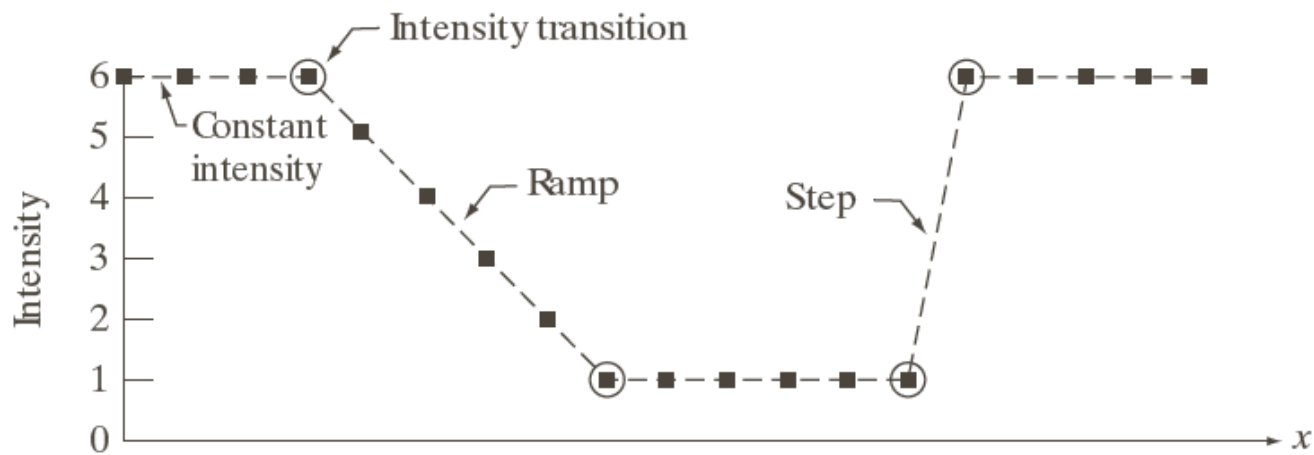
Properties of Derivatives

- First Derivative

- Must be zero in the areas of constant intensity
- Must be non-zero at the onset of an intensity ramp or step
- Must be *non-zero* along ramps

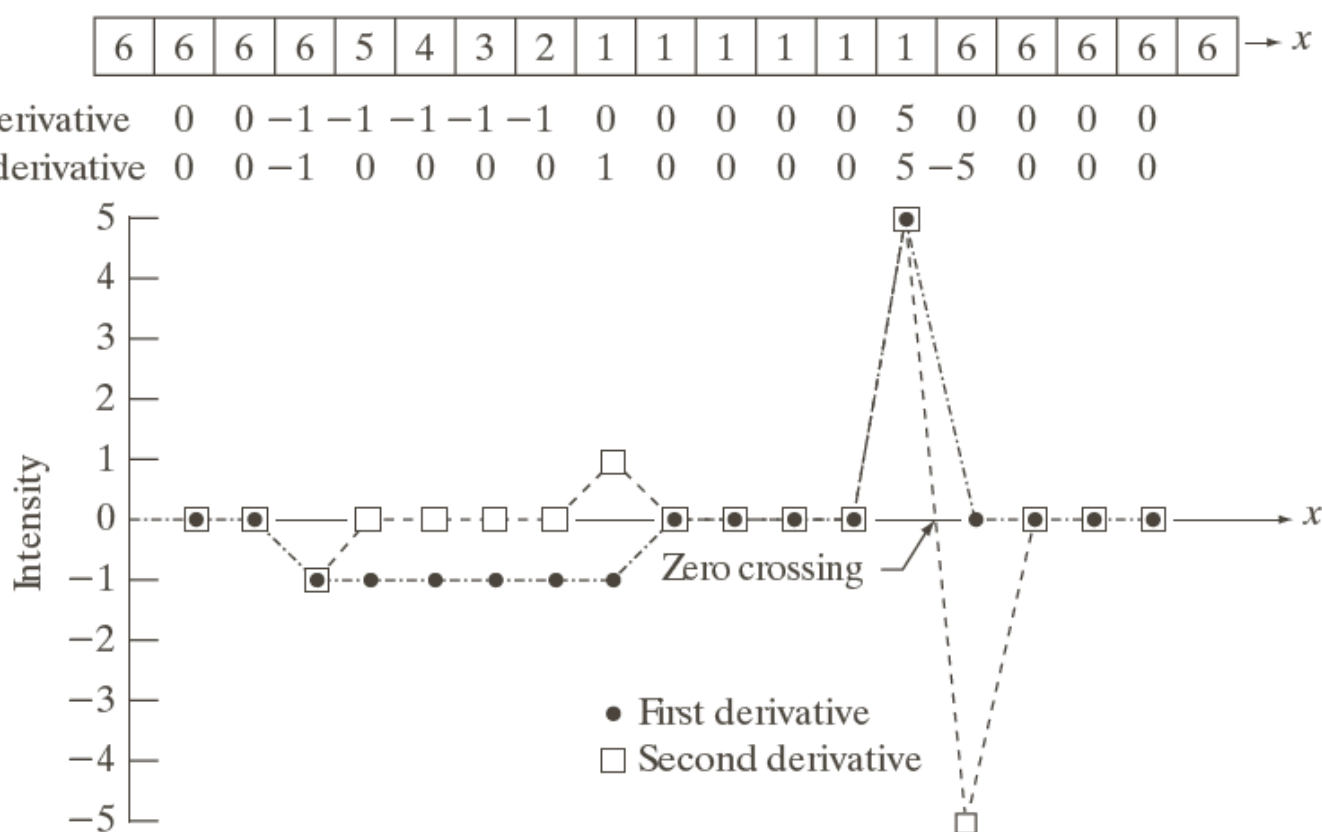
- Second Derivative

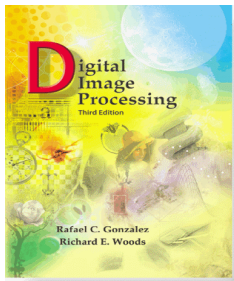
- Must be zero in the areas of constant intensity
- Must be non-zero at the onset *and end* of a ramp or step
- Must be *zero* along ramps of *constant slope*



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.





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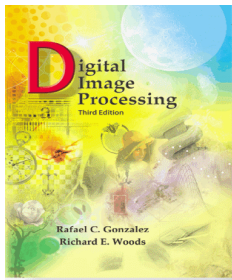
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Intensity Transformations & Spatial Filtering

LAPLACIAN



Chapter 3

Intensity Transformations & Spatial Filtering

- Laplacian
 - Isotropic
 - Rotation Invariant

0	1	0
1	-4	1
0	1	0

$$\nabla^2 f = \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2$$

$$\partial^2 f / \partial x^2 = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\partial^2 f / \partial y^2 = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

0	-1	0
-1	4	-1
0	-1	0

1	1	1
1	-8	1
1	1	1

-1	-1	-1
-1	8	-1
-1	-1	-1

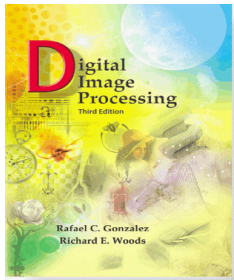
a b
c d

FIGURE 3.37

(a) Filter mask used to implement Eq. (3.6-6).

(b) Mask used to implement an extension of this equation that includes the diagonal terms.

(c) and (d) Two other implementations of the Laplacian found frequently in practice.



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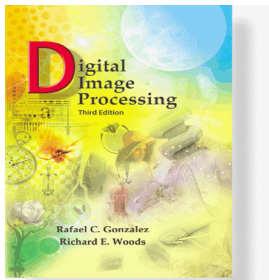
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Chapter 3

Intensity Transformations & Spatial Filtering

- Sharpening Filters

$$g(x,y) = f(x,y) + c [\nabla^2 f(x,y)]$$



ed.

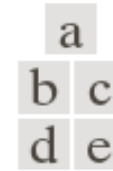


FIGURE 3.38

(a) Blurred image of the North Pole of the moon.

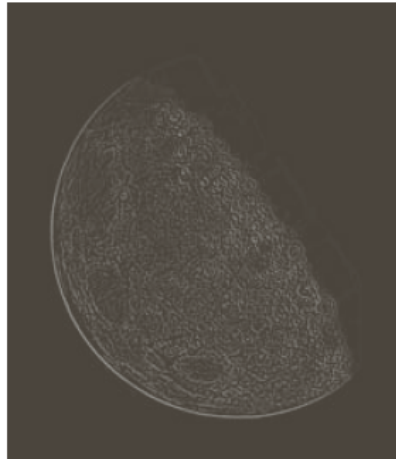
(b) Laplacian without scaling.

(c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a).

(e) Result of using the mask in Fig. 3.37(b).

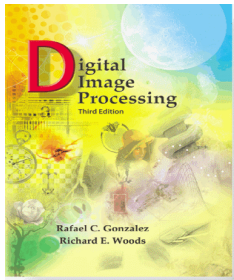
(Original image courtesy of NASA.)

0	1	0
1	-4	1
0	1	0



1	1	1
1	-8	1
1	1	1





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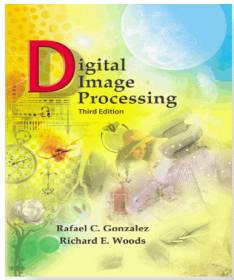
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UNSHARP MASKING & HIGHBOOST FILTERING



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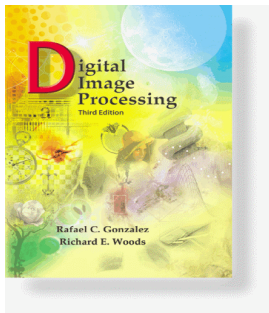
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Chapter 3

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- Unsharp Masking & Highboost Filtering
 - Blur (unsharp) the original image
 - Subtract blurred image from original to get a mask
 - Add the mask to the original



Chapter 3

Intensity Transformations & Spatial Filtering

- Unsharp Masking & Highboost Filtering

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

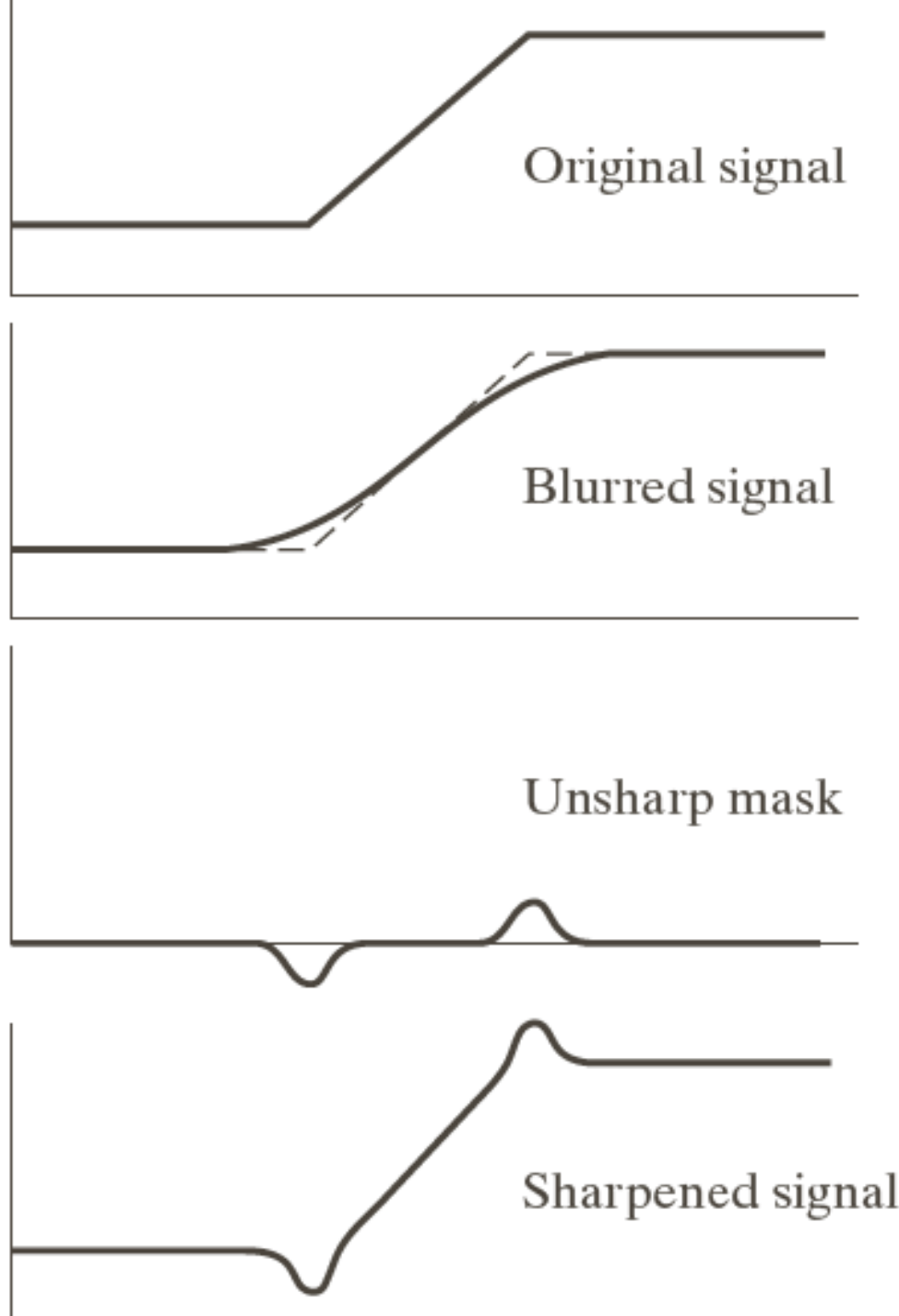
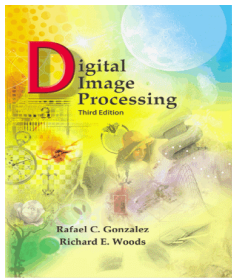
$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

$\bar{f}(x,y)$: Blurred $f(x,y)$

$k=1$: Unsharp Masking

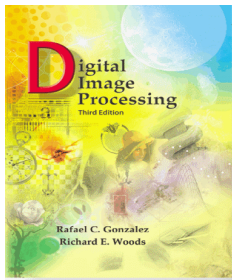
$k>1$: Highboost Filtering

$k<1$: De-emphasized Unsharp Masking



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).



Digital Image I

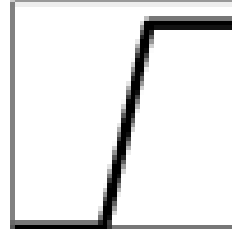
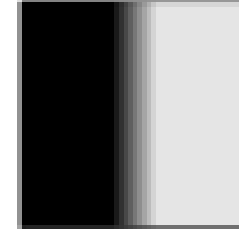
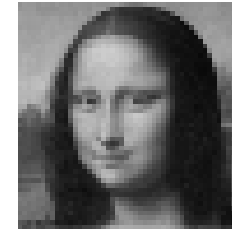
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Ch 1 Intensity Transform

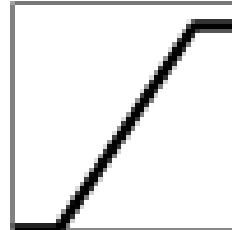
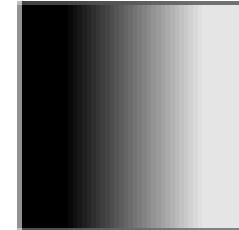
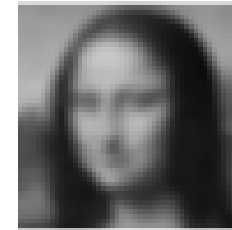
$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

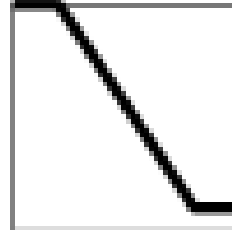
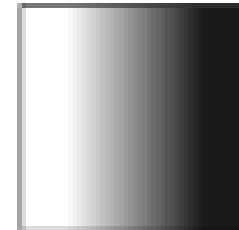
1. Original



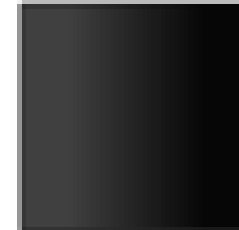
2. Blurred



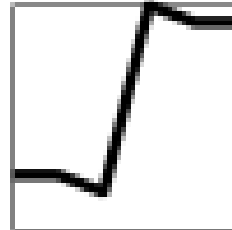
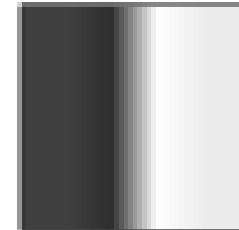
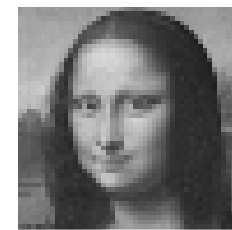
3. Blurred/
inverted



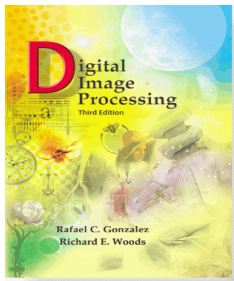
4. Blurred/
inverted/
scaled



5. Blurred/
inverted/
scaled +
Original



Source: https://en.wikipedia.org/wiki/Unsharp_masking



DIP-XE

a
b
c
d
e

DIP-XE

DIP-XE

DIP-XE

DIP-XE

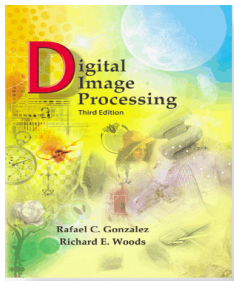
FIGURE 3.40

(a) Original image.

(b) Result of blurring with a Gaussian filter.

(c) Unsharp mask. (d) Result of using unsharp masking.

(e) Result of using highboost filtering.



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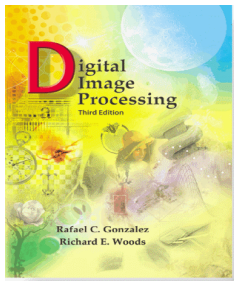
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$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$



Source: https://en.wikipedia.org/wiki/Unsharp_masking



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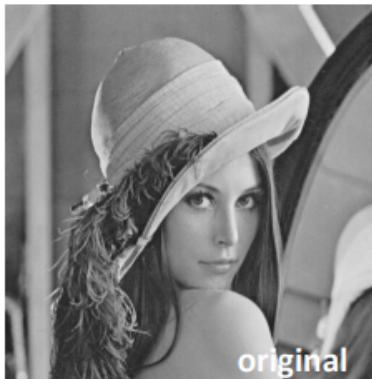
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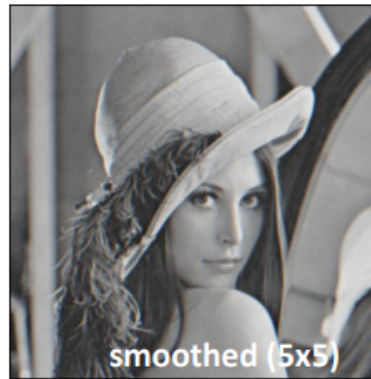
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- What does blurring take away?



—



=



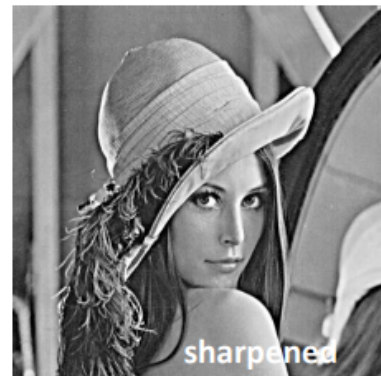
- Let us add it back:



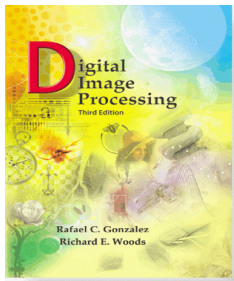
+ α



=



Source: http://www.cs.cornell.edu/courses/cs6670/2011sp/lectures/lec02_filter.pdf



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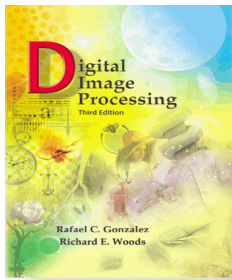
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Intensity Transformations & Spatial Filtering

IMAGE GRADIENTS



Chapter 3

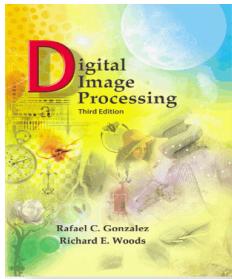
Intensity Transformations & Spatial Filtering

First Order Derivatives: Gradient Magnitude

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \end{bmatrix}$$

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

$$M(x, y) \approx |g_x| + |g_y|$$



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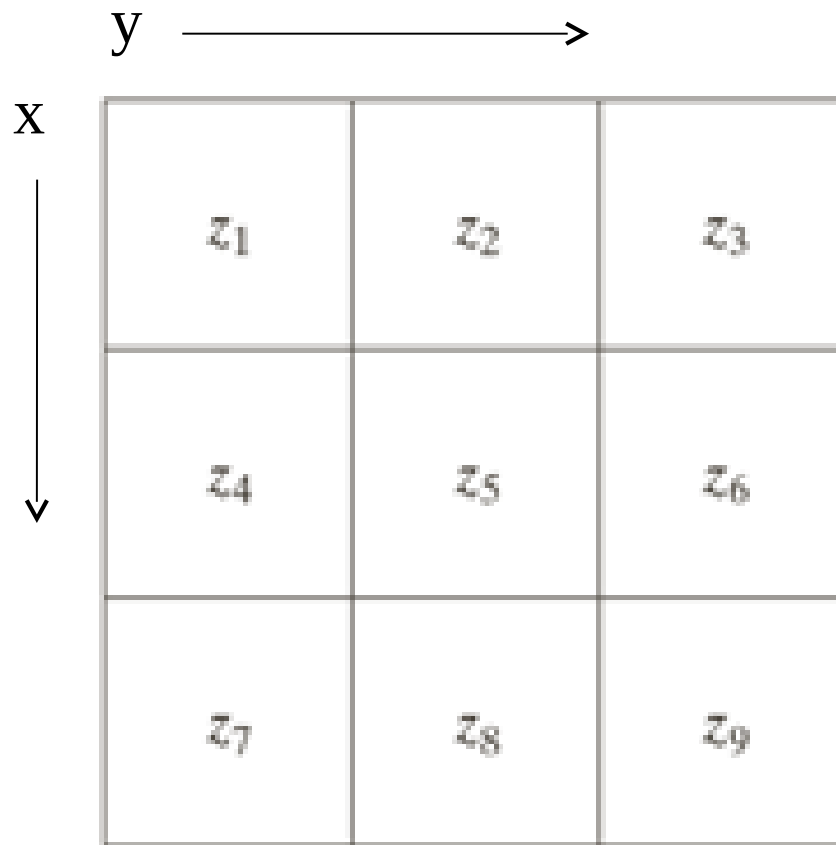
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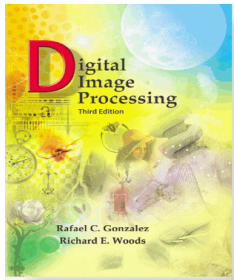
Intensity Transformations & Spatial Filtering



- Gradient Mask
 - Simple Approximation

$$g_x = z_8 - z_5$$

$$g_y = z_6 - z_5$$



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Intensity Transformations & Spatial Filtering

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

A 2D grid with 2 rows and 2 columns. The x-axis is labeled 'x' and the y-axis is labeled 'y'. The values in the cells are:

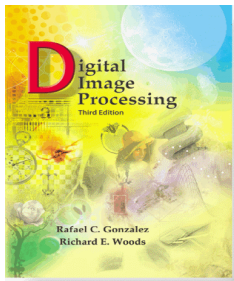
-1	0
0	1

Gradient Mask: Robert's Operator

$$g_x = z_9 - z_5, g_y = z_8 - z_6$$

$$M(x,y)=\sqrt{(z_9-z_5)^2+(z_8-z_6)^2}$$

$$M(x,y) \approx |z_9 - z_5| + |z_8 - z_6|$$



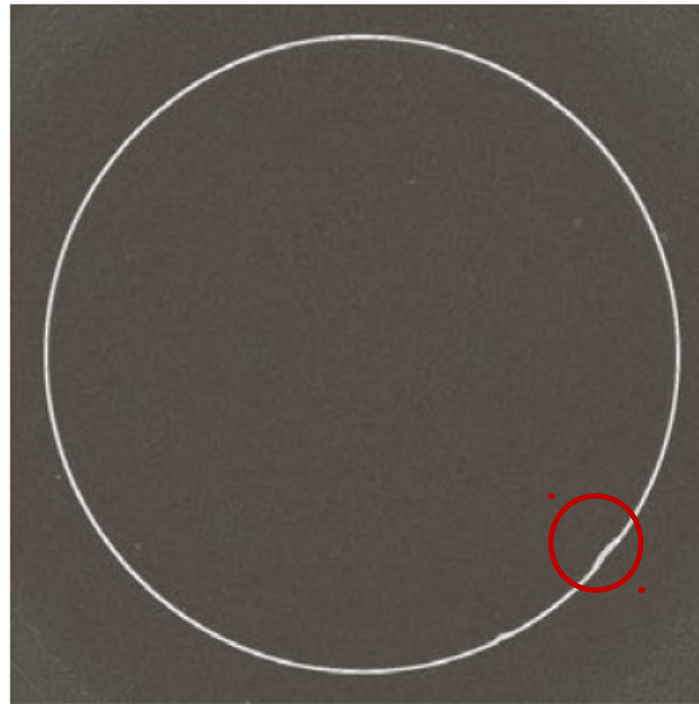
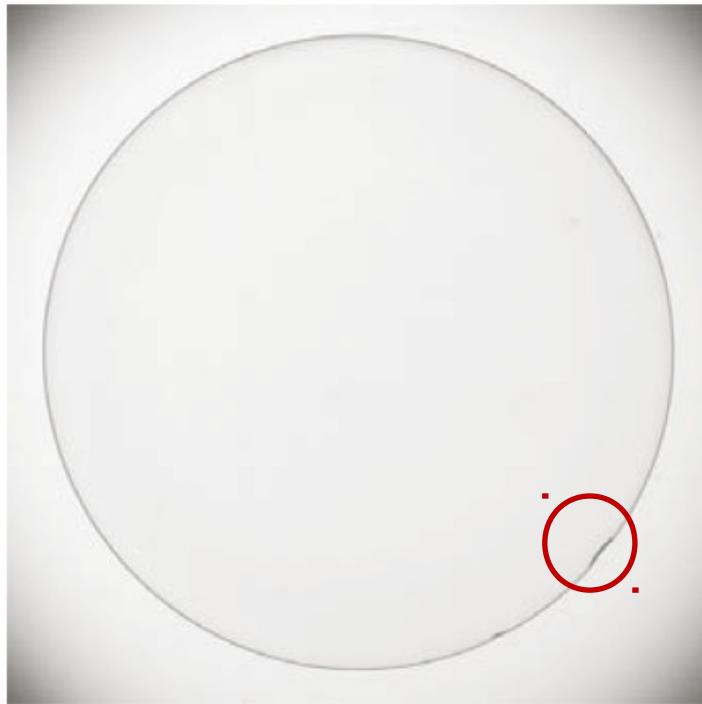
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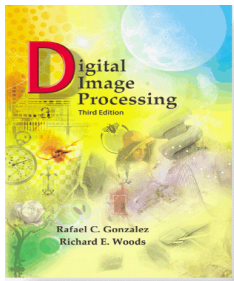
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a b

FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Pete Sites, Perceptics Corporation.)

Sobel: Example Application



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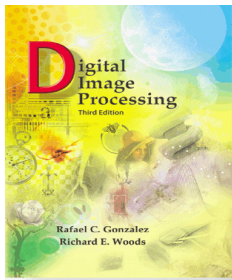
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EDGE DETECTION



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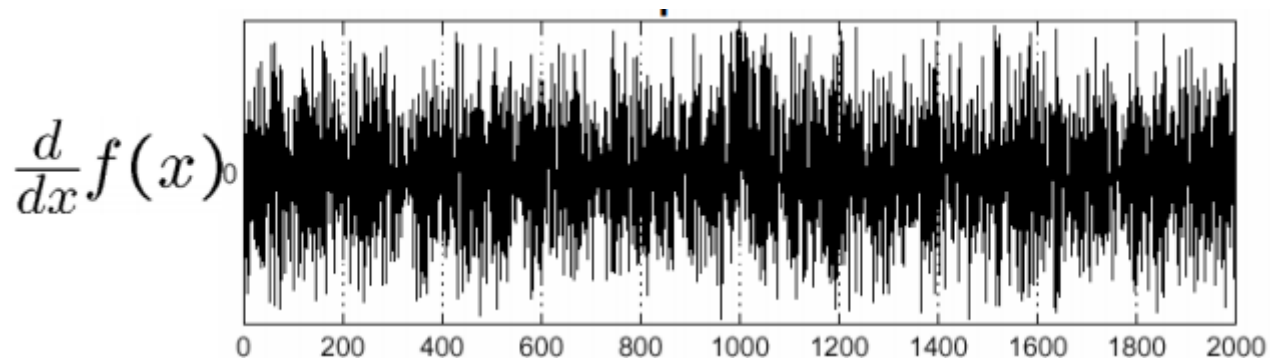
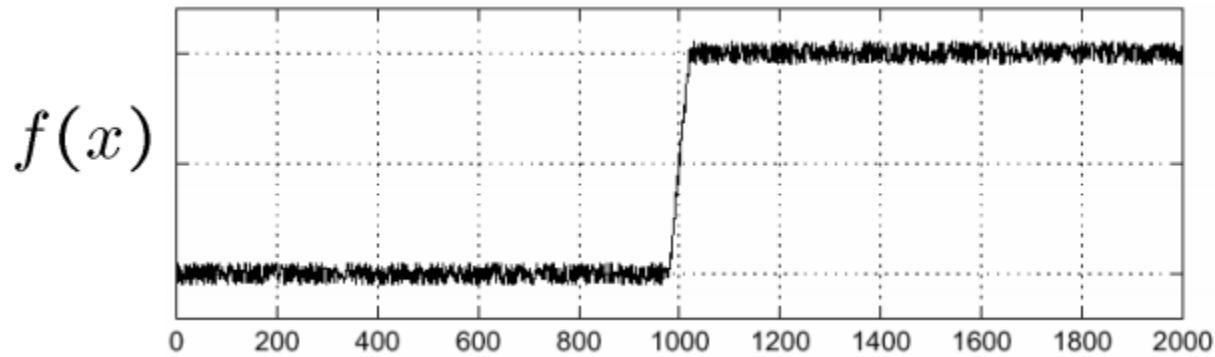
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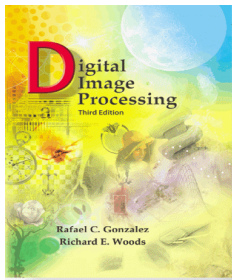
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- Where is the edge?





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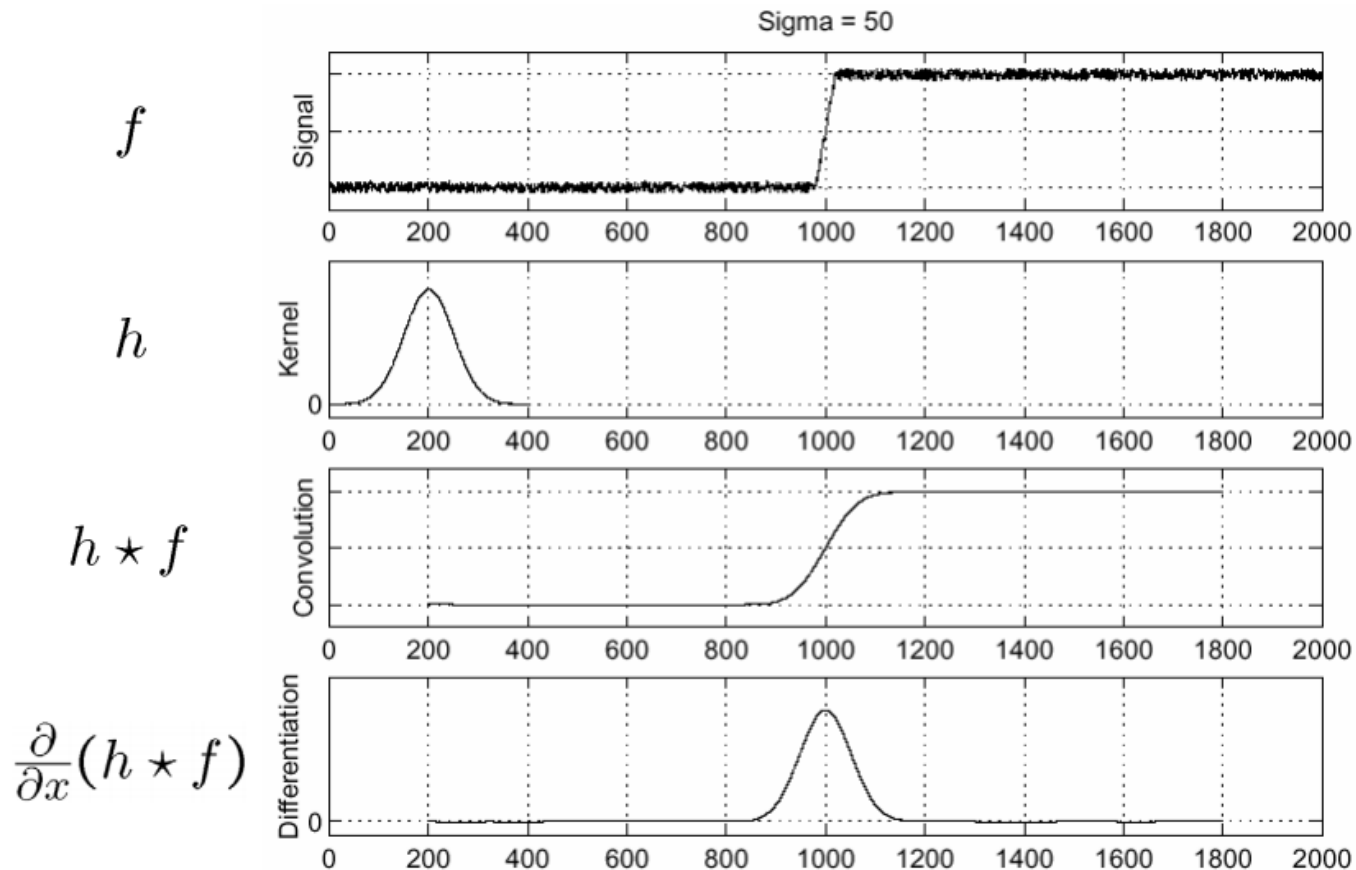
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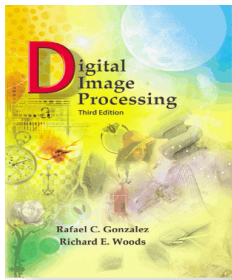
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- Smooth first





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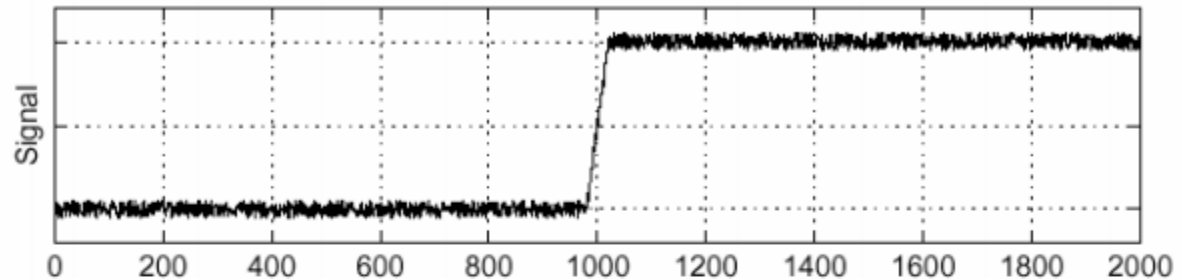
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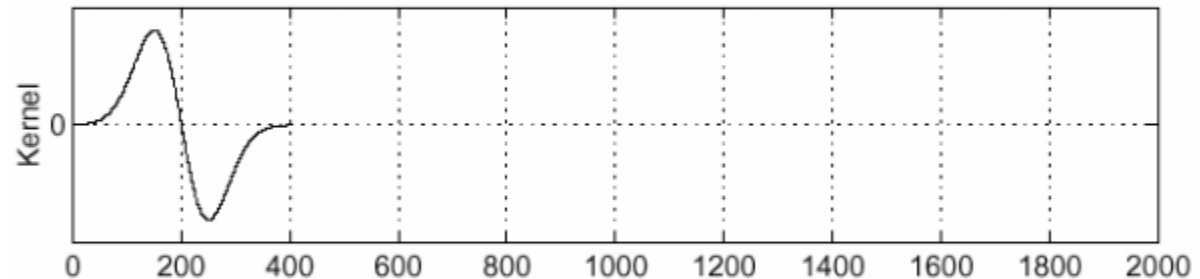
Intensity Transformations & Spatial Filtering

- Short-cut Computation

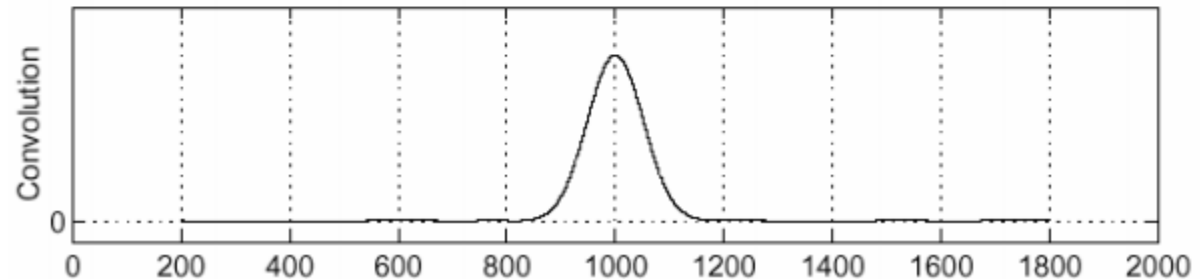
f

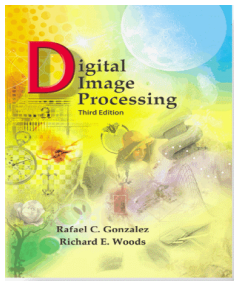


$\frac{\partial}{\partial x} h$



$(\frac{\partial}{\partial x} h) \star f$





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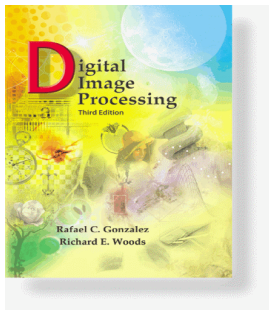
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LAPLACIAN OF GAUSSIAN



Chapter 3

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• LoG: Laplacian of Gaussian

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 p_r(r_0) = 1.33 [1]$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 p_r(r_0) + 7 p_r(r_1) = 3.08 [3]$$

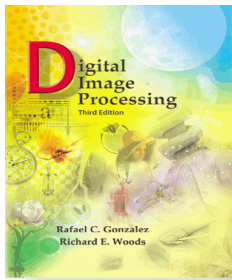
$$s_2 = 4.55 [4], s_3 = 5.67 [5], s_4 = 6.23 [6],$$

$$s_5 = 6.65 [7], s_6 = 6.86 [7], s_7 = 7.00 [7]$$

$$\nabla^2 G(x,y) = \partial^2 G(x,y) / \partial x^2 + \partial^2 G(x,y) / \partial y^2$$

$$\text{[Empty Box]} = \frac{1}{2\pi\sigma^2} \frac{\partial}{\partial x} \left[\frac{-x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \right] + \frac{1}{2\pi\sigma^2} \frac{\partial}{\partial y} \left[\frac{-y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \right]$$

$$\text{[Empty Box]} = \frac{1}{2\pi\sigma^2} \left[\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2} \right] e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{1}{2\pi\sigma^2} \left[\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2} \right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Chapter 3

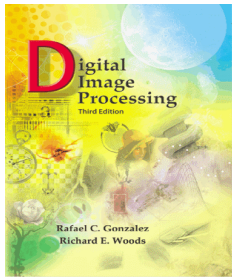
Intensity Transformations & Spatial Filtering

- Laplacian of Gaussian (LoG)

$$\nabla^2 G(x,y) = \frac{1}{2\pi\sigma^2} \left[\frac{x^2+y^2-2\sigma^2}{\sigma^4} \right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$
$$\text{LoG} = -\frac{1}{\pi\sigma^4} \left[1 - \frac{x^2+y^2}{2\sigma^2} \right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Mexican Hat

- positive central term
- surrounded by an adjacent negative region
 - a function of distance
- zero outer region



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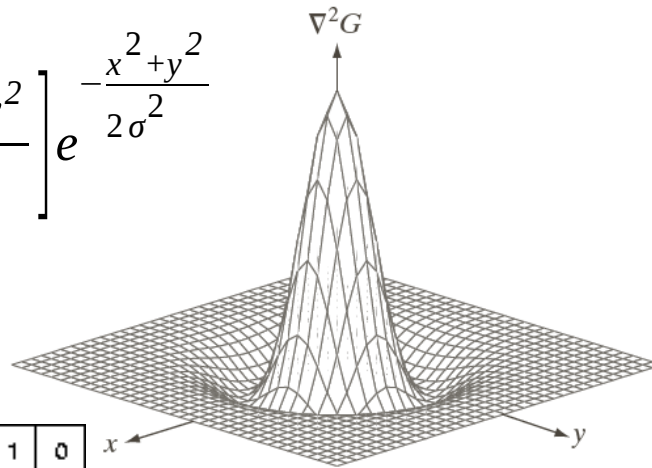
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Chapter 3

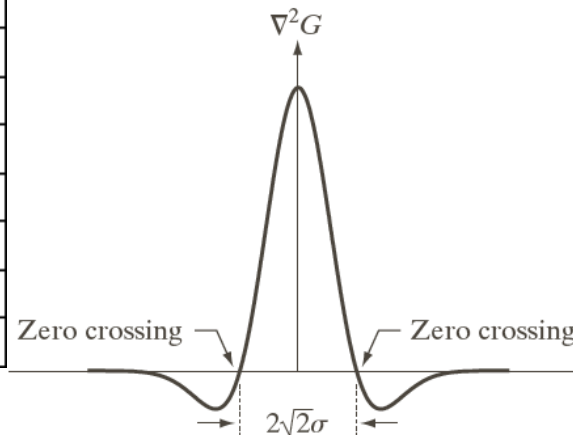
Intensity Transformations & Spatial Filtering

$$\nabla^2 G(x, y) =$$

$$-\frac{1}{\pi\sigma^4} \left[1 - \frac{x^2 + y^2}{2\sigma^2} \right] e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



0	1	1	2	2	2	1	1	0
1	2	4	5	5	5	4	2	1
1	4	5	3	0	3	5	4	1
2	5	3	-12	-24	-12	3	5	2
2	5	0	-24	-40	-24	0	5	2
2	5	3	-12	-24	-12	3	5	2
1	4	5	3	0	3	5	4	1
1	2	4	5	5	5	4	2	1
0	1	1	2	2	2	1	1	0

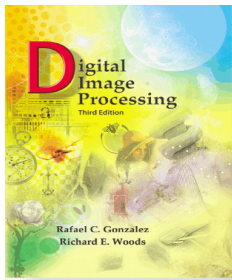


0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

a b
c d

FIGURE 10.21

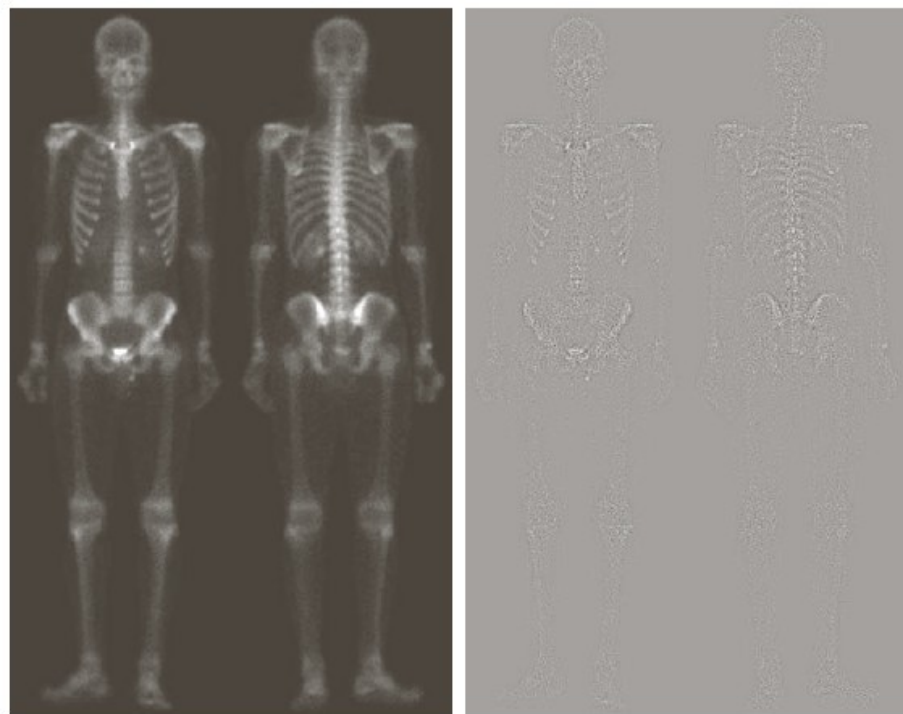
(a) Three-dimensional plot of the *negative* of the LoG. (b) Negative of the LoG displayed as an image. (c) Cross section of (a) showing zero crossings. (d) 5×5 mask approximation to the shape in (a). The negative of this mask would be used in practice.



Chapter 3

Intensity Transformations & Spatial Filtering

- Composite Spatial Enhancement Example:
 - Laplacian to enhance fine details
 - Gradient to enhance prominent edges
 - Smooth gradient image
 - Product of Laplacian and Gradient as mask
 - Add back to original image
 - Power-law transformation to stretch gray levels

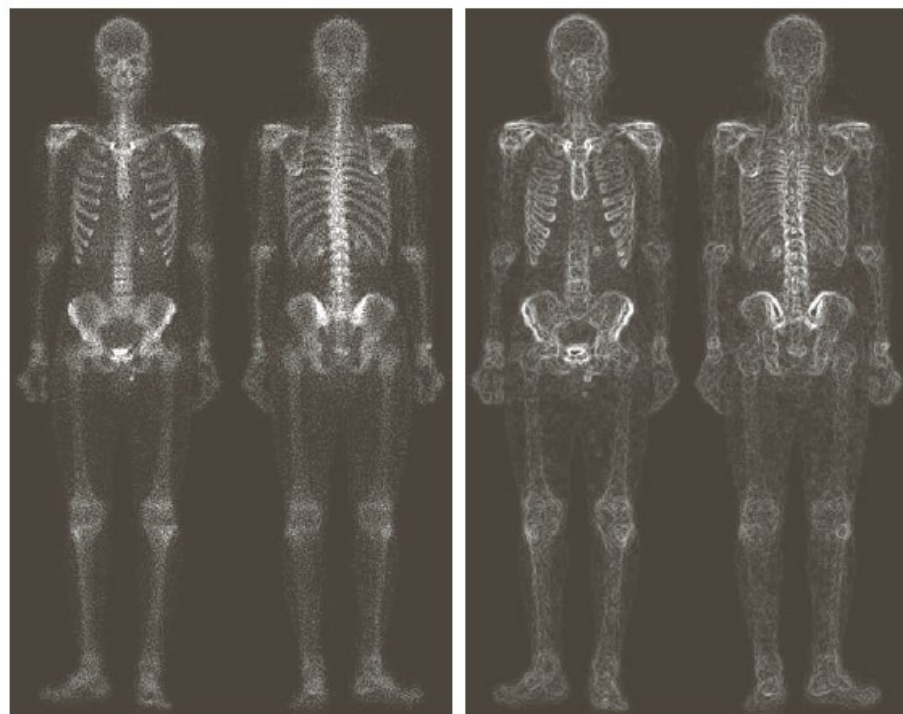


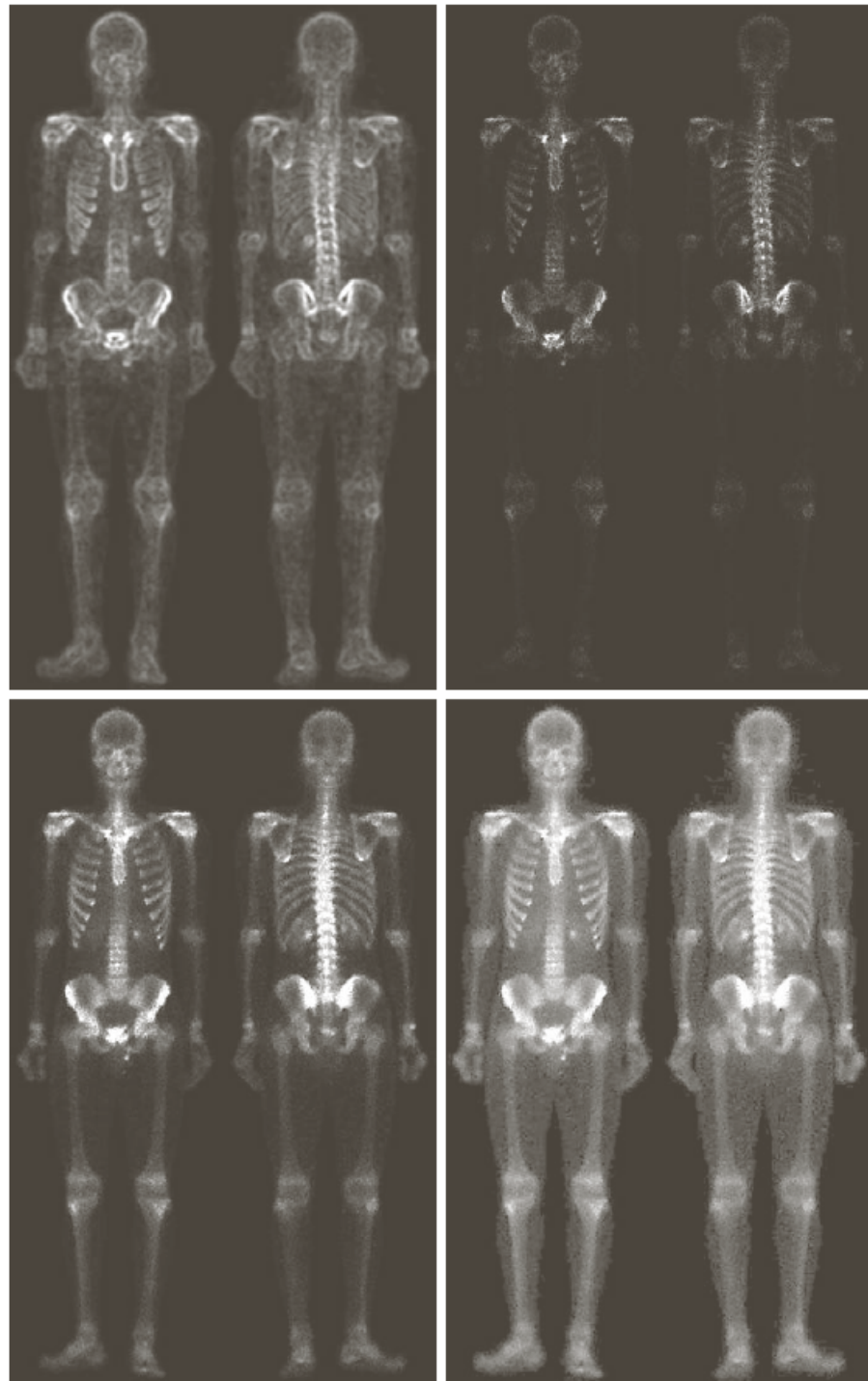
a	b
c	d

FIGURE 3.43

(a) Image of whole body bone scan.

(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).





e f
g h

ed.

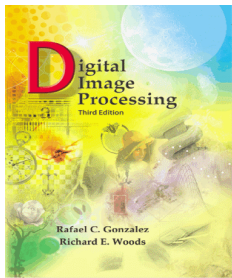
FIGURE 3.43

(Continued)

(e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e).

(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

tering



Chapter 3

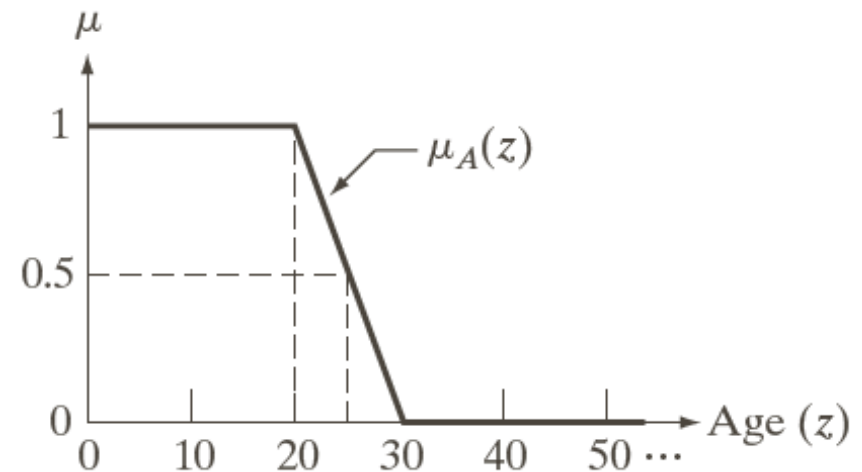
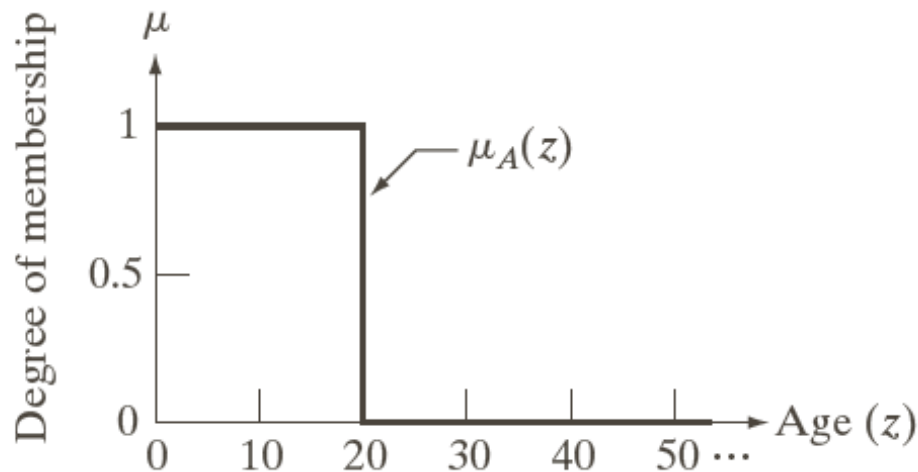
Intensity Transformations & Spatial Filtering

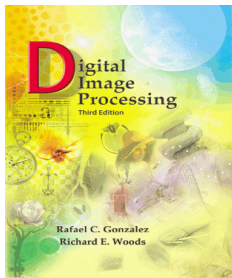
a b

- Using Fuzzy Sets for Enhancement :

FIGURE 3.44

Membership functions used to generate (a) a crisp set, and (b) a fuzzy set.





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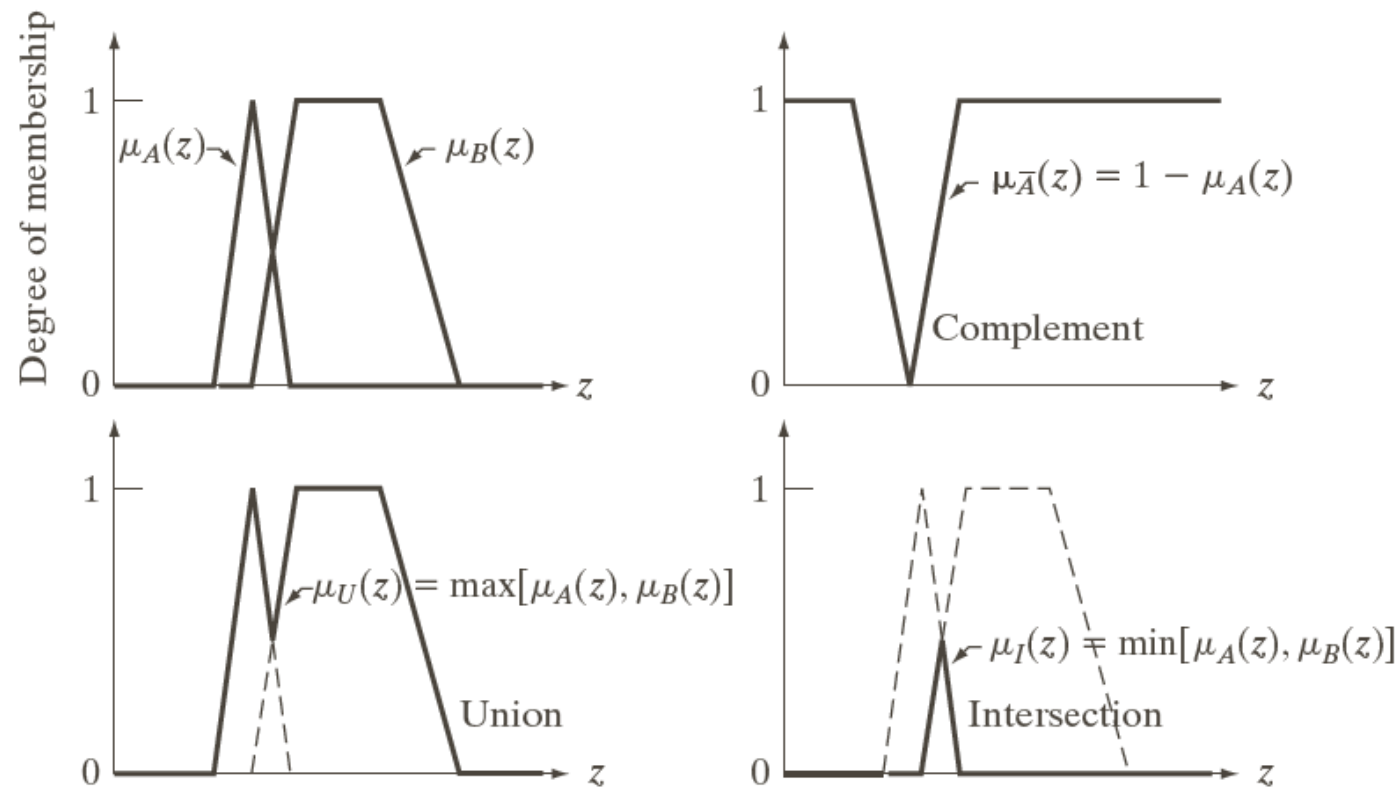
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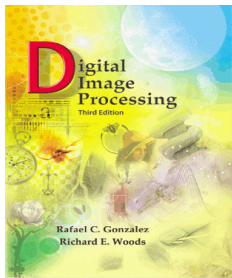
Intensity Transformations & Spatial Filtering



a	b
c	d

FIGURE 3.45

(a) Membership functions of two sets, A and B. (b) Membership function of the complement of A. (c) and (d) Membership functions of the union and intersection of the two sets.



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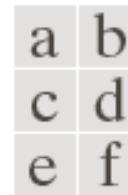
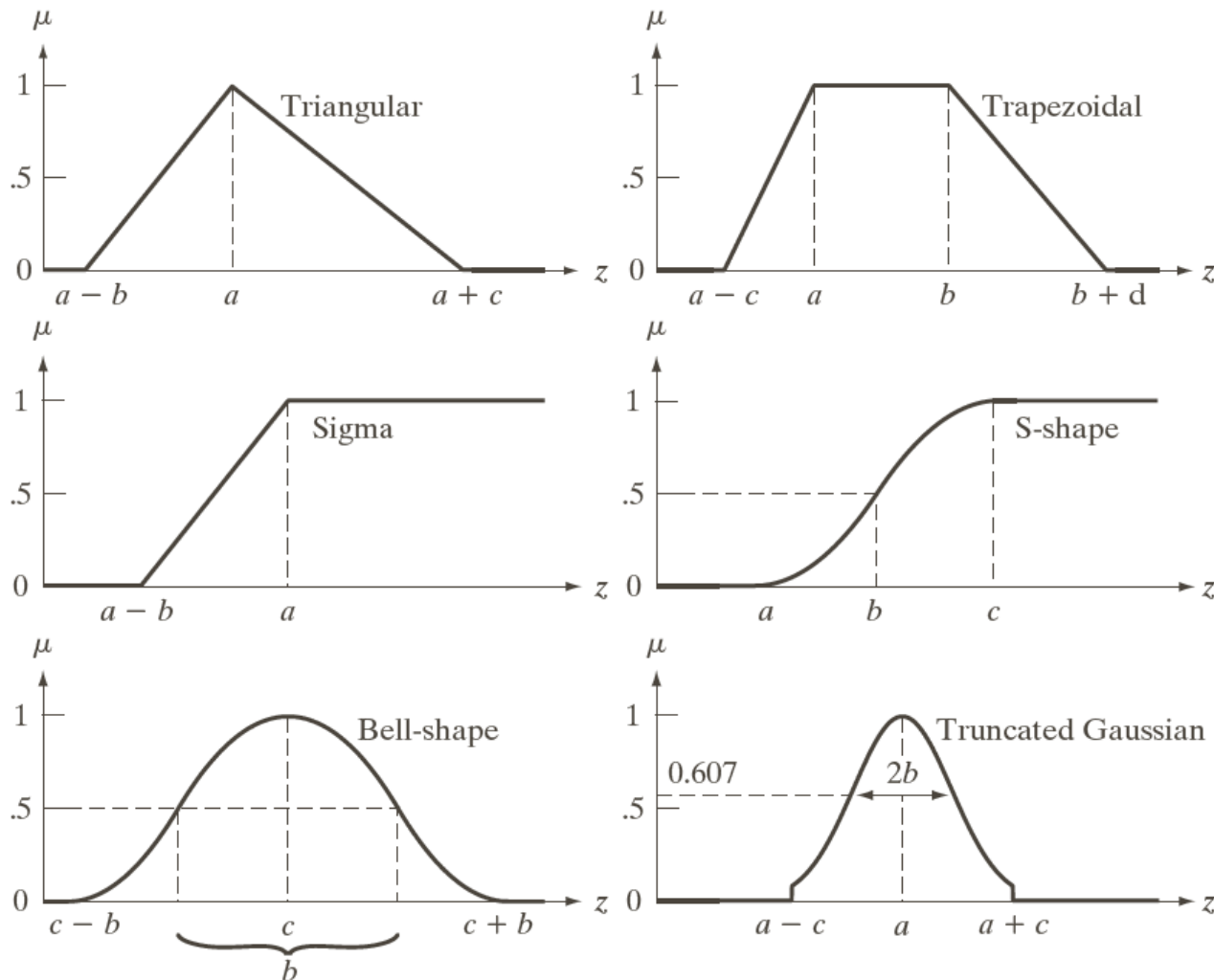
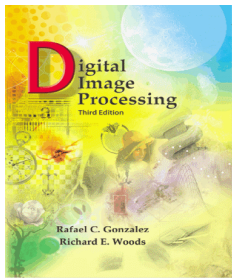
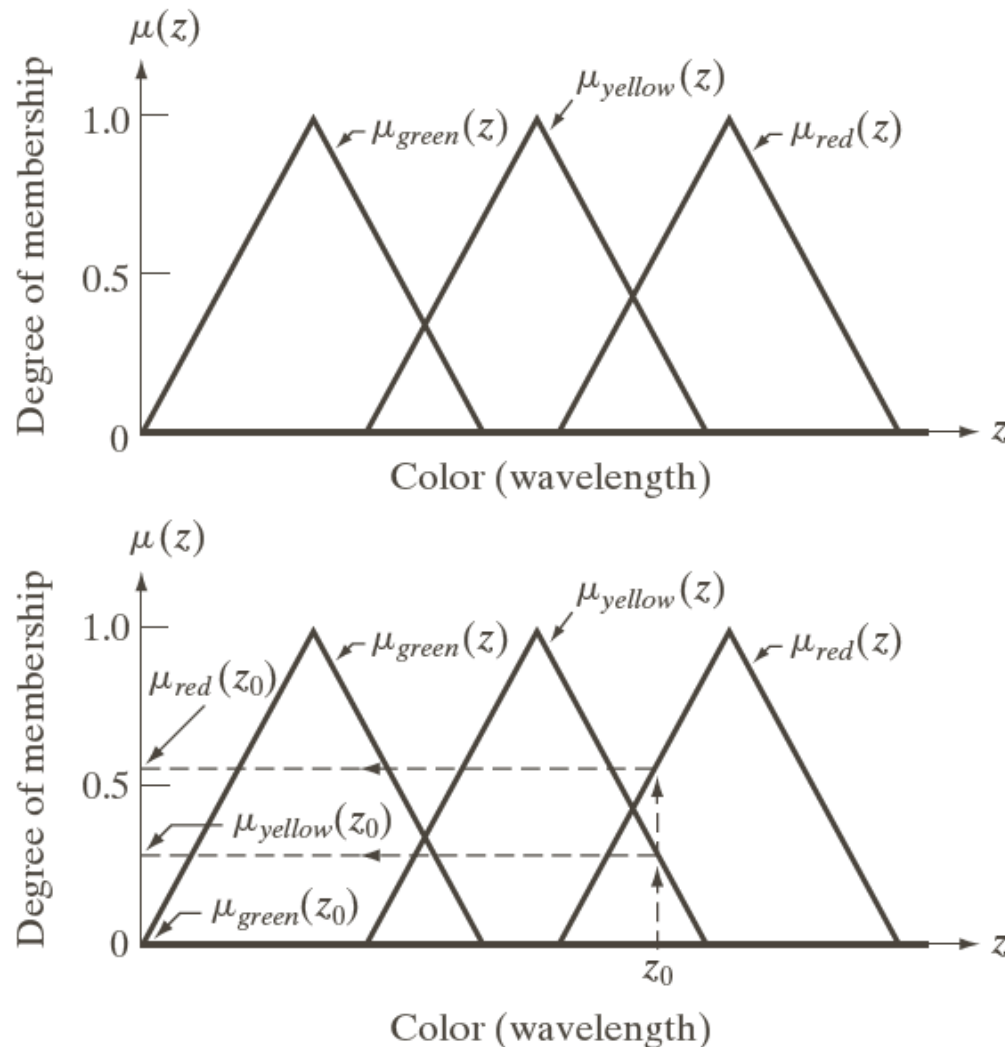


FIGURE 3.46
Membership functions corresponding to Eqs. (3.8-6)–(3.8-11).



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Intensity Transformations & Spatial Filtering



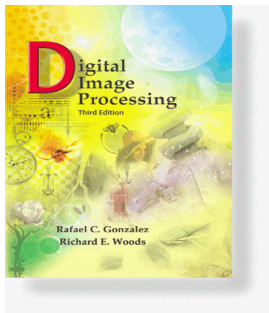
a
b

Fuzzy Inference

FIGURE 3.47

(a) Membership functions used to fuzzify color.

(b) Fuzzifying a specific color z_0 . (Curves describing color sensation are bell shaped; see Section 6.1 for an example. However, using triangular shapes as an approximation is common practice when working with fuzzy sets.)



Chapter 3

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- Fuzzy IF-THEN Rules:

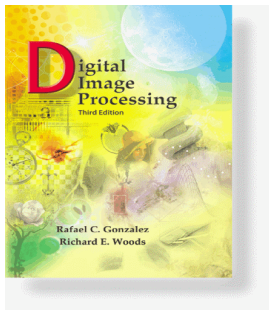
R_1 : IF the color is *green*, THEN the fruit is *verdant*.

OR

R_2 : IF the color is *yellow*, THEN the fruit is *half-mature*.

OR

R_3 : IF the color is *red*, THEN the fruit is *mature*.



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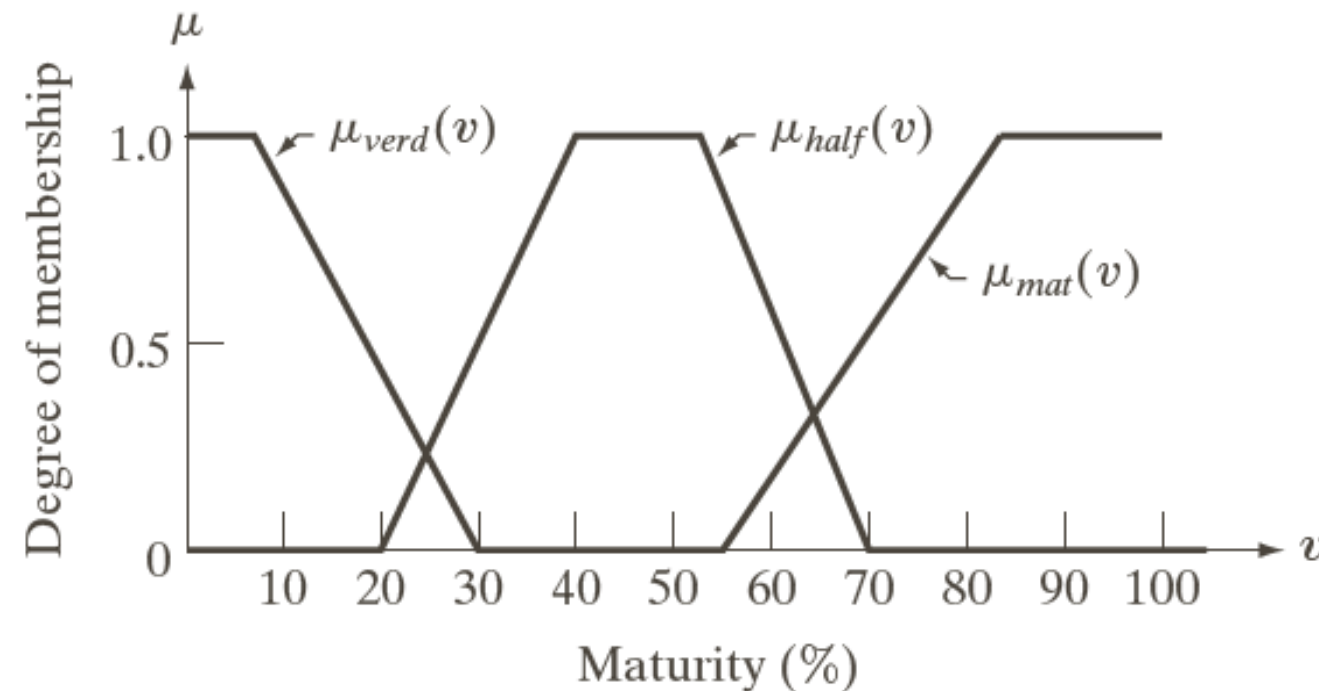


FIGURE 3.48
Membership functions characterizing the outputs *verdant*, *half-mature*, and *mature*.

a	b
c	d

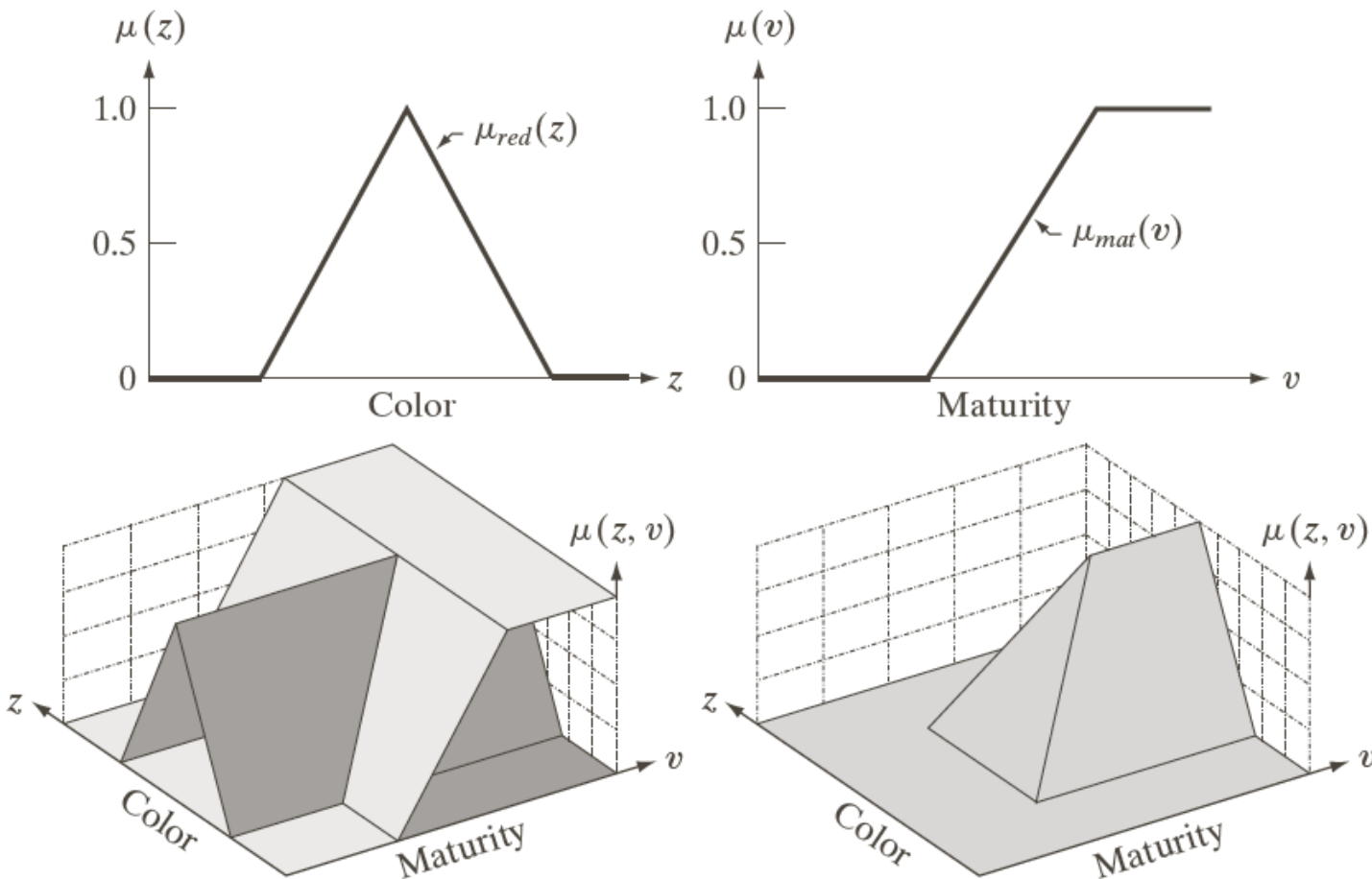
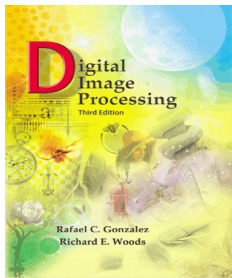


FIGURE 3.49
 (a) Shape of the membership function associated with the color red, and
 (b) corresponding output membership function. These two functions are associated by rule R_3 .
 (c) Combined representation of the two functions. The representation is 2-D because the independent variables in (a) and (b) are different.
 (d) The AND of (a) and (b), as defined in Eq. (3.8-5).

$$\mu_3(z, v) = \min \left\{ \mu_{red}(z), \mu_{mat}(v) \right\}$$



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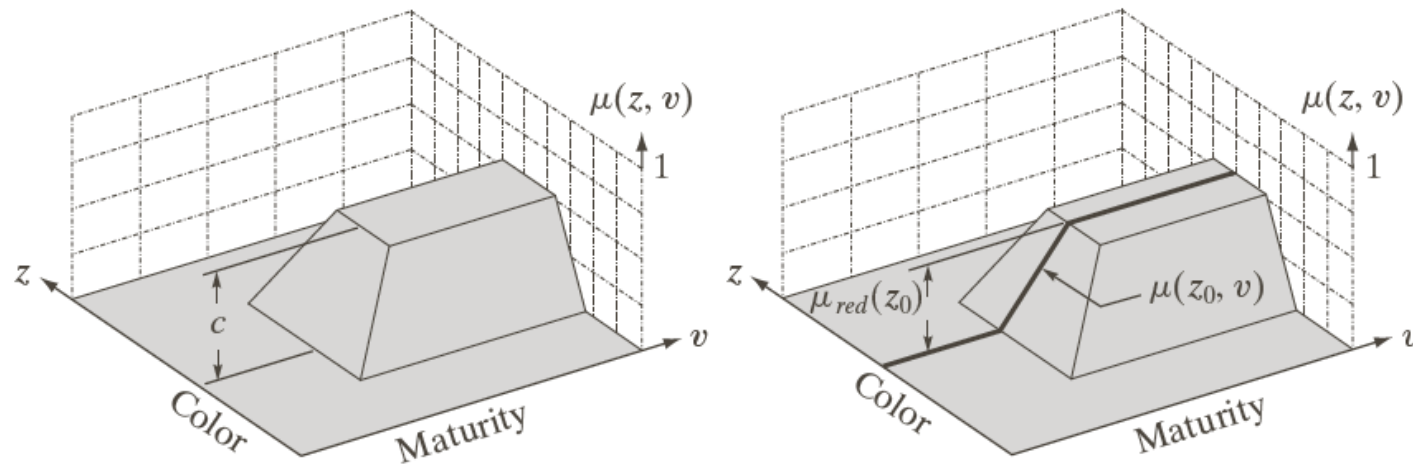


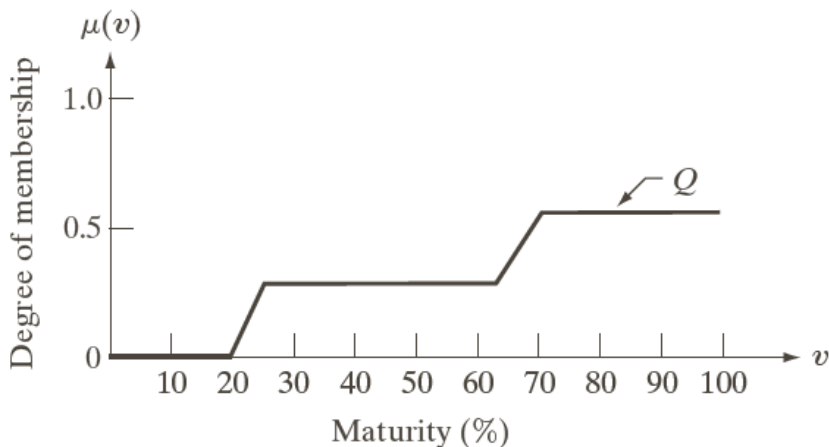
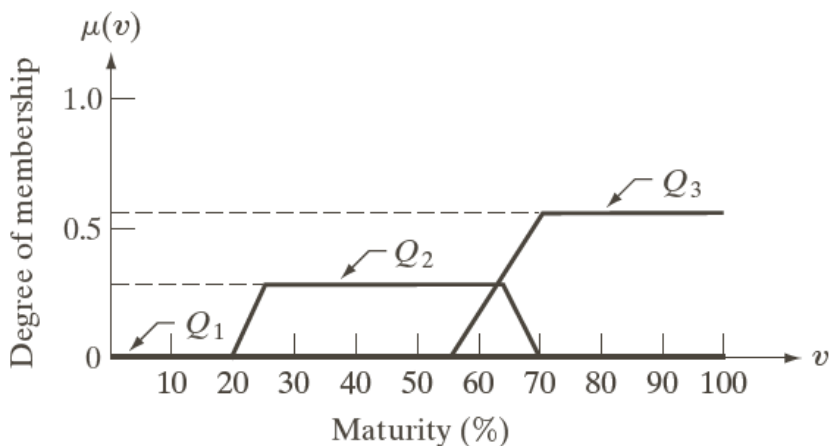
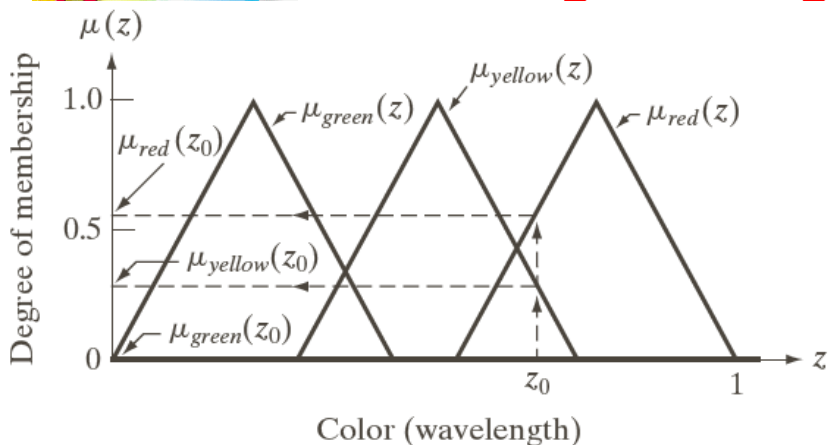
FIGURE 3.50

(a) Result of computing the minimum of an arbitrary constant, c , and function $\mu_3(z, v)$ from Eq. (3.8-12). The minimum is equivalent to an AND operation. (b) Cross section (dark line) at a specific color, z_0 .

$$Q_3(v) = \min \left\{ \mu_{red}(z_0), \mu_3(z_0, v) \right\}$$

$$Q_2(v) = \min \left\{ \mu_{yellow}(z_0), \mu_2(z_0, v) \right\}$$

$$Q_1(v) = \min \left\{ \mu_{green}(z_0), \mu_1(z_0, v) \right\}$$



$$\left\{ S \left\{ \mu_s(z_0), \mu_r(z_0, v) \right\} \right\}$$



a
b
c

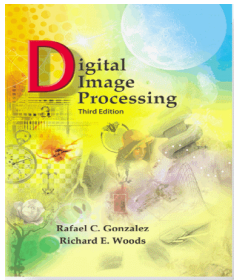
Defuzzification

$$v_0 = \frac{\sum_{v=1}^K vQ(v)}{\sum_{v=1}^K Q(v)}$$

FIGURE 3.51

(a) Membership functions with a specific color, z_0 , selected.

(b) Individual fuzzy sets obtained from Eqs. (3.8-13)–(3.8-15). (c) Final fuzzy set obtained by using Eq. (3.8-16) or (3.8-17).



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- Fuzzy Rule-based Logic:

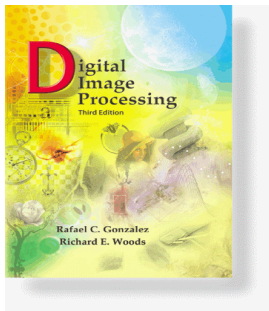
R_1 : IF (z , *green*) THEN (v , *verdant*).

OR

R_2 : IF (z , *yellow*) THEN (v , *half-mature*).

OR

R_3 : IF (z , *red*) THEN (v , *mature*).



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- Fuzzy Rule-based Logic:

IF (z_1, A_{11}) AND (z_2, A_{12}) AND ... AND (z_N, A_{1N}) THEN (v, B_1) .

IF (z_1, A_{21}) AND (z_2, A_{22}) AND ... AND (z_N, A_{2N}) THEN (v, B_2) .

...

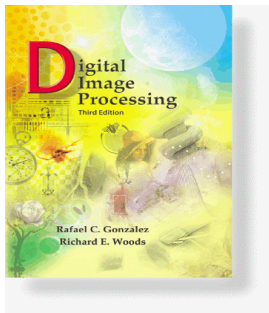
IF (z_1, A_{M1}) AND (z_2, A_{M2}) AND ... AND (z_N, A_{MN}) THEN (v, B_M) .

ELSE (v, B_E) .

$$j = 1, 2, \dots, M$$

$$\mu_{A_{ij}}(z_j)$$

$$\lambda_i = \min$$



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- Fuzzy Rule-based Logic:
 - Fuzzify the inputs
 - Perform any required fuzzy logical operations
 - Apply an implication method
 - Apply an aggregation method
 - Defuzzify the final output

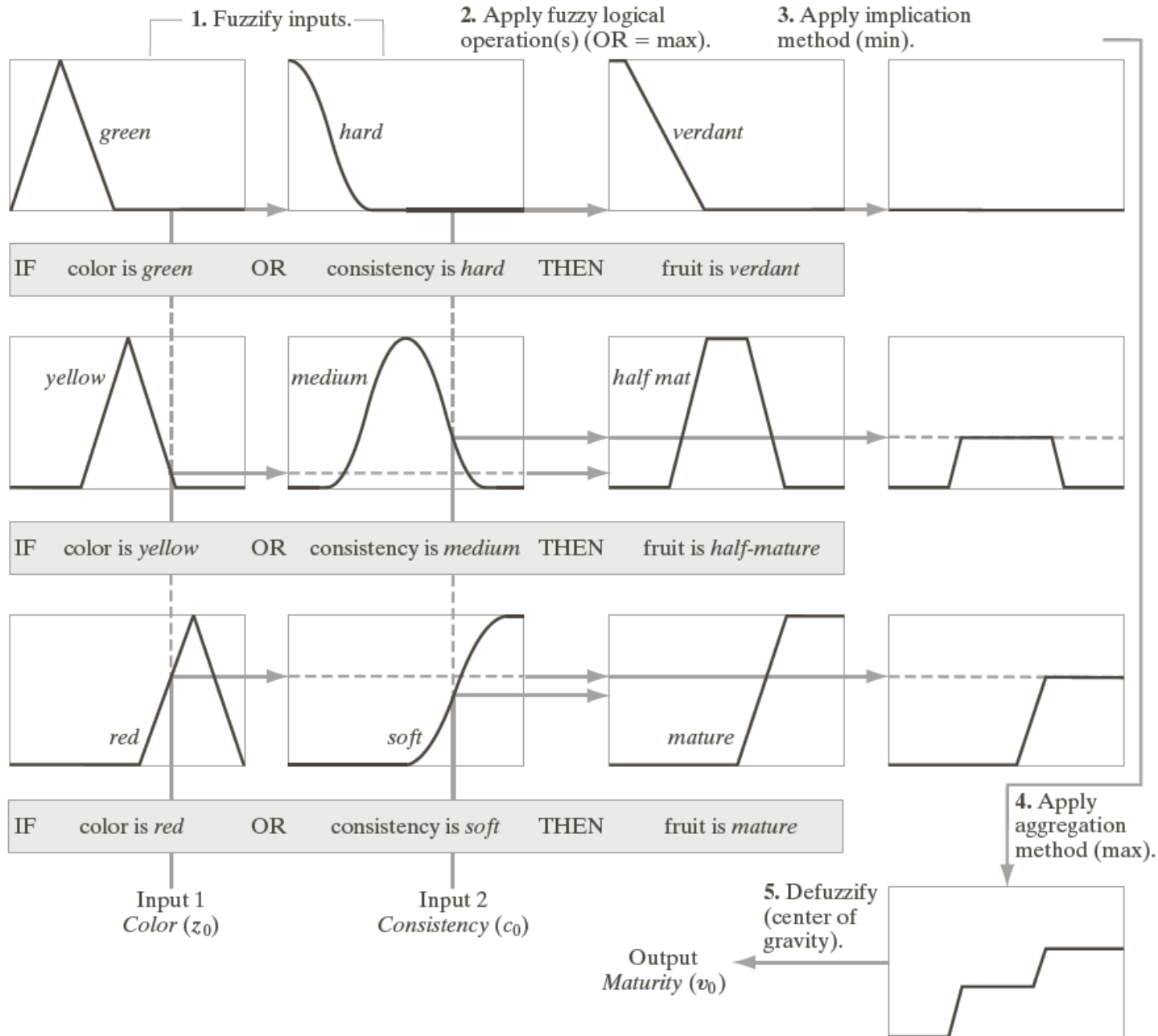
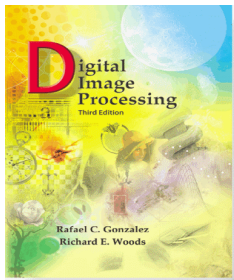


FIGURE 3.52 Example illustrating the five basic steps used typically to implement a fuzzy, rule-based system: (1) fuzzification, (2) logical operations (only OR was used in this example), (3) implication, (4) aggregation, and (5) defuzzification.



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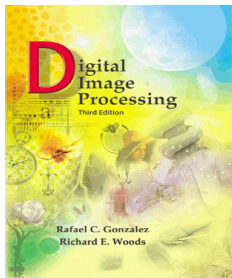
- Using Fuzzy Sets for Intensity Transformation:

IF a pixel is dark THEN make it darker.

IF a pixel is gray THEN make it gray.

IF a pixel is bright THEN make it brighter.

$$v_0 = \frac{\mu_{dark}(z_0) \times v_d + \mu_{gray}(z_0) \times v_g + \mu_{bright}(z_0) \times v_b}{\mu_{dark}(z_0) + \mu_{gray}(z_0) + \mu_{bright}(z_0)}$$



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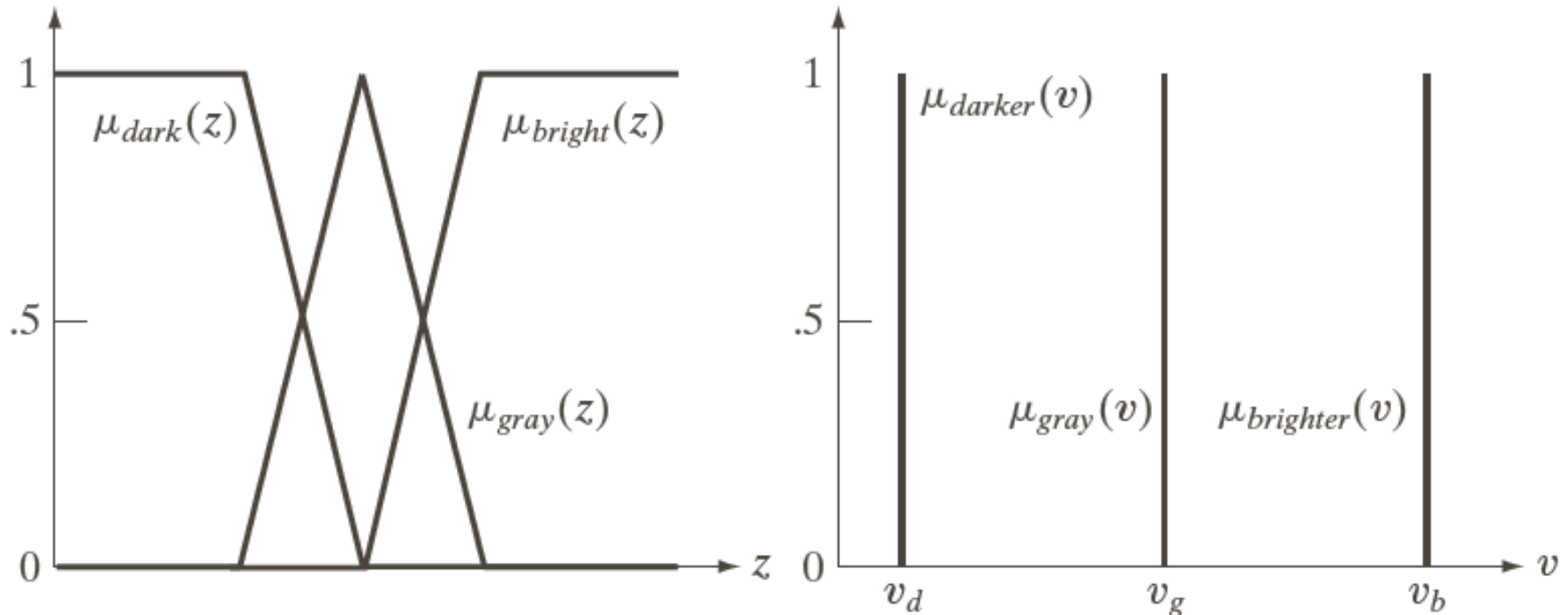
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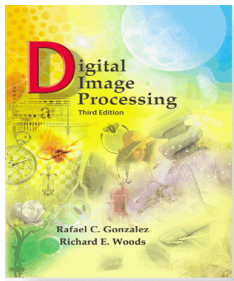
a b

FIGURE 3.53
(a) Input and
(b) output
membership
functions for
fuzzy, rule-based
contrast
enhancement.



$$v_0 = \frac{\mu_{dark}(z_0) \times v_d + \mu_{gray}(z_0) \times v_g + \mu_{bright}(z_0) \times v_b}{\mu_{dark}(z_0) + \mu_{gray}(z_0) + \mu_{bright}(z_0)}$$

$$v_d = 0 \text{ (black)}, v_g = 127 \text{ (mid-gray)}, v_b = 255 \text{ (white)}$$



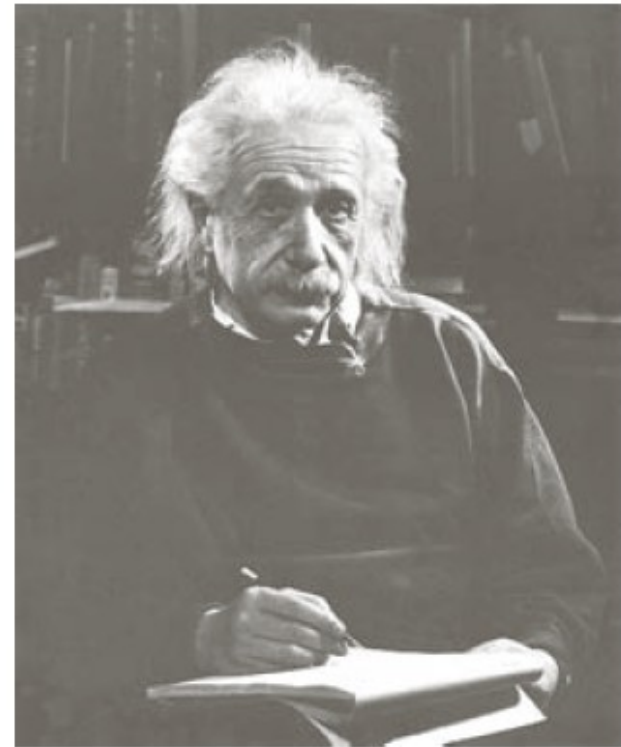
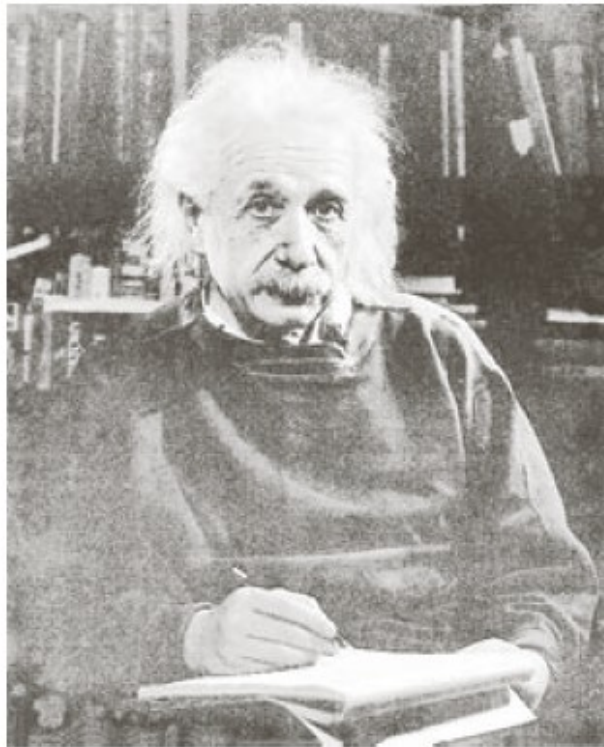
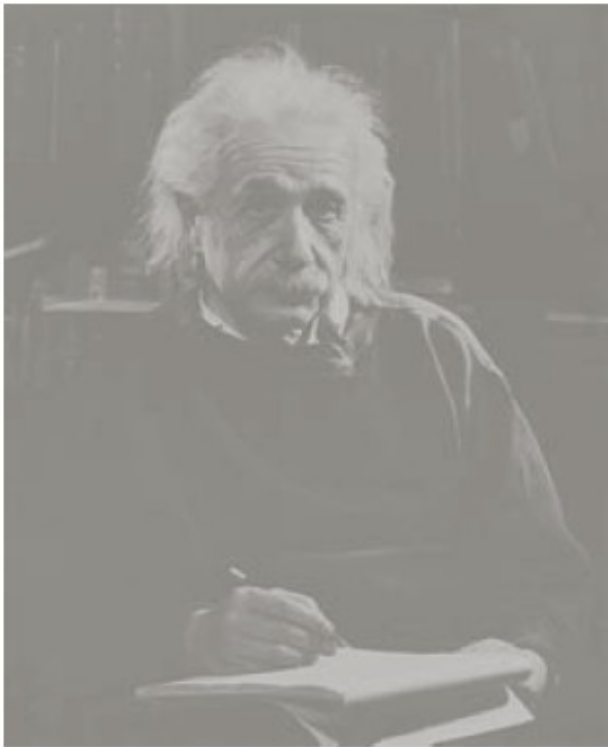
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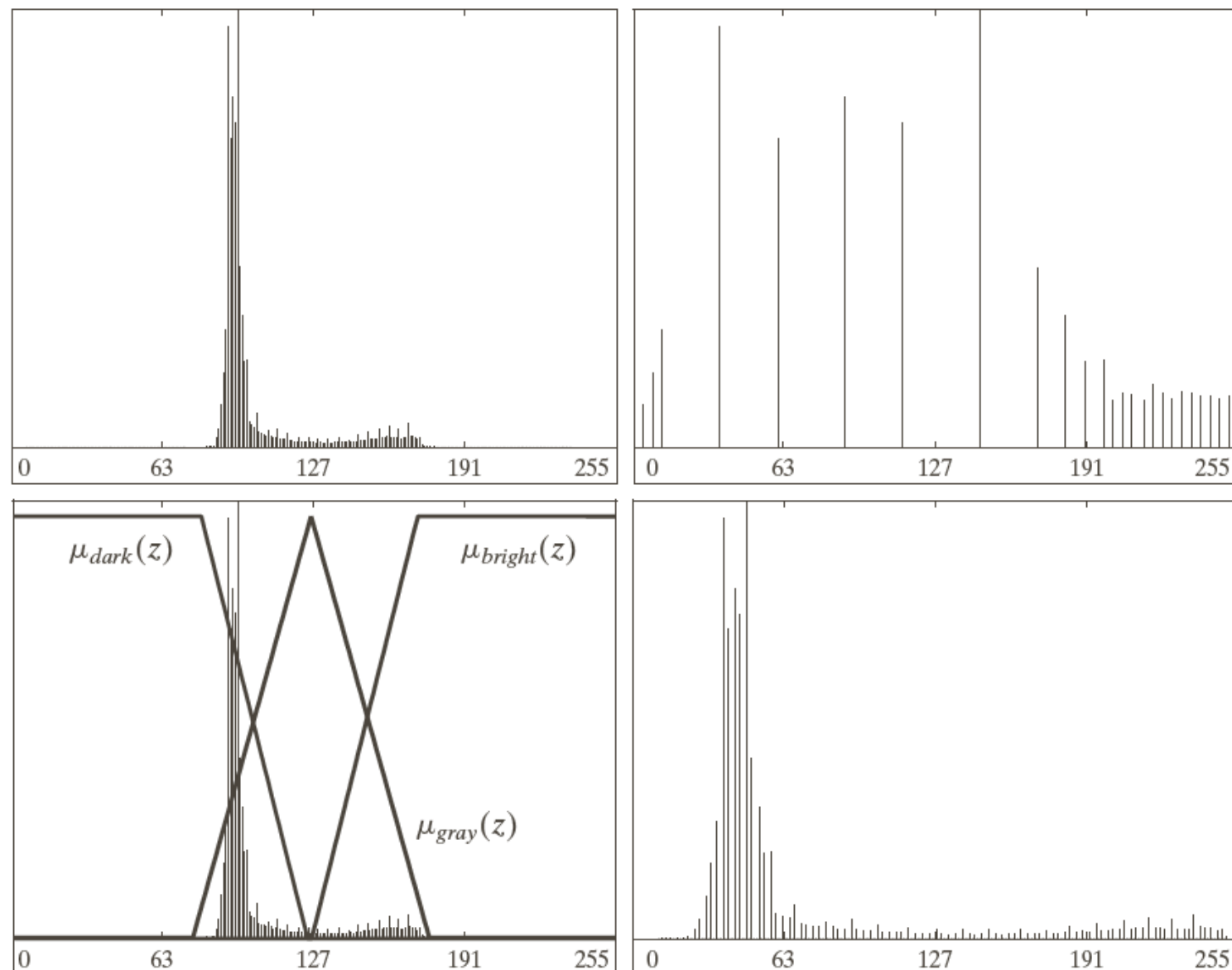
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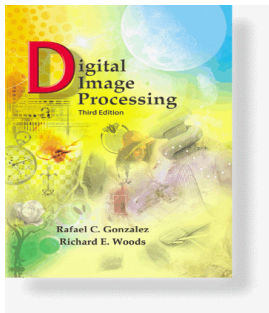
a b c

FIGURE 3.54 (a) Low-contrast image. (b) Result of histogram equalization. (c) Result of using fuzzy, rule-based contrast enhancement.



a b
c d

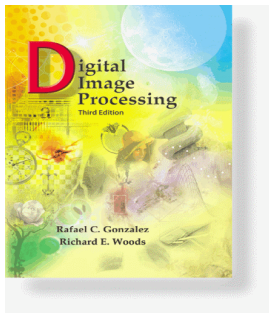
FIGURE 3.55 (a) and (b) Histograms of Figs. 3.54(a) and (b). (c) Input membership functions superimposed on (a). (d) Histogram of Fig. 3.54(c).



Chapter 3

Intensity Transformations & Spatial Filtering

- Histogram Equalization over-exposes
 - Forces extreme darkness
 - Forces extreme whiteness
- Fuzzy Approach
 - Provides moderate shift to dark & white peaks
 - Computationally more expensive
- Histogram Specification can improve speed using the histogram obtained from the fuzzy approach



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Intensity Transformations & Spatial Filtering

- Using Fuzzy Sets for Spatial Filtering:

“If a pixel belongs to a uniform region, then make it *white*; else make it *black* where *black* & *white* are fuzzy sets”

IF d_2 is *zero* AND d_6 is *zero* THEN z_5 is *white*.

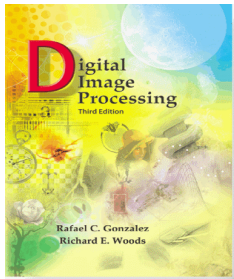
IF d_6 is *zero* AND d_8 is *zero* THEN z_5 is *white*.

IF d_8 is *zero* AND d_4 is *zero* THEN z_5 is *white*.

IF d_4 is *zero* AND d_2 is *zero* THEN z_5 is *white*.

$d_i = z_i - z_5$

ELSE z_5 is *black*.



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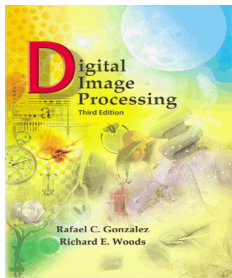
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z_1	z_2	z_3	d_1	d_2	d_3
z_4	z_5	z_6	d_4	0	d_6
z_7	z_8	z_9	d_7	d_8	d_9
Pixel neighborhood			Intensity differences		

a b

FIGURE 3.56 (a) A 3×3 pixel neighborhood, and (b) corresponding intensity differences between the center pixels and its neighbors. Only d_2 , d_4 , d_6 , and d_8 were used in the present application to simplify the discussion.



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a b

FIGURE 3.57

(a) Membership function of the fuzzy set *zero*.
(b) Membership functions of the fuzzy sets *black* and *white*.

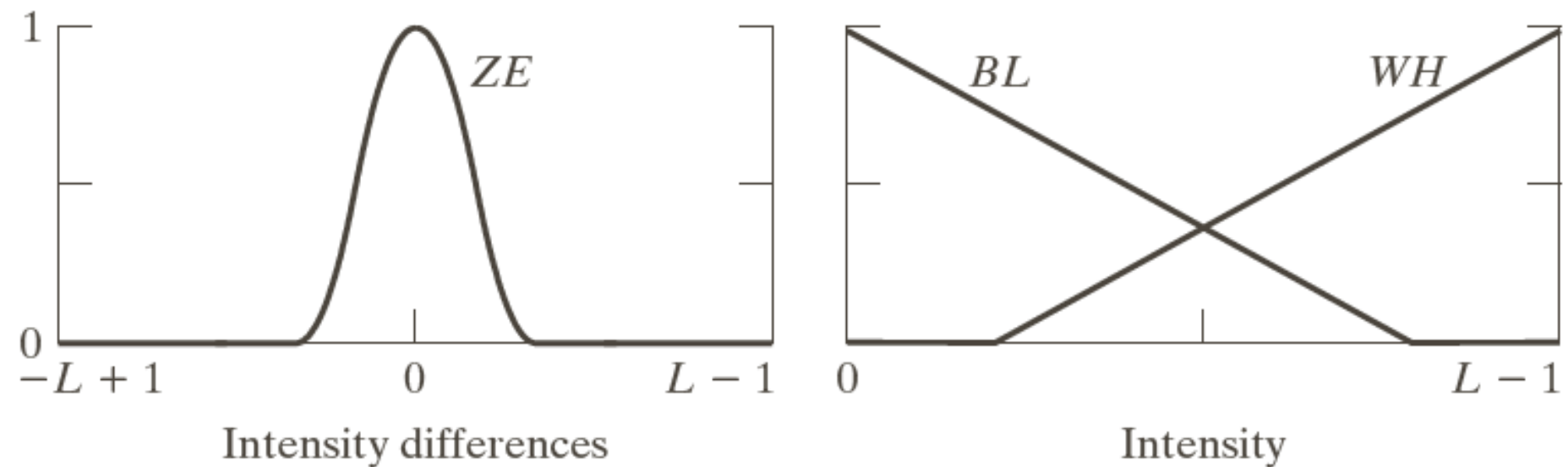


FIGURE 3.58
Fuzzy rules for
boundary
detection.

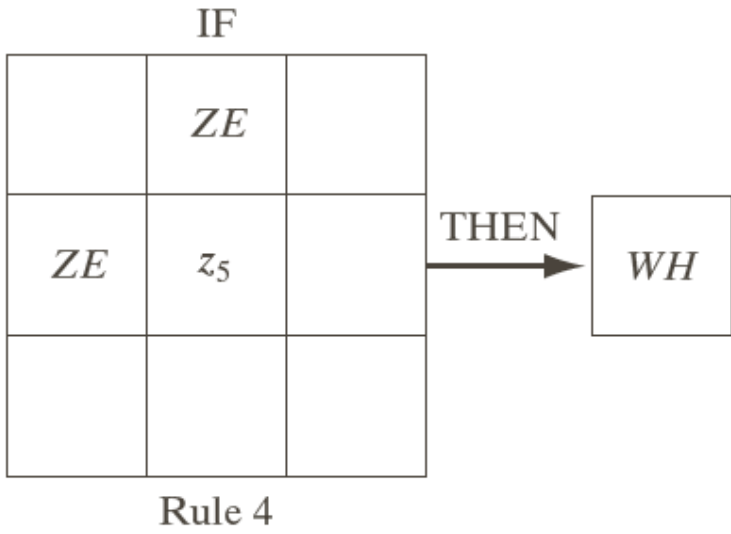
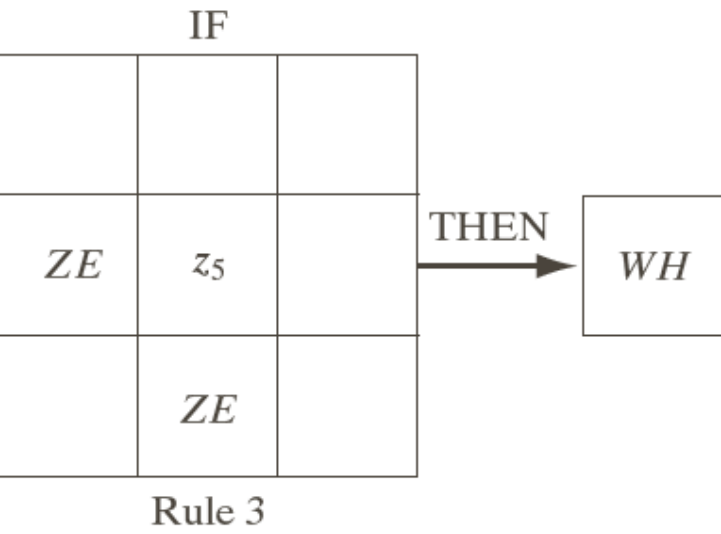
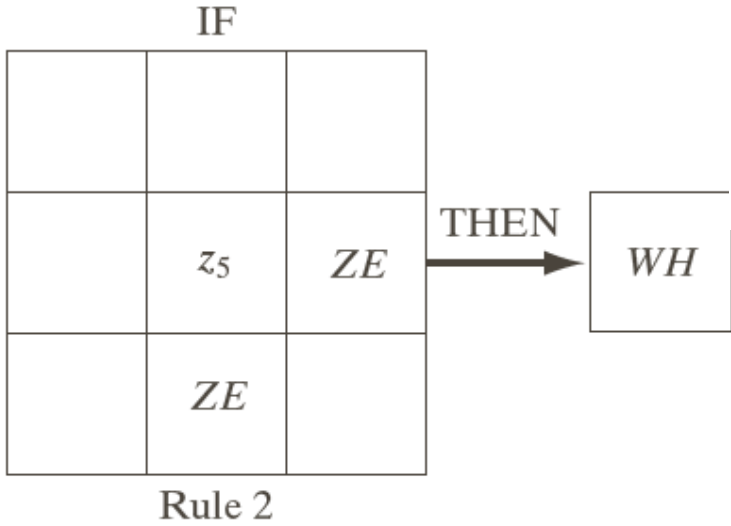
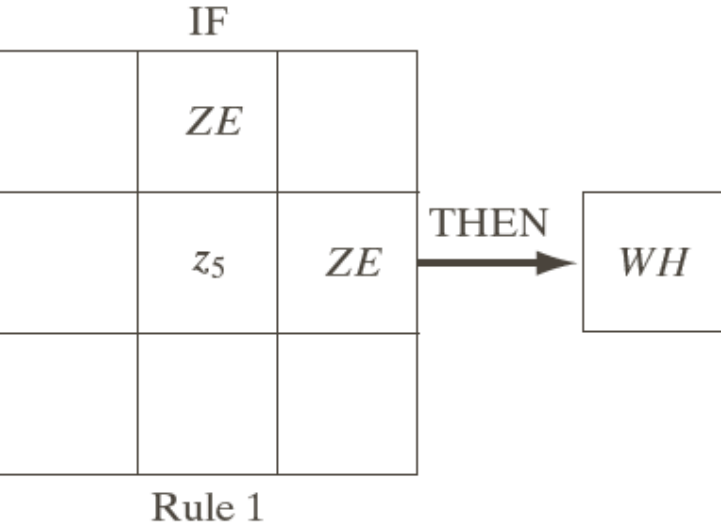




FIGURE 3.59 (a) CT scan of a human head. (b) Result of fuzzy spatial filtering using the membership functions in Fig. 3.57 and the rules in Fig. 3.58. (c) Result after intensity scaling. The thin black picture borders in (b) and (c) were added for clarity; they are not part of the data. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Scaling:

$$f_m = f - \min(f)$$

$$f_s = K[f_m / \max(f_m)]$$